FORMATION OF ACOUSTIC IMAGES FROM SPATIAL COHERENCE FUNCTIONS: PROCESSING BY SUPERRESOLUTION TECHNIQUES

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Radioastronomy deals with radiowaves emitted by different kinds of sources. These sources are very far from the receiving antenna ; the distance R is very large compared with the wavelength. In acoustic imagery of noise sources the conditions are quite different.

The acoustic waves are scalar waves (pressure waves). The wave vector is very like the POYNTING vector related to transverse electromagnetic waves. The wavelength $\lambda = c/\nu$ is, in most cases, within 10 cm or a few meters, (frequency ν). The distance R from the sources to the antenna lies between :

$$100 \leq R \leq 3\ 500 \text{ m}$$

The main problem is related to the mean power distribution of the incoming waves as a function of frequency and angle as shown by figure 1. An important feature of the interferometric receiver is that the signals are wideband ones [1]. The relative bandwidth may be of the order of 1/3 to 1/2 and 1 octave. In a few cases the bandwith may reach 2 or 3 octaves. The derivation of the receiver's performances never uses the assumption of narrow bandwidth. The array and the sensors have properties very like those of wideband antennas [2] [3].

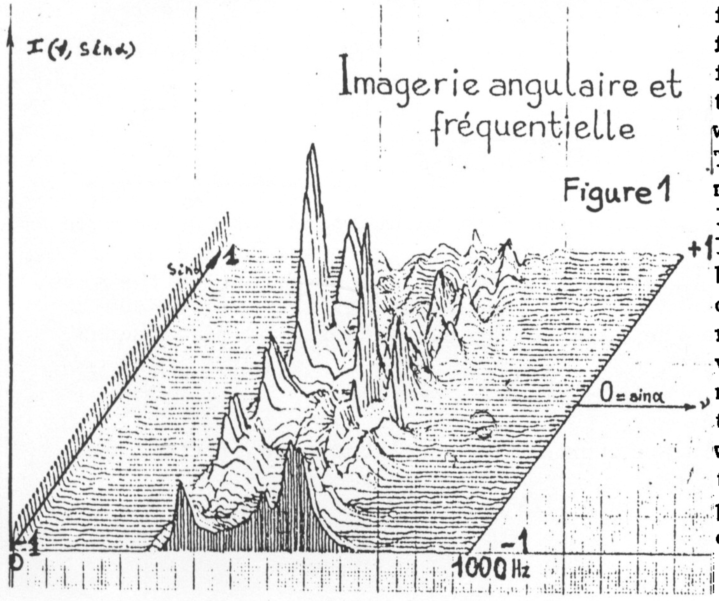

Figure 1

C. van Schooneveld (ed.), Image Formation from Coherence Functions in Astronomy, 299–313.

1. RANDOM PROPAGATION OF SOUND IN FREE TURBULENT ATMOSPHERE

Propagation of sound waves in free atmosphere is a random propagation. The fluctuations of air temperature and density generate turbulences. These turbulences modulate, in all places, the speed of sound. The local turbulent speed and the fluctuations of speed of sound affect the properties of the transmitted wave. The wave vector $\vec{k}$:

$$|\vec{k}| = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c_0}$$

is distorded by two effects :

- rotation
- modulus variation

Figure 2 displays these two effects. For a long range path (R $\simeq$ 1000m) the effects of transmission in a turbulent medium transform a pure sine wave, the frequency of which is ν_0, into a narrow band random gaussian wave :

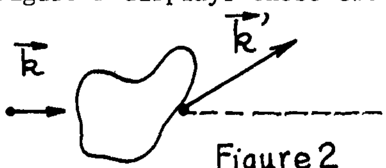

Figure 2

$$A_0 \sin(2\pi\nu_0 t + \varphi_0) \longrightarrow A(t)\sin(2\pi\nu_0 t + \Phi(t)) = X$$

X(t) is a stationnary gaussian process. The covariance function $\Gamma_X(t,\tau)$ is given by :

$$\Gamma_X(t,\tau) = \Gamma_X(\tau) = \sigma_A^2 \, e^{-\frac{|\tau|}{\tau_0}} \cos 2\pi\nu_0\tau$$

The constant τ_0 is such that : $\tau_0\nu_0^q = cst$, $1 < q \leq 2$

The random medium may be represented by a "random parameter filter" whose transfer function $h(t,\nu)$ is such that :

$$h(t,\nu) = \mathrm{Re}\,h(t,\nu) + i\,\mathrm{Im}\,h(t,\nu), \quad \Gamma_{Reh}(\tau,\nu_0) = K\,e^{-\frac{|\tau|}{\tau_0}}$$

The mean power spectrum density $\gamma_{Reh}(\nu,\nu_0)$ exhibits the spectral widening effect :

$$\Gamma_{Reh}(\tau,\nu_0) \underset{\nu}{\overset{\tau}{\rightleftharpoons}} \gamma_{Reh} = \frac{K}{2\tau_0}\,\frac{1}{1+4\pi^2\nu^2\tau_0^2}$$

as shown by figure 3.

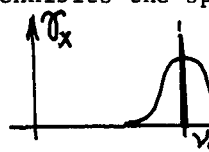

Figure 3

All along the propagation path turbulences act on the wave as a "space time" filter. Each turbulence, the correlation radius of which L is larger than λ, produces a "diffraction" effect on the wave. If we consider two paths R1 and R2 (see figure 4) the effects due to the turbulences will be the same if the two sensors, or microphones, are very near each other :

$$(k - j)\,\Delta < L$$

and these effects are independent or uncorrelated ones if the two sensors are spaced such that :

$$(k - j)\,\Delta \gg L$$

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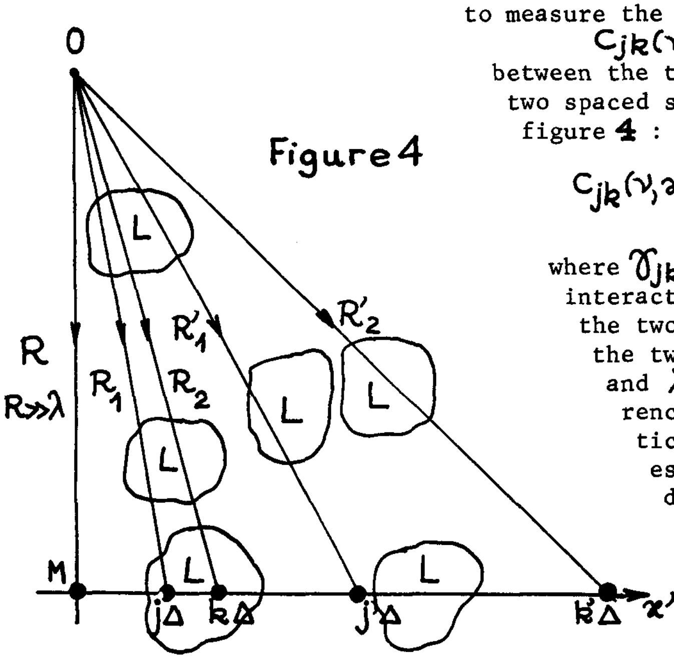
Many experiments done during the past three years have tried to measure the coherence function $C_{jk}(\nu, x)$, $(k-j)\Delta = x$ between the two received signals on two spaced sensors as shown in figure 4 :



$$C_{jk}(\nu, x) = \frac{|\gamma_{jk}(\nu, x)|}{\sqrt{\gamma_j(\nu)\gamma_k(\nu)}}$$

where $\gamma_{jk}, \gamma_j, \gamma_k$ are the mean interaction power density, and the two mean power densities of the two received signals $X_j(t)$ and $X_k(t)$. Such a coherence function is a statistical moment, which was estimated on a long duration of time. This duration was determined to avoid bias and give a good statistical accuracy for the estimate [5].

For instance :

The conditions : T observation time $T = 1500\,s$

$\Delta\nu$ spectral broadening bandwith $\Delta\nu \simeq 1\,Hz$

$C_{jk} = 0{,}4$ correspond to : $\varepsilon_{\hat{c}} = \frac{\sigma_{\hat{c}}}{E\{c\}} \simeq 5.10^{-2}$ [5]

$\hat{c}$ estimate of $c(x, \nu)$ and $\sigma^2_{\hat{c}}$ variance of the estimate.

All the experiments done on many different places and with many meteorological conditions lead to the result :

$$c(\nu, x) \simeq e^{-\frac{|x|}{n_\perp \lambda}}$$

$8 \leqslant n_\perp \leqslant 25$ $\qquad$ $400_{Hz} \leqslant \nu \leqslant 2\,400$ Hz

$n_\perp$ is a peculiar constant related to the turbulent effects of the medium on the transverse coherence. Such a result is an important one; we have to take it into account when we are deriving the angular and frequency response of the interferometric receiver.

2. ACOUSTIC IMAGERY BY VARIABLE BASELINE INTERFEROMETRY [5] [6]

M sources are radiating sound waves which are independant random ones. The wavelength λ is such that :

$$\lambda_m \leqslant \lambda \leqslant \lambda_M$$

The array or antenna is built with N sensors or microphones, which are "point sensors" compared with the values of λ . The spacing between

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two sensors Δ is : $\Delta \leqslant \frac{\lambda m}{2}$

in agreement with the sampling theorem. Cross correlation between all the sensors in the array gives : $\Gamma_{jk}(x,\tau) = \sum_{\ell=1}^{M} \Gamma_\ell\left(\tau - \frac{x}{c_0}\sin\alpha_\ell\right) = \Gamma(\tau, x)$

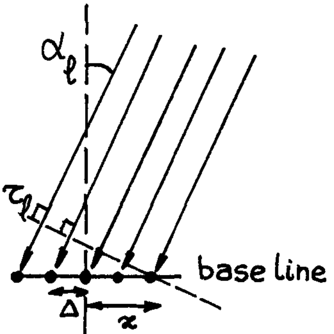

Figure 5

Γ_ℓ is the correlation function of the random wave coming from the ℓ^{th} source in the direction α_ℓ as shown by figure 5.

The FOURIER transform of $\Gamma(\tau, x)$ is $\gamma(\nu, f)$ such that :

$$\gamma(\nu,f) = \int_{R^2} \Gamma(\tau,x)\, e^{-2i\pi(\tau\nu + fx)}\, d\tau\, dx$$

We may write it in the following form by a very easy derivation :

$$\gamma(\nu,f) = I(\nu,a) = \sum_\ell \gamma_\ell(\nu)\, \delta\left(f - \frac{\nu}{c}\sin\alpha_\ell\right)$$

$\gamma_\ell(\nu)$ is the power density spectrum of the ℓ^{th} random wave coming from α_ℓ direction and $\delta(\ )$ is the so called "DIRAC function" or DIRAC distribution. This result implies that, as figure 6 shows, in the (ν, f) plane the surface $I(\nu, f)$ have main traces such :

$$f = \frac{\nu}{c_0}\sin\alpha_\ell = \nu a$$

In fact, this result has to be modified to take into account two effects. The first one is the weighting effect of time window $F(\tau)$ multiplying the output of the correlator, and the second is the "coherence loss" effect expressed by the function $C_\perp'(x,\nu) = C_\perp(x,\lambda) = C(x)$ [5].

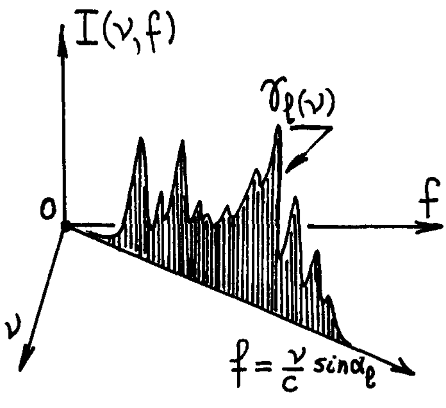

Figure 6

$$I(\nu,a) = \int_R \sum_{\ell=1}^{M} \gamma_\ell(n)\, w(\nu - n)\, c(na_\ell - \nu a)\, dn$$

$$f(\nu) \rightleftharpoons W(\tau),\ C(x) \rightleftharpoons c(\nu),\ a = \frac{\sin\alpha}{c_0}$$

a is the result of division between a wave number (m^{-1}) by a frequency $(Hz \sim s^{-1})$. $c(f)$ is the coherence weighting function :

$$c(f) \overset{f \to x}{\rightleftharpoons} C(x)$$

In most cases the maximum value of the delay $|\tau_M| \leqslant \frac{N\Delta}{c_0}$ is such that : $f(\nu)$ is a "pulse" function compared with $g(\nu a)$ which is the weighting function related to C(f), as shown by figure 6. We may express $I(\nu, a)$ in an easier way :

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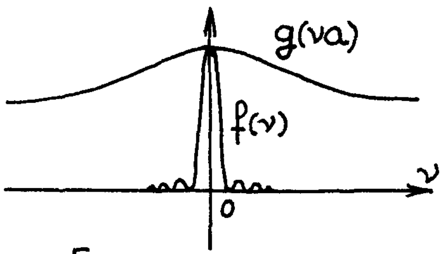


Figure 7

$$I(\nu,a)=\sum_{\ell=1}^{M} g(\nu(a_\ell-a))(\Gamma_\ell * f)_{(\nu)}$$

under the previous assumption that f is very near a Dirac distribution.

This is the most frequent situation in noise imagery. [5].
The total length of the array is:

$$2N\Delta = 2N\frac{\lambda_m}{2} = 2D_M$$

The usual conditions are such that: $|x| \leqslant x_M$ becomes

$$x_M = p\frac{\lambda_m}{2}\ ,\quad \zeta(x,\nu) \rightarrow G(x) = \Pi_{2x_M}(x)\cdot e^{-\frac{|x|}{n_\perp\lambda}}$$

Π is the "rectangle" or "gate function" such that :

$$\Pi(x) = 1\ ,\quad 0 \leqslant |x| \leqslant \tfrac{1}{2}$$
$$\Pi(x) = 0\ ,\quad |x| \notin]0,\tfrac{1}{2}[$$

The FOURIER transform of G(x) is :

$$g(-\nu a) \overset{-\nu a\quad x}{\rightleftharpoons} \Pi_{2x_M}\zeta(x,\lambda)$$

$$g(-\nu a) = \frac{2n_\perp\lambda\,(1-e^{-p/2n_\perp}[\cos(\pi p\sin\alpha)-2n_\perp\sin(\pi p\sin\alpha)]}{1+4\pi^2 n_\perp^2\sin^2\alpha}$$

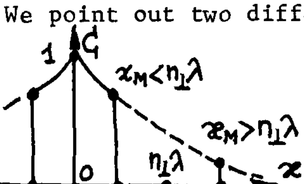
$$a = \frac{\sin\alpha}{c_0}$$

We may define the angular resolution α' by:

$$\frac{1}{2}g(0) = g\left(-\nu\frac{\sin\alpha'}{c_0}\right)$$

We point out two different situations :

Figure 8



a) $x_M = p\frac{\lambda_m}{2} < n_\perp\lambda_m$

b) $n_\perp\lambda_m < x_M = p\frac{\lambda_m}{2}$

In case a) we obtain : $e^{-\frac{x_M}{n_\perp\lambda}} \simeq 0\ ,\quad x_M \simeq 4n_\perp\lambda_m$

and in case b) : $e^{-\frac{x_M}{n_\perp\lambda}} \simeq 1\ ,\quad x_M \simeq \frac{n_\perp\lambda_m}{3}$

as shown by figure 8. In the last case we may assume that :

$$\zeta(x)\Pi_{2x_M}(x) \simeq \Pi_{2x_M}(x) \rightarrow g(-\nu a) \simeq \frac{\sin 2\pi\nu a x_M}{\pi\nu a}$$

α_c' is such that:

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$$g\left(\frac{-\nu \sin\alpha'_c}{c_0}\right) = \frac{1}{2}\, g(0)$$ as a function of $n_\perp$ and p:

$$p \leqslant 15 \quad , \quad n_\perp \simeq 8 \text{ to } 25$$

The main result is that α'_c is a decreasing function with p and the different values of $n_\perp$ have very low effects on the function $\alpha'_c(p, n_\perp)$.

Practical constraints limit the usual values of p. If we deal with sources emitting random sine waves inside the "63 Hz octave" the wavelength λ lies between :

$$3m = \lambda_m \leqslant \lambda \leqslant \lambda_M = 6m$$

$$\Delta = \frac{\lambda_m}{2} = 1{,}5m \quad , \quad p = 10 \rightarrow x_M = 15m\,!$$

The same computation in the case of "31 Hz OCTAVE" implies that : x_M = 30 m, p = 10 (i.e. large values for x_M and p).

Such constraints on the array have drastic limitations on x_M and p. In many cases :

$$x_M = p\,\frac{\lambda_m}{2} \leqslant 5\lambda_m$$

The array is designed as a "gracious array" very near the optimum solutions derived by F.BIRAUD and J.C. RIBES [8].

3. ANGULAR RESPONSE OF THE RECEIVER, COHERENCE LOSS AND ANGULAR RESOLUTION

The angular response of the receiver may be derived as a "Green function" under the assumption that a unique point source emits a pure tone wave and is located in the direction α_ℓ. This angular response is :

$$g(\nu a_\ell - \nu a) \;, \quad G(x) = \Pi_{2x_M}(x)\, \mathcal{C}(x)$$

We note that : $\mathcal{C}(x_M) \simeq 1 \,, \quad x_M \leqslant 5\lambda_m$

This result may be explained by the properties of the instantaneous phase $\Phi_{12}(x,t)$ between two signals S_1 and S_2 received on two sensors located in the positions as shown by figure 10. The experimental set described on figure 10 provides an estimate of $\Phi_{12}(x,t)$, S_0 the emitted wave being a sine wave the frequency of which ν_0 : $\nu_0 = 1$ KHz. $\hat{\Phi}_{12}(x,t)$ is a randomly sampled function, the mean sampling frequency is ν_0 , and :

$$\sigma_{\nu_S} \ll \nu_0 \qquad \nu_S \text{ sampling frequency}$$

A rather easy derivation shows that a low pass filter allows a very accurate restoration of $\Phi_{12}(x,t)$ [6]. The statistical properties of $\Phi_{12}(x,t)$ are such that :

$$\Gamma_{\Phi_{12}}(\tau) = E\left\{\Phi_{12}(x,t)\,\Phi^*_{12}(x,t-\tau)\right\} \rightleftharpoons \gamma_\Phi(\nu)$$

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$\gamma_{\Phi}(\nu)$ is a "low pass" spectrum density, the cutoff frequency of which is very near 1 Hz. If $x \leqslant 5\ \lambda_m$ we obtain :

$$E\{\Phi(x,t)\} \equiv 0,\quad \sigma_{\Phi} \leqslant 0{,}2^{\circ},\quad x \leqslant 5\lambda_m$$

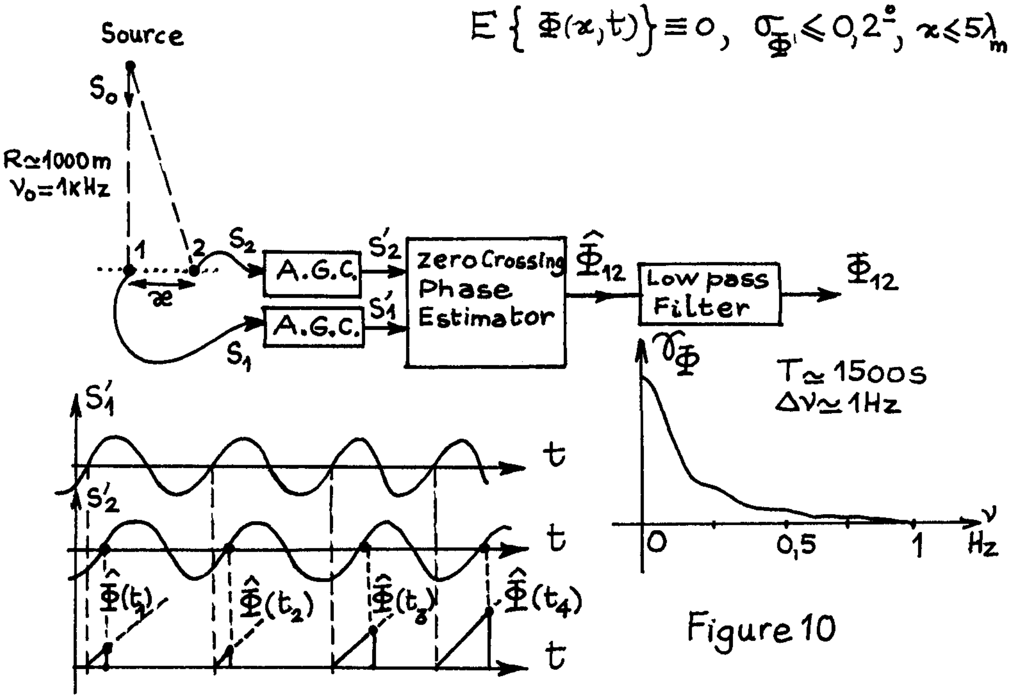

Figure 10

The coherence function describes the random modulation effects due to turbulences. These effects are modulating the group delay of the medium and modulate the path delay or the time of arrival of each wave. All these effects appear in the instantaneous phase of two signals received on two sensors distance x in the array. [6]

The coherence loss C(x) limits drastically the angular resolution of the interferometric receiver. A solution of the finite angle resolution is the technique of super-resolution and mainly "deconvolution". If we try to use the deconvolution technique we need a main information which is the angular response :

$$g(\nu(a_\ell - a))$$

We pointed out, in the last section, that we are facing two cases :

a) $n_\perp \lambda_m < x_M \quad , 8 \leqslant n_\perp \leqslant 20$

b) $x_M < n_\perp \lambda_m$

In the case a) the angular response is $g(\nu a) \rightleftharpoons G(x) = C\Pi_{2x_M}$
C(x) is the coherence function. We only deal with estimates of $\hat{C}(x)$

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and in every situation the condition described as case a) implies we have to estimate C(x). This is a very drastic limitation and may be a drawback in many cases.

On the other hand in case b) the assumption is :

$$x_M = p\frac{\lambda_m}{2} < n_\perp \lambda_m$$

and :

$$\Pi_{2x_M}(x)\, C(x) \simeq \Pi_{2x_M}(x)\ ,\quad x_M \simeq \frac{n_\perp}{3}\lambda_m$$

In such a case the deconvolution technique may be used with the realistic assumption that :

$$g(-\nu a) \simeq \frac{\sin 2\pi\nu a x_M}{\pi\nu a} \qquad x_M \leqslant 5\lambda_m$$
$$10 \leqslant n_\perp \leqslant 25$$

4 . ACOUSTIC IMAGE PROCESSING BY SUPER-RESOLUTION TECHNIQUES

a - practical constraints and a priori information on acoustic images:

The processing of the interferometric image needs a resolution criterion. This criterion tends to define and use some "a priori" informations about the object or the sources. The image, or object's observation, or estimate, is such that :

$$I(\nu,a) = \int_R O(\nu,a')\, g(\nu a - \nu a')\, da' + b(\nu,a)$$

O is the object and $b(\nu,a)$ the estimation noise due to the random modulations in the medium and the finite integration time. In many cases the signal to noise ratio may be very high. O is a positive variable or density function and we get :

$$O(\nu,a) \geqq 0\ ,\quad \hat{I}(\nu,a) = \int_R O(\nu,a')\, g(\nu(a-a'))\, da' + \overbrace{b(\nu,a)}^{\text{NOISE}}$$

This is an important a priori information. We have to point out that a is such that :

$$-\frac{1}{c_0} \leqslant a \leqslant \frac{1}{c_0}$$
$$-\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2}$$

The usual acoustic interferometric image deals with angular information in the horizontal plane. $O(\nu,a)$ is a "finite length" object.

In many cases the noise sources are point sources emitting narrow band signals or pure sine waves. All these a priori informations are taken into account to get an efficient image processing.

b - estimation of intensity and location of point sources emitting pure tone waves:

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In many cases the radiated field is generated by a few point sources, which may be considered as uncorelated sources. In such a case under this assumption, the estimation deals with a finite number of source parameters.

The data about the source location may be extracted from the cross spectrum matrix which describes the interaction spectra of all the signals received by the array's sensors. The number of point sources which may be identified is directly related to the number of sensors in the array. This problem was first described by H. MERMOZ, who uses the whole data to improve the propagation model under the assumption that the number of sources M is such that [9] : $M \leq N-1$.

If M point sources are radiating stationnary uncorrelated random waves, and if the antenna is built with N sensors, we may describe the k^{th} output of the array as the output of a "linear filter" processing the signal emitted by the m^{th} source. The transfer function is : $\xi_{km}(\nu)$

The cross spectrum density between the k^{th} and the l^{th} output $\Gamma_{k\ell}$ is such that :

$$\Gamma_{k\ell}(\nu)=\sum_{m=1}^{M}\Gamma_m(\nu).\xi_{km}(\nu)\,\xi_{\ell m}^{*}(\nu)$$

If the propagation medium is an isotropic, stable and non dispersive, one, $\xi_{k\ell}(\nu)$ may be expressed as :

$$\xi_{k\ell}(\nu)=e^{-2i\pi\nu\tau_{k\ell}}\,h_k(\nu)$$

$\tau_{k\ell}$ is the propagation delay related to the m^{th} signal received by the k^{th} sensor, $h_k(\nu)$ is the complex transfer function of this sensor.

One may define "directional vectors" $\xi_m(\nu)$, which are k dimensional ones, related to one source, given the N sensors located in the array : [10][11]

$$\xi_m(\nu)=\begin{bmatrix}\xi_{1m}\\ \vdots\\ \xi_{Nm}\end{bmatrix}$$

The cross spectrum matrix $\Gamma(\nu)$ is easily derived :

$$\Gamma(\nu)=\sum_{m=1}^{M}\Gamma_m(\nu)\,\xi_m(\nu)\,\xi_m^{+}(\nu)$$

ξ_m^{+} is the conjugate transposed vector.

If : $\quad M < N$

the cross spectrum matrix $\Gamma(\nu)$ is a M dimension square matrix which has (N - M) non zero eigenvalues. Computing these eigenvalues leads to the number of sources.

One may define, in the same way, a "directional matrix" $\mathbf{X}(\nu)$ which is a M x N dimension one, such that : $\mathbf{X}(\nu)=\left[\xi_{km}(\nu)\right]$

The cross spectrum matrix is a function of $\mathbf{X}$:

$$\mathbf{\Gamma} = \mathbf{X}\,\mathbf{S}\,\mathbf{X}^{+}$$

with : $\mathbf{S} = [\gamma_m]$ which is a diagonal matrix.

H. MERMOZ defined the "source vectors" $y_m(\nu)$ by : [9]

$$y_m(\nu) = \sqrt{\gamma_m}\,[\xi_{km}] = \sqrt{\gamma_m}\,\xi_m$$

and a "source matrix" such that :

$$\mathbf{Y}_{(\nu)} = [y_1(\nu), \cdots, y_k, \cdots y_M(\nu)]$$ which is an N x P order matrix

$$\mathbf{\Gamma} = \mathbf{Y}\mathbf{Y}^{+}$$

To identify the M sources, we have to compute the mean power spectrum densities and the directional vectors from the eigenvalues and the eigenvectors of the $\mathbf{\Gamma}$ matrix. The solution of such problems re - quires in most cases a rather cumbersome algorithmic solution [12] .

When dealing with a rectilinear array and with remote sources, the solution may be easily derived. The source parameter to be estimated is the angle α_m and the mean power density $\gamma_m(\nu)$. The cross spectrum density related to the k^{th} and l^{th} sensor is :

$$\gamma_{k\ell}(\nu) = \sum_{m=1}^{M} \gamma_m(\nu)\, e^{i(k-\ell)\,2\pi\frac{\Delta}{\lambda}\sin\alpha_m}$$

The cross spectrum matrix $\mathbf{\Gamma}$ is now a TOEPLITZ one. Estimation of the quantities $\gamma_{1\ell}(\nu)$ is a sufficient estimation to derive the whole matrix. In such a case a possible estimation of the source parameters uses the PISARENKO method [13] .

A peculiar case of interest is, when dealing with two sources located in very close directions (such as double sources in radioastronomy). For a rectilinear antenna built of N equally spaced sensors, the lower bound (as defined by CRAMER RAO) of the angular resolution is given by [14] [15]:

$$\delta\alpha = \frac{\lambda}{\pi\Delta}\left[\frac{45}{N(N^2-1)(2N-1)(8N-11)\mathcal{R}}\right]^{1/4}$$

$\delta\alpha = \alpha_1 - \alpha_2$ difference between the angular parameters, such that $\delta\alpha^2$ equals the variance of the estimate. This result deals with two sources the powers of which are the same. $\mathcal{R}$ is defined as :

$$\mathcal{R} = \frac{s^2}{\sigma^2}$$

$\mathcal{S}$ is the mean power of the source, σ is the standard deviation of the crossspectrum estimates, which are supposed to be uncorrelated with the same variance σ^2 .

A super-resolution gain (defined by reference to RAYLEIGH's criterion), about 20, needs for a three sensor antenna (M = 3) an accurate estimation of the cross spectrum densities such that :

$$\mathcal{R} \simeq 760 \ , \quad \frac{\sigma}{\mathcal{S}} \simeq 3{,}5\ 10^{-2}$$

This solution requires that there are two and only two sources to be estimated from the data given by the three sensors. Under this assumption the solution describes the mean power density of each source by estimation of $\gamma_m(\nu)$, $\nu_{min} \leqslant \nu \leqslant \nu_{max}$. Even if the three sensors are such that their transfer functions have different modulus the solution may be derived under the assumption that the phases of these transfer functions are the same, by computation of the complex coherence matrix :

$$\mathbb{C} = \left[\frac{\gamma_{k\ell}(\nu)}{\sqrt{\gamma_k \gamma_\ell}} \right]$$

This C matrix is a TOEPLITZ one and the diagonal values are all equal to 1 ; the computation deals only with the two complex data C_{12} and C_{13} in order to estimate the four parameters : $\alpha_1, \alpha_2, \gamma_1, \gamma_2(\nu)$ $P_1 = \mathcal{S} = P_2$, (M = 2, N = 3). A straight forward solution is proved to exist [16]. Under the same assumptions, defined above, the angular resolution $\delta\alpha$ is :

$$\delta\alpha = \frac{\lambda}{\pi\Delta} \left(18\,\mathcal{R}\right)^{-1/4}$$

Such an estimation is not a true efficient one as defined by the CRAMER-RAO bound. Nevertheless this result is <u>very near</u> the optimum one :

$$\delta\alpha_{min} = \frac{\lambda}{\pi\Delta} \left(\frac{104}{3}\,\mathcal{R}\right)^{-1/4}$$

The RAYLEIGH criterion defines the angular resolution as : $\lambda/2\Delta$ then the super resolution gain is : $\eta = \frac{\pi}{2} \left(18\,\mathcal{R}\right)^{1/4} = 3{,}24\ \mathcal{R}^{1/4}$

If $\mathcal{R} = 760$, $\frac{\sigma}{\mathcal{S}} \simeq 3{,}5\,10^{-2}$: $\eta = 17$

This proves that the superresolution gain needs a very accurate estimation of the cross spectrum densities or of the coherence function.

<u>c - positive deconvolution of acoustic images related to "finite length" objects [17] :</u>

In many cases the assumption of a few uncorrelated point sources cannot be taken into account. We are faced with the deconvolution problem. As explained before we deal with an image

$$I(\nu, a) = (g * O)_{(\nu, a)} \quad , \quad O(\nu, a) \geq 0$$

$$I(\nu, a) = \int_R O(\nu, a')\, g(\nu a - \nu a')\, da'$$

$a = \frac{\sin\alpha}{c}$, $0 \leq |\sin\alpha| \leq 1$ The "object" is a finite length one, represented by a function which is a non zero one only if :

$$-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$$

Y. BIRAUD has shown that a possible positive deconvolution method may use an iterative restoration whose first order solution is the so called "BRACEWELL'S principal solution". The iterative computation tries to get new informations about the FOURIER transform of $O(\nu,a)$ outside of a given bandwidth $(\mathcal{B})$, under the constraint : $O(\nu,a) \geq 0$

$$J \rightleftharpoons j \, , \qquad O(\nu, a) = |J(\nu, a)|^2 \underset{}{\overset{\nu a, x}{\rightleftharpoons}} \sigma(x) = (j * j)(x)$$

The iterative process deals with a function $j_n(x)$ such that :

$$\varepsilon_n^2 = \int_{(\mathcal{B})} \left(j_n^{*2} - j_o \right)^2 dx \longrightarrow (\varepsilon_n^2)_m \quad \text{minimum of } \varepsilon_n^2$$

where $j_o(x)$ is the BRACEWELL'S SOLUTION, j_n being an hermitian function, $J \in \mathbb{R} \to j = j^*(-x)$. An energetic constraint ensures that :

$$j_n(0) = j_o(0)$$

The next step $j_{n+1} = j_n(x) + \omega_n(x)$ where $\omega_n(x)$ is an hermitian "perturbation pulse function". This pulse function is located at :

$$x = k x_s$$

, where x_s is the sampling period of $j_n(x)$. The solution has to compute ω_n such the two constraints which were defined previously are fullfilled. The iterative process goes on till the error ε_n^2 is such that :

$$\varepsilon_n^2 \simeq \sigma_N^2$$

where σ_N^2 is the variance of the "noise" process added to the signal.

To compute the BRACEWELL'S solution we have to define the equivalent bandwidth $(\mathcal{B})$ and the sampling rate related to the impulse response :

$$g(\nu a) \overset{\nu a, x}{\rightleftharpoons} G(x)$$

Under the assumptions previously defined we may assert that :

$$g(\nu a) = \frac{\sin 2\pi \nu a x_M}{\pi \nu a}$$

or we may choose a weighting function such that :

$$g(\nu a) = \left(\frac{\sin 2\pi\nu a x_M}{\pi\nu a}\right)^2 \rightleftharpoons \left(\Pi_{2x_M} * \Pi_{2x_M}\right)(x)$$

Some experiments with such a response show that a deconvolution solution may be found even if the Fourier transform of $g(\nu a)$ equals a "square" function[17].

The signal to noise ratio cannot be defined in an easy way. The effects of "turbulent fluctuations" are modulation effects. These effects are very near the so called "scintillation" effects. The previous results on "random propagation" in turbulent atmosphere show that they are very low frequency effects. A way to describe easily the sources is to estimate a first image, then to have a "deconvolution" solution. An other estimate computed on an uncorrelated sample, a few minutes after, may be processed to have a new solution, and so on. We are now trying to do systematic experiments with different sources and environmental conditions.

5. CONCLUSION

Incoherent imagery deals with uncorrelated sources, the statistical properties of which are only second order moments. In such a case the space and frequency coherence function provides an easy way to describe the properties of the far field. A rectilinear array with N sensors receives the incoming field.

The cross spectrum matrix may be processed to estimate the directions and the power spectra of the M sources, $M < N$. The length of the receiving array $2x_M = 2p\lambda/2$ has to be lower than $2n_\perp\lambda$, where $n_\perp$ is a peculiar parameter which describes the coherence loss due to the random propagation medium. Such an assumption $x_M < n_\perp\lambda$ allows to describe the space and frequency response of an interferometric image processor as the FOURIER transform of a constant function inside the "total length" $x_M = p\lambda/2$. The angular resolution is limited by the random propagation effect and mainly by the coherence loss. A solution to improve the frequency and angular resolution is to process the image by super resolution techniques such as source estimation and deconvolution. A priori informations such positiveness, finite length, finite number of sources may provide constraints leading to super resolution effects under the assumption that the estimation accuracy is a convenient one.

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Reference 7 may be obtained at ICPI, Laboratoire Traitement du Signal, 25 Rue du Plat, 69288 Lyon-Cedex 1.
Reference 16 is edited by Dr. Filippi, 31 Chemin J. Aiguier, 13274 Marseille.

DISCUSSION

Comment D.B. SHAFFER
Are you trying to understand the atmosphere or do you study acoustic sources in the atmosphere, like thunder or airplanes?
Reply B. ESCUDIE
We try to identify the properties of the random channel (atmosphere). These random properties are related to effects such as "coherence loss" on the array. Taking these data into account we make an image (estimate the brightness) of noise sources such as powerplants, factories, etc., and in the future of vehicles such as cars, trains and maybe aircraft.

Comment T.A. CLARK
In addition to the other non-linear effects, do you also have to consider

dispersion in your medium?
Reply B. ESCUDIE
Theoretically speaking the medium is a dispersive one. In fact the random modulation effects are the main ones, as far as we know now. These modulation effects are frequency dependent. The spectrum width is a function of frequency ν:

$$\Delta\nu = k\,\nu^{q}\ ;\ 1 \leq q \leq 2$$

This may also be called a frequency dependent effect or a dispersive effect.

Comment C. VAN SCHOONEVELD
Can the situation be summarized in the following way? In a frozen, but granulated medium, a point source can always be identified as such. If sufficiently rapid fluctuations occur, this is no longer possible and a point source can not be distinguished from an extended one, unless a-priori knowledge can be used in a deconvolution.
Reply B. ESCUDIE
This is perfectly correct! We have to use the a-priori information to do the "deconvolution" processing. The random modulation effects may be interpreted as a variable medium, the impulse response of which is a random one. The impulse response of the interferometric receiver is a highly medium dependent response. If we estimate the image of a "true point source" with the interferometric receiver, the estimated image will tend to a "pulse function" only if the integration time T_i is very high compared with the inverse of spectrum width ΔV. This pulse function is not very "narrow" one.