SPECIAL ORTHOGONAL LATIN SQUARES OF ORDER 10

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The orthogonal latin squares displayed in [1] and [2] have the property that their row permutations are transformed amongst themselves by a permutation of order 7. In this note I present three examples of orthogonal latin squares of order 10 whose row permutations are transformed amongst themselves by a permutation of order 9.

We suppose the rows labelled 0 to 9 from top to bottom and the columns labelled 0 to 9 from left to right, and that the entries in each row of the latin squares under consideration are the integers 0, 1, ..., 9. Each row is a permutation of these symbols. If R_i and R_i' are the ith row permutations of two orthogonal latin squares of order 10, we require that

$$R_i = P^{-i} R_o P^i, \quad R_i' = P^{-i} R_o' P^i \quad (i = 0, 1, \ldots, 8),$$

where $P = (012345678)$, while R_9 and R_9' are powers of P. The conditions are satisfied by the row permutations of the three pairs of orthogonal latin squares shown below. Thus, in Fig. 1, $R_o = (125387946)$, $R_o' = (18)(2965)(347)$, $R_9 = P^6$, $R_9' = P^4$.

These figures have other special features. The squares in Fig. 1 are transposes (in the matric sense) of one another. One of the squares in Fig. 2 is symmetric, and the columns of one square form a permutation of the columns of the other.

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Fig. 3 may be derived from Fig. 1 by the following rule: If x, y is the entry in the *i*th row and *j*th column of Fig. 1, then i, x is the entry in the *j*th row and *y*th column of Fig. 3. While the two figures are isomorphic, it is noteworthy that Fig. 3 has the involutory property: If x, y is the entry in the *i*th row and *j*th column, then i, j is the entry in the *x*th row and *y*th column.

00	28	59	84	67	32	15	93	71	46
82	11	30	69	05	78	43	26	94	57
95	03	22	41	79	16	80	54	37	68
48	96	14	33	52	89	27	01	65	70
76	50	97	25	44	63	09	38	12	81
23	87	61	98	36	55	74	19	40	02
51	34	08	72	90	47	66	85	29	13
39	62	45	10	83	91	58	77	06	24
17	49	73	56	21	04	92	60	88	35
64	75	86	07	18	20	31	42	53	99

Fig. 1.

96	64	41	13	38	87	72	25	59	00
69	97	75	52	24	40	08	83	36	11
47	79	98	86	63	35	51	10	04	22
15	58	89	90	07	74	46	62	21	33
32	26	60	09	91	18	85	57	73	44
84	43	37	71	19	92	20	06	68	55
70	05	54	48	82	29	93	31	17	66
28	81	16	65	50	03	39	94	42	77
53	30	02	27	76	61	14	49	95	88
01	12	23	34	45	56	67	78	80	99

Fig. 2.

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00	65	18	52	96	29	47	81	34	73
45	11	76	20	63	97	39	58	02	84
13	56	22	87	31	74	98	49	60	05
71	24	67	33	08	42	85	90	59	16
69	82	35	78	44	10	53	06	91	27
92	79	03	46	80	55	21	64	17	38
28	93	89	14	57	01	66	32	75	40
86	30	94	09	25	68	12	77	43	51
54	07	41	95	19	36	70	23	88	62
37	48	50	61	72	83	04	15	26	99

Fig. 3.

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