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ON A PLAUSIBLE PHYSICAL MECHANISM LINKING THE MAUNDER MINIMUM TO THE LITTLE ICE AGE

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ABSTRACT. To understand better the Earth's climate, we need to know precisely how much radiation the Sun generates. We present here a simple physical mechanism describing the convective processes at the time of low sunspot activity. According to this model, the kinetic energy increased during the Maunder Minimum, causing a decrease of the solar radiation that was sufficient to produce a little Ice Age.

INTRODUCTION

Much effort has been made to find the signature of solar activity in the Earth's atmospheric records (see a summary of recent work by Labitske & Van Loon 1990). Evidence for a statistically significant 11-year periodicity in various atmospheric indices is increasing. However, the time sequence (a few 11-year solar cycles) is usually too short, so that some aliasing effect may result (Teitelbaum & Bauer 1990). Further, we are lacking a physical interpretation of the link between the 11-year solar cycle and the properties of the Earth's atmosphere.

Maunder (1894) reported some relation between long series of solar cycles and climatic events. In particular, a severe cold period was coincident with decreased sunspot activity during the second half of the 17th century. Although the coincidence is intriguing, the relation remains to be established.

In this paper, we offer a plausible mechanism linking the Maunder Minimum to the coincident mini-Ice Age. Our understanding of the Maunder Minimum is based on observations of the present solar cycle as well as on historical data. We shall first summarize our present knowledge about the 11-year solar cycle.

THE 11-YEAR SOLAR CYCLE: PRESENT KNOWLEDGE

During the course of the 11-year solar cycle, the Sun's magnetic field oscillates between a poloidal component, which is dominant at sunspot minimum, and toroidal components, a manifestation known as the "butterfly diagram" of the active regions.

For the α-ω dynamo to hold, two ingredients are required: 1) non-uniform rotation that produces the toroidal field from the poloidal field; 2) some convective motions that regenerate the poloidal field through the "α" effect (Roberts & Stewartson 1974).

Several solar phenomena are related to the solar magnetic cycle:

1. Large-scale circulation in the form of "azimuthal rolls" has been discovered (Ribes & Mein 1984; Ribes & Bonnefond 1990). These giant rolls are cycle-dependent, and are at their maximum, three per hemisphere, in the ascending phase of the cycle. Their upper and lower borders run along 60° and 35°, respectively. After the sunspot maximum, there are two rolls in each hemisphere, with edges at 15°–20°. No rolls are visible any longer at sunspot minimum. Periods of strong solar activity favor the two-roll configuration (Ribes & Bonnefond 1990). One interesting property of the rolls is that they modulate the surface rotation (Ribes, Mein & Mangeney 1985).

2. Helioseismological studies have given some insight into the internal rotation of the Sun (Brown & Morrow 1987). The convective zone (from 0.7 $R_\odot$ up to the surface) exhibits the same latitudinal rotation throughout, whereas the radiative zone (below 0.6 $R_\odot$) rotates like a solid body.
3. The apparent solar diameter varies during the solar cycle, reaching minimum at the time of the sunspot maximum (Laclare 1987). This has been confirmed independently from the frequency shift of acoustic oscillations through the solar cycle (see Fossat *et al.* (1987) for a summary of observations).
4. The total irradiance monitored by ACRIM (on board the Solar Maximum Mission Satellite) varies, and is maximum when the sunspot number is maximum (Willson & Hudson 1991). The changes in the apparent solar diameter and total irradiance observed during cycle 21 (1975–1986) are 2.10^{-4} and 10^{-3}, respectively.

One astrophysical question of interest is the relation between these various observations.

In the framework of the (α-ω) dynamo, the toroidal field is produced from the poloidal field by a shear in the angular velocity. Such a shear exists at the interface between the radiative and the convective zones (Brown & Morrow 1987). Further, a positive shear of the angular velocity is confined to the surface latitude belt of ± 30°, thus being responsible for the distribution of active regions through the solar cycle, "the butterfly diagram" (Ribes 1990a; Belvedere, Proctor & Lanzafame 1991). The amplitude of the surface differential rotation, which varies from one 11-yr cycle to the other, might reflect the strength of the sunspot cycles. Then, the azimuthal rolls could be the regulator of the internal positive shear and, thus, of the surface rotation.

It is likely that large-scale circulation in the convective zone contributes to the energy transport from below, leading to a structural change in the solar envelope (diameter and luminosity). On the one hand, though sunspots and bright faculae may easily explain the short-term fluctuations present in the total solar irradiance, there must be some phenomenon to account for the 11-yr trend (Willson & Hudson 1991). On the other hand, the good anticorrelation between apparent diameter and solar constant data suggests that the azimuthal roll pattern might be the cause of the trend present in both apparent diameter and solar constant data (Ribes & Laclare 1988). The modulation of the surface rotation can be used as a tracer of the thermal response caused by periodic emergence of magnetic fields. In this context, the Maunder Minimum, characterized by a lack of sunspots, gives a first-order constraint to the models.

THE MAUNDER MINIMUM: OBSERVATIONAL DATA

The Maunder Minimum is a well-documented period. A unique, 50-year record of solar observations has been made at the Observatoire de Paris. Philippe La Hire (1718) observed and recorded the solar diameter over the period, 1683 to 1718, using the same instrument, thus preserving the homogeneity of the data set. His observations have been analyzed (Ribes, Ribes & Barthalot 1987; Ribes *et al.* 1990; Ribes 1990b), and several facts have been noted:

1. The apparent solar diameter expanded, mainly when there were few sunspots, *i.e.*, between 1683 to 1700, and then slowly came back to normal after 1705. We know that 17th-century optics were poor, and systematically enlarged the apparent diameter. However, Ribes *et al.* (1990) estimated a 0.2% increase of the apparent radius from La Hire's 36 years of homogeneous observations, and calibrated these observations with the solar diameter as derived from the solar eclipse timed by Halley, in 1715. Moreover, the observed apparent diameter exhibits a strong 11-yr modulation, despite the little sunspot activity. Some variability, of magnetic origin, is also present in the abundance of cosmo-

genic isotopes, such as ^{14}C (Kocharov 1986) and ^{10}Be (J. Beer, personal communication). The similarity in the variability of the apparent solar diameter and the cosmogenic isotope abundance indicates that the solar envelope responds to the cyclic magnetic fields. If one accepts that the observed changes of the apparent diameter are "*in situ*," the Sun's luminosity might have changed as well.

2. About 100 sunspots were observed during the second half of the Maunder Minimum, and these were confined within the latitude range of ± 18°. The rate of the surface rotation has been obtained by measuring the displacement of these sunspots in their transit across the solar disk. Ribes, Ribes and Barthalot (1987) found that the surface rotation was much more differential than it is now: the equatorial rate was 2% smaller, and the rate at 18° latitude was 6% smaller than at present. In other words, the 18° latitude belt had the same rotation rate as the present 30° latitude belt.

CONVECTIVE PATTERN DURING THE MAUNDER MINIMUM

The model we develop here consists of estimating the contribution of the large-scale convection to the solar luminosity. We make two assumptions:

1. The large-scale rolls inferred from the meridional circulation have a depth comparable to their width, which is the depth of the convective zone (Ribes & Laclarė 1988).
2. We assume that the large-scale convective pattern is the main cause of the modulation of the surface sunspot rotation through the Coriolis forces. In particular, we neglect the Lorentz forces in the dynamic equations; this assumption is supported by the fact that the observed large-scale magnetic field is weak (Hoeksema 1991).

We shall make some comments related to these assumptions. The physical conditions for exciting large-scale modes of convection, the rolls, can be achieved in the convective zone, as suggested by the presence of rolls. The critical Rayleigh number from which the large-scale convective mode develops is reached under the action of the magnetic field on the turbulent (small-scale) plasma.

The Rayleigh number, which measures the importance of the buoyancy force relative to the stabilizing effects of diffusion, is proportional to the temperature gradient and to the depth of the convective layers, and is inversely proportional to the thermal diffusivity, χ, and the kinematic viscosity, ν. In the presence of magnetic field, one usually believes that convection is inhibited, reducing χ and ν. If large-scale convective rolls are thus decoupled from small-scale turbulent plasma, one can imagine a way to reach the critical Rayleigh number for large-scale convection, denoted hereafter as $Ra_{c,r}$: the small-scale (and stronger field) decreases χ and ν, thus increasing $Ra_{c,r}$; the larger the magnetic field, the larger the number of rolls. This explains, qualitatively at least, the change of the roll pattern from no roll (or a single roll per hemisphere) at sunspot minimum up to a three-roll pattern when the sunspot activity reaches its maximum.

We shall examine whether this model is able to estimate the change of solar luminosity through the 11-yr cycle and for longer cycles, of the Maunder Minimum type. In the present paper, we apply our model to the Maunder Minimum, leaving the 11-yr cycle for a separate study.

In principle, the large-scale convection pattern can be derived from the meridional motions of sunspots. The magnitude of the flow is of the order of 10 to 15 m s^{-1}, which was not easily measurable with 17th century sunspot observations. However, the surface rotation profile was strongly modulated in the second half of the 17th century, suggesting that giant convective motions did exist.

A simple equation in which the Coriolis force is balanced by the turbulent diffusivity, in an incompressible fluid, can be obtained from the azimuthal component of the momentum conservation

$$\frac{\partial}{\partial t}\left\{\frac{\delta\Omega}{\Omega_0}\right\} + 2\left\{\frac{w}{r} + \frac{v\cos(\theta)}{r\sin(\theta)}\right\} = \frac{1}{t_R}\frac{\delta\Omega}{\Omega_0} \tag{1}$$

where Ω_0 is the equatorial rotation without meridional circulation, $\delta\Omega$ the deviation from this rotation rate, w and v the vertical and horizontal components of the meridional circulation, r and θ the radial coordinate and the colatitude, and t_R the viscous damping time, $\sim l^2/\nu$, $l \sim 0{\cdot}3R_\odot$ being a typical scale of the circulation pattern, and $\nu \sim 10^{12}$–10^{13} cm^{2}s^{-1} the turbulent viscosity; with these typical parameters, t_R is a few years. A number of effects have been neglected in this equation, for example, the Lorentz force due to the magnetic field, an assumption which would not really be justified if the magnetic field were larger than a few thousands gauss. However, because we aim only to illustrate the basic consequences of a meridional circulation, we shall neglect all effects besides the Coriolis force and the viscosity. The best justification for doing this is that the order of magnitude for the differential rotation one expects from this balance

$$\frac{\delta\Omega}{\Omega_0} \sim \frac{t_R}{R_\odot} V_{circ}$$

is comparable to what is observed, if any credit is given to the standard estimates of the turbulent viscosity.

To keep with the level of sophistication of Equation (1), we do not study a fully self-consistent problem, but, rather, we consider a steady state with a prescribed meridional circulation in a spherical shell of inner and outer radii, R_1 and R_2, $l = R_2 - R_1$.

We introduce a velocity potential $\Psi(r,\theta)$ such that

$$\frac{1}{\sin(\theta)}\frac{\partial}{\partial\theta}\sin(\theta)\frac{\partial}{\partial\theta}\Psi = -\frac{\partial r^2 w}{\partial r} \tag{2}$$

$$v = \frac{1}{r}\frac{\partial\Psi}{\partial\theta}\ . \tag{3}$$

In the particular model we consider here

1. $\Psi(r,\theta) = H(r)F(\theta)$, with H(r) having the simplest form compatible with the conditions, $w(R_1) = w(R_2) = 0$

$$H(r) = -\frac{V_c}{R_2 l}\left\{4r^2 - 3r(R_1 + R_2) + 2R_1R_2\right\} \tag{4}$$

(V_c being a typical meridional velocity).

2. $F(\theta)$ is a superposition of a small number of even Legendre polynomials of $x = \cos(\theta)$, so that the meridional circulation is symmetric about the equator

$$F(\theta) = \text{constant} \pm (P_2(x) + \lambda P_4(x) + \mu P_6(x)) \ . \tag{5}$$

The value of the constant is irrelevant.

This form of the velocity potential produces a pattern of symmetric meridional cells, with the number of cells depending on the value of the parameters, λ and μ. Figure 1 shows an example of such a pattern. In the steady state, the differential rotation pattern at the surface ($r = R_2$) obtained from equation (1) is

$$\frac{\delta\Omega}{\Omega_0} = \frac{t_R V_c}{R_2}\{f(\lambda,\nu,\theta)\} \tag{6}$$

in which f is a superposition of even Legendre polynomials, with coefficients depending on λ and μ.

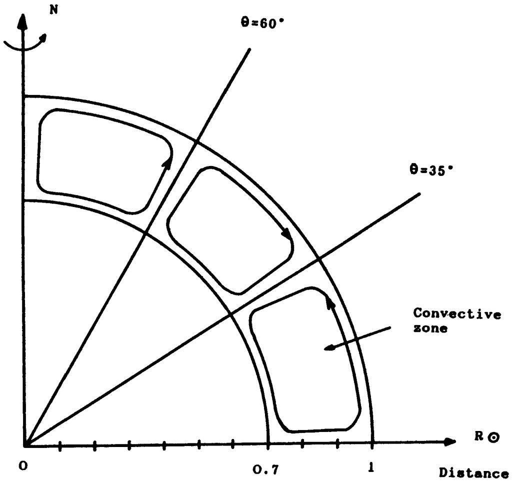

Fig. 1. $\mu = 1.4$; $\lambda = 1.2$; Three-cell meridional circulation corresponding to the ascending phase of the solar cycle 21 (1976–1986)

On the other hand, observations of the differential rotation can be fitted with a combination of Legendre polynomials providing Ω, V_c, λ and μ, parameters that characterize the meridional flow. For the Maunder Minimum, the best polynomial fit to the observations gives one roll per hemisphere, with

$$\Omega_0 \sim 13.98\text{deg/day}, \tag{7}$$

and

$$V_c \sim 0{\cdot}3\frac{R_\odot}{t_R} \ . \tag{8}$$

Ω_0 is of the order of the rotation rate of the radiative interior. For comparison, the same fit for 1984, near sunspot minimum, also gives one roll per hemisphere, with

$$\Omega_0 \sim 14{\cdot}4\text{deg/day}, \tag{9}$$

and

$$V_c \sim 0.7\frac{R_\odot}{t_R} \quad . \tag{10}$$

In 1979, near sunspot maximum, there are three rolls per hemisphere, with

$$\Omega_0 \sim 13{\cdot}45\text{deg/day}, \tag{11}$$

and

$$V_c \sim 0{\cdot}07\frac{R_\odot}{t_R} \quad . \tag{12}$$

If the viscous damping time, τ_R, is a few years, the convective velocities deduced from the solar rotation modulation were ranging from 10 ms^{-1} for the Maunder Minimum and the year, 1984, to 1 ms^{-1} for the sunspot maximum of 1979.

How do we relate these variations in V_c to luminosity variations?

One possible line of argument is:

At the base of the convective zone, the flux is constant over the time scales under consideration. On the other hand, the partitioning between large and small scale of motions in the convective zone may change with time. In particular, the energy flux going into large-scale motions can be estimated as:

$$E = M_c\, V_c^2 \,/\, \tau_{conv} \,,$$

where M_c is the mass of the convective zone (10^{31} g). If the observed meridional flow traces deep convective rolls, one can estimate τ_{conv}, the turnover convective time of the roll (ratio between the depth of the convective zone (2.10^8 m) and the meridional velocity (~10m s^{-1})), to be of the order of one year. Thus, the kinetic energy excess is 10^{30} ergs s^{-1}, *i.e.,* 0.1 times the observed change of solar irradiance, suggesting that the efficiency of the thermal driving of the large-scale convection is of the order of 0.1. Our mechanism explains the inverse correlation between V_c and the δ L, as observed.

To obtain more accurate estimates of the luminosity variations, solar envelope models are needed. This will be done in a separate study.

CONCLUSION

We have shown that a single-roll pattern is capable of reducing the surface rotation in a way shown by the 17th-century observations of the surface rotation. Such a pattern persisted over a period of several decades, at the time of low sunspot activity. The convective velocity associated with the roll is almost an order of magnitude larger than the velocity of the rolls in 1979, at sunspot maximum. We can speculate that the increase of the energy stored in the large-scale convection results in a decrease of solar luminosity, if a constant energy flux is assumed at the base of the convective zone. For the present 11-yr cycle, the solar irradiance changes by one part per thousand between solar maximum and solar minimum. A similar decrease in solar irradiance can be deduced during periods of low sunspot activity, such as the Maunder Minimum. The decrease of luminosity might have been larger: some time intervals of almost 10 years were characterized by a complete lack of sunspots. So, one conjectures that the convective instability threshold for large-scale rolls had not been reached, causing a more drastic luminosity decrease. The present study does not model this. However, the conservative estimate of the solar irradiance decrease of at least 0.3 Watt m^{-2} at the top of the Earth's atmosphere and persisting over several decades, could account for some climatic changes.

The solar luminosity decrease inferred from the large-scale circulation in the convective zone is corroborated by the reported expansion of the solar diameter during the Maunder Minimum (Ribes *et al.* 1990). No satisfactory mechanism explains the observed anticorrelation between apparent diameter and solar irradiance (see Spruit 1991 for a review of the mechanisms). The expansion of the apparent solar diameter was strongly modulated with an 11-yr periodicity, at least during the period spanned by the Paris observers. The expansion was up to two arcseconds every ten years, and fell to a much smaller value five years later. The modulation exhibits the same characteristics as the present modulation of the apparent diameter, except that the amplitude was ten times larger. We have no measure of the solar output at that time. However, if the ratio between solar irradiance and apparent radius (a ratio of five for the previous 11-yr solar cycle) were the same during the Maunder Minimum, it would correspond to a maximum reduction of 0.1 to 1.10^{-2} of the solar output, at a rhythm of ten years. This would result in a mean variation of the solar constant of 0.5% over several decades. Such a solar radiation forcing corresponding to a decrease of one Watt m^{-2} at the top of the Earth's atmosphere is sufficient to produce a little Ice Age (H. Le Treut, personal communication).

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