a *differential polynomial equation* defines a closed set in the Kolchin topology, the differential analogue of the Zariski topology. In particular, we focus on differentially closed fields (close analogues of algebraically closed fields that enjoy many good model-theoretic properties, including quantifier elimination).

Our question, first looked at in essentially this form by E. R. Kolchin (1974) and later by W. Y. Pong (2000), is this: If V is a projective δ-variety over a differentially closed field K of characteristic zero and W is an δ-variety over K, then does the projection map $\pi : V \times W \to W$ take Kolchin-closed sets to Kolchin-closed sets?

The answer is "not necessarily," but Pong showed one reason for the counterexamples: δ-completeness requires a variety to have *finite rank* (in one of several equivalent senses). In this thesis, we take the basic model-theoretic and algebraic setup used by Pong and further develop it into several strategies for attacking the δ-completeness problem. Our work includes new examples of complete δ-varieties. Importantly, we also give the first example of an incomplete finite-rank projective δ-variety; hence, we now know that this class is not empty.

Our methods are as follows:

1. Modify Pong's "valuative criterion" for δ-completeness and produce alternative versions in terms of the Kolchin closure of the image of the projection as well as differential elimination ideals. We use these results to give multiple explicit elimination algorithms proving completeness of several new varieties.
2. Reduce from the differential setting to the algebraic by showing how the modified valuative criteria transfer the problem to a sequence of complex algebraic varieties. This enables one to use tools from analysis or standard (nondifferential) commutative algebra on the δ-completeness problem.
3. More speculatively, we isolate two conjectural properties (interesting in their own right as questions of algebraic geometry) of these complex varieties, both depending on the notion of generically perturbing the coefficients of the associated systems of equations. We explain how asymptotic properties of the complex varieties might imply δ-completeness of the original variety, given the above conjectures.

The thesis concludes with an evaluation of these methods and their prospects for classifying complete δ-varieties. We also discuss applications of δ-completeness. The appendices contain a new case of the differential algebraic phenomenon of nontrivial projective δ-varieties contained entirely in a single affine chart, as well as algorithmic elimination proofs of δ-completeness.

[1] E. R. Kolchin, *Differential equations in a projective space and linear dependence over a projective variety*, ***Contributions to Analysis*: *A Collection of Papers Dedicated to Lipman Bers*** (L. V. Ahlfors, I. Kra, B. Maskit, and L. Nirenberg, editors), Academic Press, New York, 1974, 195–214.

[2] W. Y. Pong, *Complete sets in differentially closed fields*. ***Journal of Algebra***, vol. 224 (2000), pp. 454–466.

(Adapted from the original thesis abstract by the author).

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Filippo Cavallari, ***Regular Tree Languages in the First Two Levels of the Borel Hierarchy***, University of Turin, Italy, and University of Lausanne, Switzerland, 2018. Supervised by Luca Motto Ros (Turin) and Jacques Duparc (Lausanne). MSC: 03D05, 03E15, 03E75. Keywords: Descriptive Set Theory, Automata Theory, regular languages, Wadge hierarchy, Rabin-Mostowski index.

Abstract

The thesis focuses on a quite recent research field lying in between Descriptive Set Theory and Automata Theory (for infinite objects). In both areas, one is often concerned with subsets of the Cantor space or of its homeomorphic copies. In Descriptive Set Theory, such subsets are usually stratified in topological hierarchies, like the Borel hierarchy, the Wadge hierarchy and the difference hierarchy; in Automata Theory, such sets are studied in terms of regularity, that is, the property of being recognised by an automaton or, equivalently, of being expressible in Monadic Second-Order Logic. This double point of view leads to many interesting questions about the interplay and relationship between topological complexity and regularity.

While we have a complete picture of what happens in the case of automata on words, the case of automata on trees is still a *terra incognita*. Some results have already been obtained for particular classes of languages, like Büchi languages, deterministic languages, and unambiguous languages. In this thesis we instead drop any restriction and prove some new results concerning arbitrary regular tree languages which belong to low levels of the Borel hierarchy and of the Wadge hierarchy. In particular we prove the following:

THEOREM 1. *A regular tree language L is recognised by a weak-alternating automaton that uses only two priorities if and only if it is in the first level of the Borel hierarchy.*

Then we prove some results about slightly higher levels of the Wadge hierarchy that can be summed up by the following theorem:

THEOREM 2. *The following holds*:

1. *Let Γ be a Wadge degree with finite Wadge rank. Then it is decidable if a regular tree language L belongs to Γ.*
2. *It is decidable if a regular tree language L is a Boolean combination of open sets.*
3. *It is decidable if a regular tree language L is in the Borel class Δ^0_2.*

Finally, we give a complete characterisation of the second level of the Borel hierarchy:

THEOREM 3. *It is decidable if a regular tree language L belongs to the second level of the Borel hierarchy. Moreover, a regular language L is in the second level of the Borel hierarchy if and only if it is recognised by a weak-alternating automaton that uses exactly three priorities.*

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