Enzyklopädie der mathematischen Wissenschaften, Bd. I, I Heft 6, Teil II, by H. Boerner. Darstellungstheorie der endlichen Gruppen, Stuttgart, 1967. 80 pages.

This very thorough report does not only give a clear and lucid exposition of the theory of representations of finite groups with an exhaustive list of references (between the years 1935 and 1965), but it also offers some insight into modern group theory - as it is influenced from the character theory - and some topics related to representation theory, e.g. projective representations and the representations of semi-groups.

In the first section, "Allgemeine Theorie der gewöhnlichen Darstellungen", the classical theory of semi-simple rings - as far as necessary for the theory of representations - is presented, using simultaneously the notion of matrices and modules. The case of an algebraically closed ground field is treated, and the orthogonality relations and other important theorems on characters are listed. After a section on Kroneckerproducts (i.e., tensor-products), the author reports the results on faithful representations (i.e., representations, the kernels of which are zero). The next two sections are dedicated to induction and restriction relative to subgroups and normal subgroups. This part concludes with a treatment of abelian groups. (At some places in this first section, the unexperienced reader might have some difficulties in deciding where it is necessary that the base field is a splitting field for the group).

The second part, "Gewöhnliche Darstellungen spezieller Gruppen", gives in a very distinct fashion all the important formulae for the representations of symmetric and alternating groups, and thence also touches on the finite classical groups.

In the next section, "Modulare und ganzzahlige Darstellungen", the modular theory is presented à la R. Brauer (blocks, defects and defect groups) and J.A. Green (vertices and sources), and it ends with the statement of D.G. Higman's characterization of the groups with a finite number of non-isomorphic indecomposable modular representations. The modular theory is then applied to the symmetric and alternating groups. Next the author turns to the integral representations; after some defining remarks on the terminology he lists the results on the number of non-isomorphic indecomposable integral representations of a finite group. (It might be worthwhile to mention that recently the problem under which conditions a finite group has a finite number of non-isomorphic indecomposable integral representations over the ring of algebraic integers in an algebraic number field, has been solved independently by Drozd, Ju. A. - A.V. Roiter [Izv. Akad. Nauk, SSSR 31 (1967), 783-798] and Jacobinski, H. [Acta. Math. 118 (1967), 1-31].) Next the author presents Maranda's results on the genus (for the statement of those results, page 57, the definition of a representation given on page 54, should be altered to: "If G is a finite group and R a Dedekind domain, a representation of G in R is a left RG-module of finite type, which is projective over R." The same is true for Zassenhaus' lemma, pages 54-55) and the problem of extensions of representations; the author ends this section with a list of some results on the representation algebra.

In the fourth section, the author presents a list of the very interesting results from group theory, which have been obtained with the help of representation theory (in this connection I would like to mention also W. Feit's book "Characters of Finite Groups" [New York, Amsterdam, 1967]).

The last section is occupied with the results on monomial representations, representations by collineations, representations by semi-linear transformations, and finally, representations of semi-groups.

This booklet will give a very good and even deep insight into the field of representations of finite groups to the non-specialist; it will be a helpful supplement for the specialist; and the list of references is a very valuable source of information to everybody related with this subject.

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Permutation groups by D.S. Passman. W.A. Benjamin, Inc., New York, 1968. 310 pages. U.S. $9.50 (clothbound); U.S. $4.95 (paperback).

Everyone who has taken an elementary course in the theory of representations and characters of finite groups has heard of Frobenius groups. These can be described as finite transitive permutation groups in which some non-trivial elements leave one letter fixed but only the identity leaves more than one letter fixed. One of the classical applications of character theory due to Frobenius is that in a Frobenius group the elements which move all letters, together with the identity, form a normal subgroup (called the Frobenius kernel) which complements each of the subgroups fixing one letter (called the Frobenius complements). Such groups occur often enough in the theory of finite groups to make the study of their structure worthwhile.

It is known that a finite group is a Frobenius kernel if and only if it has a fixed point free automorphism of prime order, and these groups were shown to be nilpotent by John Thompson in 1959. The facts about the Frobenius complements are less well known. Burnside knew that these groups must have all their Sylow p-groups either cyclic or (if $p = 2$) possibly generalized quaternion; but a theorem of his which purports to give their complete structure is false. In 1935 in his paper "Über endliche Fastkörper", Zassenhaus showed that a finite group is a Frobenius complement if and only if it has a faithful representation over the complex field which is fixed point-free in the sense that each non-trivial element corresponds to a matrix without 1 as an eigenvalue. He then described in detail the possible structure which such a group may have. Unfortunately, his paper contained some serious gaps, although it appears that he has since, in an unpublished manuscript, filled in these gaps. Recently two full accounts of the classification of Frobenius complements have been published. One is in the book "Spaces of Constant Curvature" (McGraw-Hill, 1967) by J.A. Wolf, and the other is in Passman's book. The two approaches differ slightly and to some extent complement each other; Wolf is interested for his applications in the fixed point-free representations whilst Passman looks at the permutation groups. The case of solvable Frobenius complements is straightforward (although not trivial), but I found the case of the non-solvable Frobenius complements a beautiful surprise. If $K$ is a metacyclic group of order prime to 30, and $SL(2,5)$ is the special linear group of order 120 (a central extension of the simple group of order 60), then $K \times SL(2,5)$ is a non-solvable Frobenius complement; conversely, every non-solvable Frobenius complement is of this form or else has a subgroup of index 2 which is of this form. In contrast to the simple statement of this result, the proof is long and tedious. [Since I wrote this review I have been told by Dr. B. Huppert that he has found several gaps remaining in Wolf's account.]

This structure theorem on Frobenius complements is one of the principal aims of Passman's book. The second is Huppert's theorem (1957) on the structure of solvable doubly transitive permutation groups. One way to construct such a permutation group is as follows. Take a finite field $GF(q^n)$ and consider all permutations of this field of the form $x \mapsto ax^{\sigma} + b$ $(a, b \in GF(q^n),\ a \neq 0,$ and $\sigma$ a field automorphism). Huppert's theorem shows that, except for six exceptional degrees, every solvable doubly transitive permutation group is essentially a subgroup of one of these particular permutation groups.