THE MICROWAVE BACKGROUND FROM CAMBRIDGE

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1. INTRODUCTION

Microwave background astronomy has arrived. Largely because of improvements in receivers, the last three years have seen the emergence of agreement concerning important measurements of the microwave background radiation (MBR):

- hard evidence that the MBR has a black-body spectrum (see Smoot in these proceedings);

- the unambiguous detection of the Sunyaev-Zeldovich decrement in clusters of galaxies (Birkinshaw et al 1984), and indeed profiles of the decrement across clusters (Birkinshaw in these proceedings);

- the lack of anisotropy in the MBR temperature T on scales of a few arcmin down to $\Delta T/T \sim 5 \times 10^{-5}$ (Uson & Wilkinson 1984; Readhead, private communication) and the consequent elimination of simple theories of adiabatic primordial fluctuations.

We are now in a position to make MBR telescopes that we can use to tackle two fundamental programmes in astronomy:

1. The mapping of the S-Z decrement in cluster gas over substantial ranges of cluster type and redshift. Such mapping leads directly to a map of cluster pressure; when combined with X-ray maps, one can produce maps of gas temperature and of gas density. We can expect to understand much more about the workings and evolution of clusters.

2. The very sensitive search ($\Delta T/T$ of from 10^{-5} to 10^{-6}) for MBR anisotropies on scales of from a few arcmin to a degree. The Rosetta Stone here of course is the detection of primordial fluctuations associated with the formation of clusters at $z \sim 1000$ and the constraining of particle physics at even earlier times; the fact that the horizon scale at $z \sim 1000$ is $\sim 1°$ makes such studies even more exciting. If it turns out that we can see back only as far as a z of 100 or 10 because of reheating (though complete erasure of primordial

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fluctuations is unlikely on energetic grounds) - well, that would be pretty exciting too.

How can we make telescopes with the required performance? Telescopes used so far for MBR observations have mostly been single-dishes; I shall argue here that interferometers have major advantages for the observations we want. I'll then describe work at Cambridge on MBR-telescopes: the present major enhancing of the 5-km and plans for a new synthetic aperture telescope, the VSA. Finally I'd like to comment on ways of evaluating the results of MBR measurements.

2. MBR MEASUREMENTS WITH SINGLE-DISHES

Single-dishes have large collecting areas and the obvious advantage of high sensitivity - provided the measurements are receiver-noise-limited. (I will argue later that even high sensitivity is not all it seems - for we really want good temperature sensitivity but bad flux sensitivity.) Sensitivity aside, there are several problems with single-dishes, as follows.

a. There are the well-known problems of ground spillover (i.e. the sidelobes pick up thermal radiation from the ground) and of interference from terrestrial sources.

b. The systematic error from spillover is best reduced by making drift scans (with the dish in fixed orientation with respect to the Earth), and subtracting the ground and sky-background contributions by beam-switching, i.e. by subtracting the signals from a pair of feed-horns near the dish focus viewing slightly different parts of the sky. However, performance using this technique is ultimately limited by differential spillover due to differences in the standing wave patterns above the two feed-horns. And minimisation of these differences gets harder the broader the observing band.

c. Single-dishes detect unwanted thermal emission from the troposphere.

d. A single-dish, even without wagging, measures a fluctuation (an RMS temperature) which contains components summed in a complicated way over the beam-switching scale as well as over the scales (0 to ~ 180°) to which the dish itself is sensitive. Relating the RMS measurement to a limit on sky fluctuations as a function of scale is hardly trivial.

e. There is confusion from radiosources in the wide single-dish beam. Of course one can reduce this problem by increasing observing frequency, but amplifiers get harder to make and the troposphere gets brighter.

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3. MBR MEASUREMENTS WITH INTERFEROMETER ARRAYS

The advantages of interferometers over single-dishes in this context seem to me to be the following.

a. The effect of interferometer operation is to place on the sky a phase-rotator operating at the celestial fringe rate f_c for the observing frequency, the baseline and the field position, and to provide phase-sensitive detection such that any output not varying at f_c is filtered out. Thus terrestrial interference and ground spillover, which have a fringe rate different from f_c, will be filtered. Now it is true that as the baseline is reduced to obtain sensitivity to larger-scale structure, f_c falls and gets closer to e.g. the spillover fringe frequency and spillover filtering becomes less efficient. However, in the map-plane such systematics (which account for a substantial amount of "correlator offset") always have a shape centred on the map phase-centre - rather than the pointing-centre. The key to optimising their removal is to arrange that the phase-centre of the map is as far away as possible from the region of interest - the pointing-centre.

b. There is no beam-switching and so no differential spillover. There is the risk of the systematic of dish-dish crosstalk, but again such a correlated signal has the wrong fringe rate and appears on the map well away from the pointing-centre.

c. An interferometer would appear to be less affected by tropospheric emission. A beam-switching single-dish subtracts out the effect of large-scale tropospheric emission but not the patches of emission that are smaller than the dish beam-size. An interferometer with the same field-of-view as the single-dish is instantaneously sensitive only to the effects of small patches that lie inside its instantaneous, fan-shaped, synthetic beam. Over the whole observation, the interferometer is thus sensitive to the effects of fewer small patches than the single-dish in the ratio synthetic beam area/field-of-view and so the signal from the tropospheric emission is smaller for the interferometer than the single-dish in the ratio (synthetic beam area/field-of-view)$^{1/2}$

d. An interferometer array is instantaneously sensitive to a wide and tuneable range of angular scale, and a map results. Much of the uncertainty present in interpreting the single-dish results is removed.

e. With an interferometer array containing some long baselines, we can of course subtract off confusing radiosources. And we can go further, designing arrays that couple well to temperature but less well to the flux of radiosources. It can be shown that, for a reasonably well sampled u,v-plane with Gaussian grading, the relation between temperature sensitivity ΔT and flux-density sensitivity ΔS of a telescope is

$$\Delta T(\mu k) = 1500\ L(m)\ \lambda(m)\ \Delta S(mJy)\ /\ \Phi(arcmin),$$

where L is the longest telescope baseline, λ the wavelength and Φ is any scale $> 1.3\lambda/L$ to which the telescope is instantaneously sensitive. Now we want $\Delta T/\Delta S$ as small as possible: comparing a single dish of diameter L with an array having largest baseline L but inter-dish spacing d, it is apparent that the array has Φ larger by L/d and thus $\Delta T/\Delta S$ smaller by d/L.

f. We can extend this argument to compare the <u>observing efficiencies</u> of single-dishes and arrays. Using the above relation and the standard formula relating ΔS to T_{system}, aperture area, integration time t and bandwidth B, one finds the single dish has $\Delta T \propto T_{sys}/(Bt)^{1/2}$ on the scale λ/L. For the array (which we take as having N dishes and $L \propto N^{1/2}$ d), $\Delta T \propto T_{sys}/(Bt)^{1/2}.(d/D)^{1/2}\ m^{-1}$, on all scales $m\lambda/L$ with m running from 1 to L/d. Thus the array has ΔT worse by a factor $(d/D)^2$ because it is poorly filled compared with the single dish, but better by 1/m due to the range of scales simultaneously sampled. And there is an additional factor because the instantaneous field-of-view of the array is larger than that of the single-dish by $(L/mD)^2$. Then the temperature sensitivity of the array thus conected for range of angular scales and for field-of-view is $\propto T_{sys}/(Bt)^{1/2}.d^2m/L^2$. So the array is more efficient than the single-dish by L^2/d^2m.

4. INTERFEROMETER ARRAYS FOR MBR OBSERVATIONS ON SCALES $\geqslant$ 3 ARCMIN

The key fact about existing arrays is that the VLA is the ideal instrument for MBR observations on scales $\leqslant$ 1 arcmin (appropriate to galaxy-sized masses at $z \sim 1000$) but it is of no use on larger scales: its minimum baseline of 45 m corresponds to a maximum scale of 2 arcmin even at 5 GHz - and for many MBR observations a higher frequency is required.

Here the Cambridge 5-km telescope has a tremendous advantage - small dishes. The minimum baseline is 18 m and angular scales of several arcmin are not resolved out. At present the 5-km has wholly inadequate sensitivity but we are engaged on a major programme of enhancement consisting of

- fitting cooled FET amplifiers for 5 and 15 GHz
- correlating all baselines
- increasing observing bandwidth from 10 to 350 MHz.

The large bandwidth produces problems of chromatic aberration and potential problems of terrestrial interference, so we are dividing the 350 MHz into channels 10 MHz wide, partly with filter-mixers and partly by cross-correlating with multiple delays and Fourier-transforming these products to synthesise the 10-MHz-wide channels. The graph (Fig. 1) shows the sensitivities of the VLA and the enhanced 5-km in a month; both reach $\Delta T/T$ of about 2×10^{-5} but the 5-km most importantly extends this sensitivity to larger scales.

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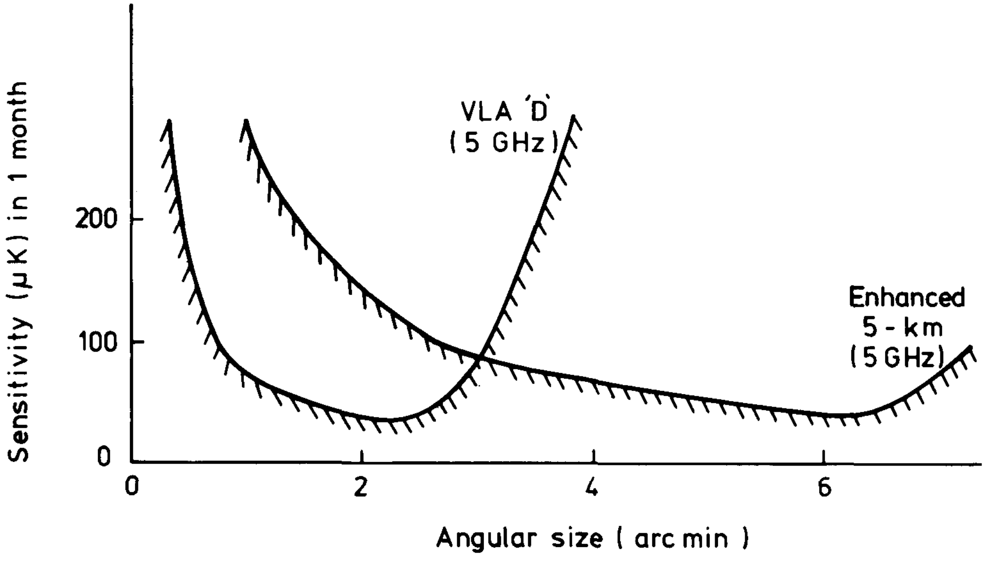

Fig. 1. Comparison of the sensitivities of the VLA and enhanced 5-km as a function of angular scale. (Observing time = 1 month.)

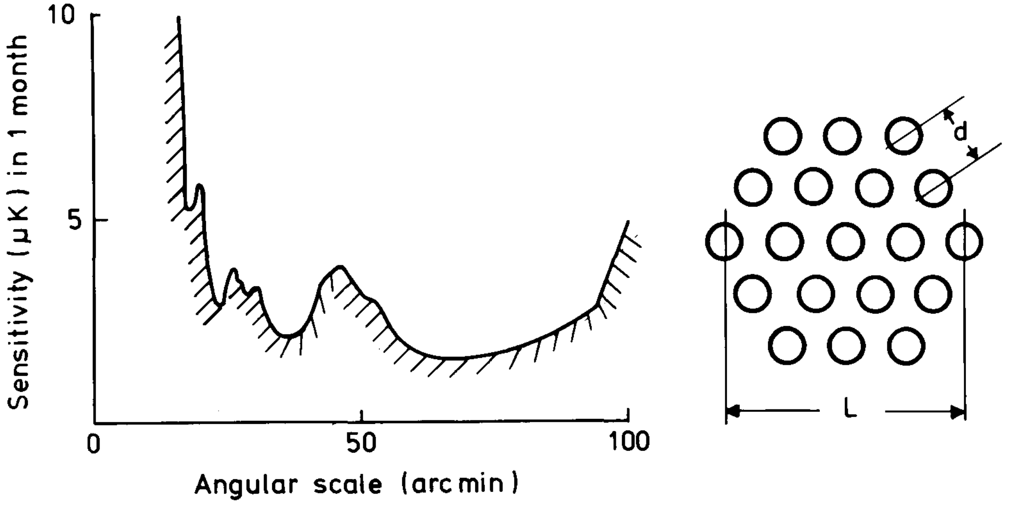

Fig. 2. Projected sensitivity of the VSA (1 month observing time) as a function of angular scale for the array configuration shown on the right. (d = twice horn diameter = 1.4m)

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The proposed VSA (Very Small Array) extends the angular scale further - up to ~ 1°. We plan a hexagonal array of some 20 feed-horns each with a collecting area of 0.5 m^2, with largest baseline ~ 10 m. Of course the horns will be moveable into A, B, C and D arrays. Operation at 15 GHz, bandwidth 500 MHz (capitalising an experience with the enhanced 5-km corelator), 1989 start, cost £1.5 M. Sensitivity - tremendous: $\Delta T/T$ of 10^{-6} in a month. (See Fig.2.)

5. ESTIMATION OF UPPER LIMITS GIVEN NON-DETECTIONS

We are searching for primordial fluctuations. We have made our observations on a particular angular scale in a particular direction on the sky and found a mean brightness temperature μ with measurement uncertainty σ. We find $\mu \ll \sigma$ - we haven't found sky fluctuations. But we're concerned with evaluating the importance of our observations and setting a limit on the level Δ of true sky fluctuations on that scale in that direction. How should we do it? The case where there is only one such μ available raises some interesting questions of principle and corresponds to the situation actually faced in measurements such as those currently being undertaken by Readhead et al. at Owen's Valley (p.c.). The extension to several μ (the more common situation) is straightforward once the principles in the single μ case have been agreed upon.

We know the data we have is a sample from a normal distribution with mean zero and variance $\Delta^2+\sigma^2$. Working in μ^2 and Δ^2 to avoid problems of sign of sky fluctuation, we can write the likelihood function for Δ^2 as

$$L(\Delta^2) = P(\mu^2|\Delta^2) = \frac{1}{\mu} \frac{1}{[2\pi(\sigma^2+\Delta^2)]^{\frac{1}{2}}} \exp\left\{\frac{-\mu^2}{2(\sigma^2+\Delta^2)}\right\}$$

where the notation $P(\mu^2|\Delta^2)$ means the probability of μ^2 given Δ^2. This likelihood <u>function</u> is <u>the</u> result of the experiment (it employs only experimental data and the assumption of Gaussian statistics). $L(\Delta^2)$ is constant, independent of Δ^2 while $\Delta^2 \ll \sigma^2$, and begins to fall as Δ^2 approaches σ^2. We should be able to agree on a couple of numbers to encapsulate the result of the observations, ie to parameterize the likelihood function. We could choose the values of Δ^2 for which $L(\Delta^2)$ has fallen to 0.5 and to 0.05 of its low Δ^2 value. These are good measures of the efficacy of the experiment and their use is a good way of comparing different attempts at setting limits on fluctuations on a given scale.

Measurement of efficacy aside, however, what we really want to know is $P(\Delta^2|\mu^2)$, the probability of Δ^2 given the data. We can get this directly from Bayes' Theorem

$$P(\Delta^2|\mu^2) = \frac{P(\mu^2|\Delta^2)\ P(\Delta^2)}{\int P(\mu^2|\Delta^2)\ P(\Delta^2)\ d\Delta^2}$$

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in which (a) we must provide the prior $P(\Delta^2)$ and (b) we may, depending on $P(\Delta^2)$, have to provide limits on Δ^2 in order that the integral in the denominator be convergent. I emphasize that Bayes' theorem is no less and no more secure than Pythagoras'; it is disputes over the choice of $P(\Delta^2)$ and the limits that increase the risk of heart disease.

The form of $P(\Delta^2)$ that represents complete uncertainty is $P(\Delta^2) \propto 1/\Delta^2$, ie P is uniform in log Δ^2 and we impose no scale on Δ^2. If we use this form in Bayes' theorem we'll need limits on Δ^2 - we might put Δ_{max} = 1K and Δ_{min} = 0.1 μk (since we have strong theoretical grounds for expecting some real sky fluctuation, not necessarily primordial, with a value above this). If we compute $P(\Delta^2|\mu^2)$ we can determine, e.g. the confidence upper limit for Δ^2. That limit is a very weak function of Δ_{min}, though it is true that the lower we set Δ_{min}, the lower is the upper limit for Δ^2. Yet this merely reflects present reality: what we expect Δ^2 to be is constrained much more by our theoretical ideas (expressed as prior knowledge) than current direct observations which, after all, don't see anything. This line of reasoning suggests a more appropriate form for $P(\Delta^2)$ might be a function cutting off at more gently at smaller Δ^2 than a step function at Δ_{min}, in agreement with the 'fuzzy' nature of our ideas about how small Δ^2 might really be. The problem, though in essence trivial, is in fact at the boundaries of our understanding of statistical method and provides a fertile area for exposing differences and problems within the alternative approaches to statistics, (Bayesian, frequentist and fiducial). Work in progress at Cambridge by Gull, Kaiser and Lasenby, on which the above discussion is based, promises to throw interesting light on this.

6. GETTING TO GRIPS WITH DETECTIONS

As systematic errors are overcome and sensitivities improve, the chances of detecting fluctuations at least from $z > 10$ will increase. Say we get an apparently significant detection - what are we to do? On the one hand we must certainly not just assume we're still playing the game of upper limits, subtract it (unless we have very good morphological or spectral information that it is a radiosource) and re-evaluate μ^2 and σ^2. On the other, we must check rigorously for systematics by mapping outside the primary beam, by observing at different frequencies and with different feeds.

I suspect that when it comes to detecting primordial fluctuations, the real challenge will be to know when we have arrived.

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