Fractal Computer Graphics as a Stimulus for the Enhancement of Perceptual Sensitivity to the Natural Environment

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Introduction

> The beauty and freshness of fractal geometry suggests that once again we are at the start of science and mathematics... women and men will look back on this era much as we look back to the early Greeks. (Barnsley, 1989, p. 5)

Such enthusiasm for a perceived new paradigm in the mathematical sciences is beginning to emerge within broader educational contexts (Devaney & Keen, 1989; Egnatoff, 1989; Geake, 1990a & 1990b). Much of the interest in fractal geometry has focussed on its ability to describe complex natural phenomena (Mandelbrot, 1983 & 1990; Pickover, 1987; Barnsley, 1988). Recent investigations into the visual perception of natural imagery have used fractal mathematics in describing the characteristics of such perception (Pentland, 1984; Field, 1987; Peli, 1990). This study examined human visual perception of the fractal form found in the natural environment. Specifically, this research project examined how exposure to a program of fractal computer graphics affected the perceptual sensitivity of primary school children to the natural visual environment. The underpinning rationale was to address a long standing challenge of Linke (1980) to develop a stronger theoretical basis for environmental education in Australia.

Fractal form

> Fractal geometry is a new language. Once you speak it, you can describe the shape of a cloud as precisely as an architect can describe a house. (Barnsley, 1988, p. 1)

The most significant recent development in the scientific disciplines has been the emergence of the new field of Chaos Theory to explain the behaviour of complex natural phenomena. Chaos is a field of applied mathematics created some twenty years ago from the pooling of remarkably similar results from nonlinear modelling in meteorology, cardiology, fluid dynamics and population dynamics (Gleick, 1987; Briggs and Peat, 1989). More recently, nonlinear modelling has been used in a wide variety of fields including astronomy (Foukal, 1990), chemistry (Ottino, 1989), biology (Haken, 1983), and physics (Berry, 1987).

https://doi.org/10.1017/S0814062600003268 Published online by Cambridge University Press

A seminal construct within the field of Chaos Theory is that of a *fractal*. Fractal geometry can explain the hitherto indescribable shapes of natural phenomena such as coastlines, clouds, flames, lightning, tree branching, and mountain range profiles (Mandelbrot, 1983). Fractal is a generic description of objects and shapes whose constituent parts have the same form as their whole, at any level of scaling. For example, a typical Slash Pine Christmas tree is actually a branch, but its branching is a scaled replica of the parent tree.

Natural fractals are said to be statistically scale invariant since the structure at each level of scaling is not an exact replica of the structure of an adjacent level. Also, natural fractals are statistically scale invariant over wide, but not unlimited, transformations of scale. There are physical restrictions on the range of scaling over which natural objects have fractal properties. The lower limit is determined by the size of the constituent particles; the upper limit is determined by the size of the object (Pentland, 1984).

One reason that natural shapes are characteristically fractal is that

> fractals ... are the end result of physical processes that modify shape through local action. Such processes will, after innumerable repetitions, typically produce a fractal surface shape. (Pentland, 1984, p. 662)

Examples of such physical processes include: erosion, e.g., formation of seascape and mountain profiles; aggregation, e.g., galaxy formation, meteorite accretion, snowflake growth; and turbulent flow, e.g., rivers, lava flows (Pentland, 1984; Schibeci, 1989).

The term "fractal" was coined by Mandelbrot (1983), and is derived from the concept of fractional dimensionality [**D**], one of the distinguishing features of a fractal. Consider the task of measuring the length of a typical convoluted coastline. The measured length will always be less than the actual length because there are always convolutions which are smaller than the measuring scale. Reducing the scale of the unit measure will not remove the problem because the number of convolutions increases as the magnification increases (Mandelbrot, 1983 & 1990; Egnatoff, 1989). Spatial intuition can be accommodated by the notion that an infinitely convoluted line, rather than being constrained within a twisted topological dimension of one, begins to occupy area. Thus for planar fractal curves, the values of **D** should lie between 1 and 2. Similarly, fractal surfaces should have **D** between 2 and 3.

There is agreement in the literature with the values of **D** that best represent natural phenomena. For profiles found in natural scenery Burrough (1981) reports **D** = 1.2 to 1.3. The coastline of Britain has a dimensionality approaching 1.26 (Mandelbrot, 1983). For planar images of natural scenery, Pentland (1984) and Field (1987) both give **D** = 2.5.

Visual perception of fractal form

Voss has argued that as a species which evolved in the natural environment we carry a predilection for recognizing natural, that is, fractal, form (Gardner, 1983). He has elaborated:

> We have evolved in a fractal world, which has a profound effect on what we appreciate - what we feel a certain resonance with. ... You rarely find people thinking that man-made shapes look like anything natural. (quoted in Smith, 1990)

Familiarity, or recognition of similarities, seems to be an important characteristic of visual processing. One of the outcomes of an information processing approach to visual perception is the necessity for redundancy within the information received. This requirement is particularly germane to natural image perception (Bullock, 1976; Watson, Barlow & Hobson, 1983; Treisman, 1986).

> ... rather than searching for features in an image, the visual system codes a given image with regard to its relation to the statistical properties of the set of natural images. Because the space of possible pictures is so great, it makes good sense to utilize naturally occurring redundancy to recode image information into a less redundant form. (Kersten, 1987, p. 2395)

Kersten concluded that we not only impose meaning to make predictions about our visual reality, but that we are able to get information from the statistical distribution of the intensity of the light signals. This is consistent with Gisbon's comment "that one must understand the nature of the environment before one can understand the nature of visual processing" (quoted in Field, 1987: 2379). Field argued "that images from the natural environment should not be presumed to be random patterns. Such images show a number of consistent statistical properties" (Field, 1987, p. 2379). His analysis of natural images showed that quite different images have a characteristically similar spectrum.

> The amplitude falls off quickly [which] is what we would expect if the relative contrast energy of the image were scale invariant (i.e., independent of viewing distance). ... This falloff ... can also be related to the fractal nature of the luminance profile of the images. (Field, 1987, p. 2385-86)

Field noted implications of fractal properties for information processing by suggesting that "coding a scale-invariant image into an array of scale-invariant sensors produces an even distribution of the information" (Field, 1987, p.

2386). Knill, Field and Kersten argued that to be efficient, "the human visual system should be tuned to the ensemble of images that it sees." (Knill, Field and Kersten, 1990, p. 1113).

Pentland (1984), using computer generated surfaces of increasing fractal dimension over the range $2 < D < 3$, showed that human perception of surface roughness (texture) is "almost perfectly" (91%) correlated with the fractal dimension of the surface. It is apparent that "the fractal dimension corresponds closely to our intuitive notion of roughness" (Pentland, 1984, p. 662). Furthermore, the image of a fractal surface retains the fractal properties of that surface. Because fractal dimension is independent of scale, its value remains constant under changing conditions of projection foreshortening and perspective gradient. Thus Pentland provided an explanation of how the reflectance function of a fractal surface matches the optimal visual signal required for its perception.

Preferential perception of fractal form, then, provides a good explanation of how our visual processing system meets the many constraints associated with providing consistency of information about an apparently variable and complex physical reality.

Perceptual sensitivity

Perceptual sensitivity was defined in this study as the measure of ability to discriminate between discrete instances of highly similar natural imagery. Perceptual sensitivity was measured by the correct short-term visual recall of target images from distractor images.

Brewer (1988) concluded that accuracy and confidence in memory recall were both associated with a high measure of visual imagery, and that variability in recall was directly associated with the "distinctiveness of the representation" (p. 87). Differences in ability at imaginal-memory recall, then, can provide an indication of differences in imaginal-distinctiveness. Baroni, Job, Peron & Salmaso (1980) suggested that when designing memory tests of natural settings, a medium rather than high level of specific focussing is best to preserve memory for structural elements. The instructions for this instrument avoided any reference to specific aspects of the target images on which the subjects might focus.

Two instruments to measure perceptual sensitivity were designed; the first to measure the effect, the second to check that the stimulus was of sufficient strength.

Perceptual sensitivity to natural form

Twenty five slides of a target set were each viewed for two seconds. The set was then randomly re-ordered and randomly distributed amongst fifty-five distractor slides. The eighty slides of the total set were then each viewed for three seconds, which included time allocated for marking a response. Respondents had to mark on an answer grid whether the viewed slide was a member of the target group or not. Correct recognition of a target slide only was scored.

The slides were original photographs of natural forms, grouped into themes: flowers, bushes, trees, mountain-scapes, seascapes, timber and masonry surfaces, textile patterns, forest scenes, garden scenes, and paintings of natural scenery. From each thematic group, one or two were chosen as targets, the remainder as distractors.

Perceptual sensitivity to fractal form

The same design as the *Perceptual sensitivity to natural form* instrument was used with a set of eighty slides of fractal computer graphics. Where a group of slides were taken of a zoom sequence [see below], one or two were chosen as targets, the remainder as distractors.

Fractal computer graphics

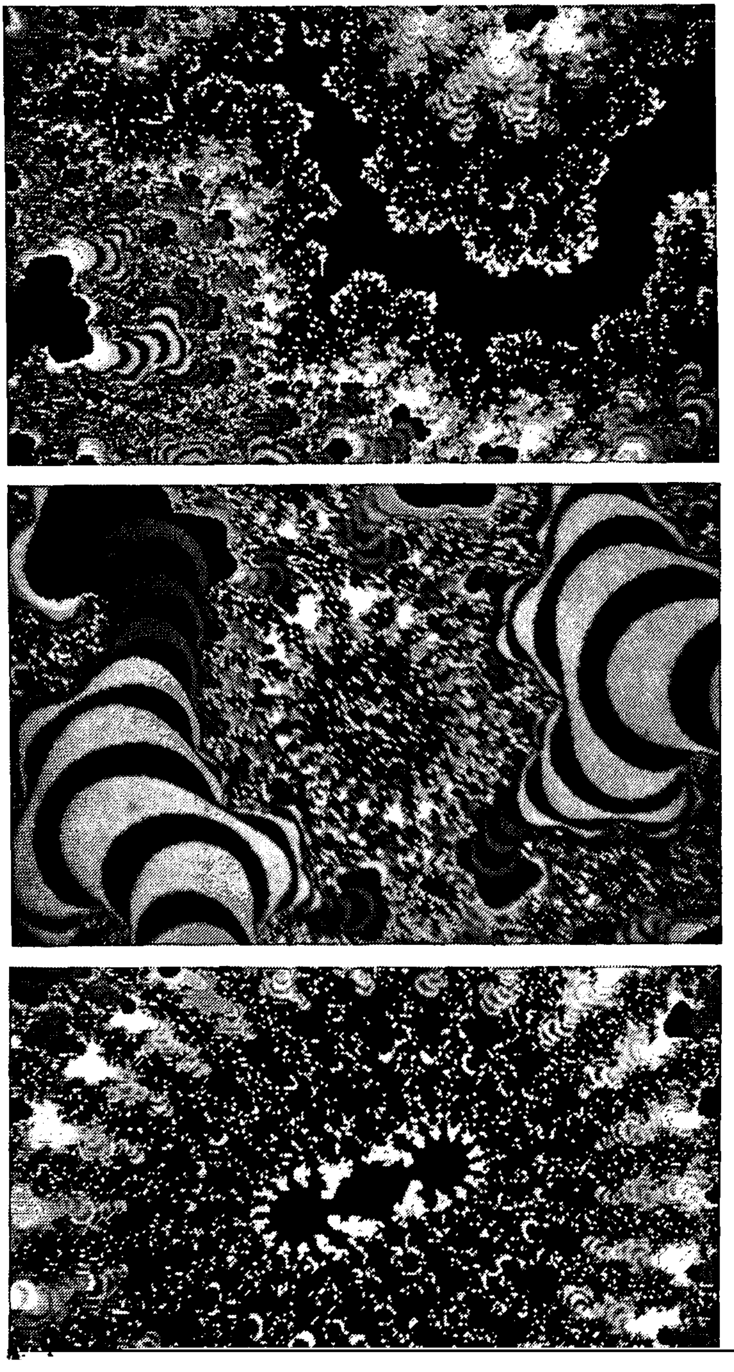
Fractal images are easily generated as computer graphics (Dewdney, 1986; Peitgen & Richter, 1986; Peitgen & Saupe, 1988). Two types of fractal imagery were used in this study, the Mandelbrot set, and Iterated Function System (IFS) fractals.

Mandelbrot set

Mathematical curves with statistical scale invariance are drawn with reference to attractors - points in the plane which the iterative values of some chosen polynomial converge to or diverge from (Peitgen and Richter, 1986). The Mandelbrot Set, {M}, is a fractal curve in the complex plane generated by iterating a simple quadratic function. {M} is a picture of the behaviour of the function for each point for some predetermined large number of iterations. By choosing different colours for the differing divergence rates of points which do not belong in {M}, a very complex picture is generated. The complexity is revealed through "zooming in" on a small section of the boundary, i.e., by repeating the plot over a smaller range of values. Surprisingly, this process can be repeated indefinitely. The images are tantalisingly suggestive of natural shapes and forms (Sorensen, 1984; Peitgen & Richter, 1986; Dewdney, 1987 & 1989; Peitgen & Saupe, 1988). A typical zoom sequence is shown in Figure 1.

The software used in this study was *Fractal Explorer* by Eclat Microproducts (Apple II) and *FractalMagic* by Sintar Software (Atari). Both programs are menu driven and have efficient algorithms for plotting the Mandelbrot set. They have a zoom box to select areas for magnification, and have a slide show capability for displaying previously plotted images. As characterized by all Mandelbrot set programs, plot times increased considerably for higher levels of magnification.

Figure 1 Zoom sequence of a boundary region of the Mandelbrot set (AMYGDALA slide set, 1989)



https://doi.org/10.1017/S0814062600003268 Published online by Cambridge University Press

Iterated function systems

Barnsley (Barnsley and Sloan, 1988) has written simple and short computer programs that can render life-like computer graphic images of natural objects. As above, the image is the attractor of an iterated function. Here the function is a matrix of affine transformations - rotations, scalings and translations of the original image. It is something like remembering the image of a tree having viewed it from many angles. The attractor is composed of a collage of the transformed shapes (Barnsley, 1988).

The software used was *Fractal Imitator*, an original program for Apple II computers. *Fractal Imitator* is a menu driven program which allows the user to plot IFS fractals by inputting the values for the affine transformation matrix. A library of successful images and their codes is available for reference. Resultant images can be magnified and/or reoriented for improvement.

Experimental design

The subjects were 197 Year 6 pupils from seven intact classes in seven Department of School Education primary schools in the North Coast Region. Only one class per school was used in order to prevent diffusion between experimental groups, and to restrict any compensatory effects for different treatments. The locations of the schools were spread throughout the region, with a diversity of socio-economic profiles in order to maximise the generalisability of the results. The gender distribution of the teachers across the treatment groups was equitable. The treatment period was over eight weeks of Term 4, 1990. A repeated measures design was employed. Three classes were allocated to the Mandelbrot set graphics group, and two classes were allocated to the IFS graphics group. Subjects used the programs during class time between normal class activities and immediately before and after class time. They usually worked in small groups to decide input values. Subjects were encouraged to label their images using names of whatever natural phenomena they thought the fractal image reminded them.

One class had an intense environmental education program, planned and delivered by their teacher, which did not involve the use of computers. The unit was based on the *Environmental Education Curriculum Statement K-12* and its accompanying *Resources for Environmental Education*, and used local environmental resources for themes on rainforest industries and coastal management. One class acted as the control group.

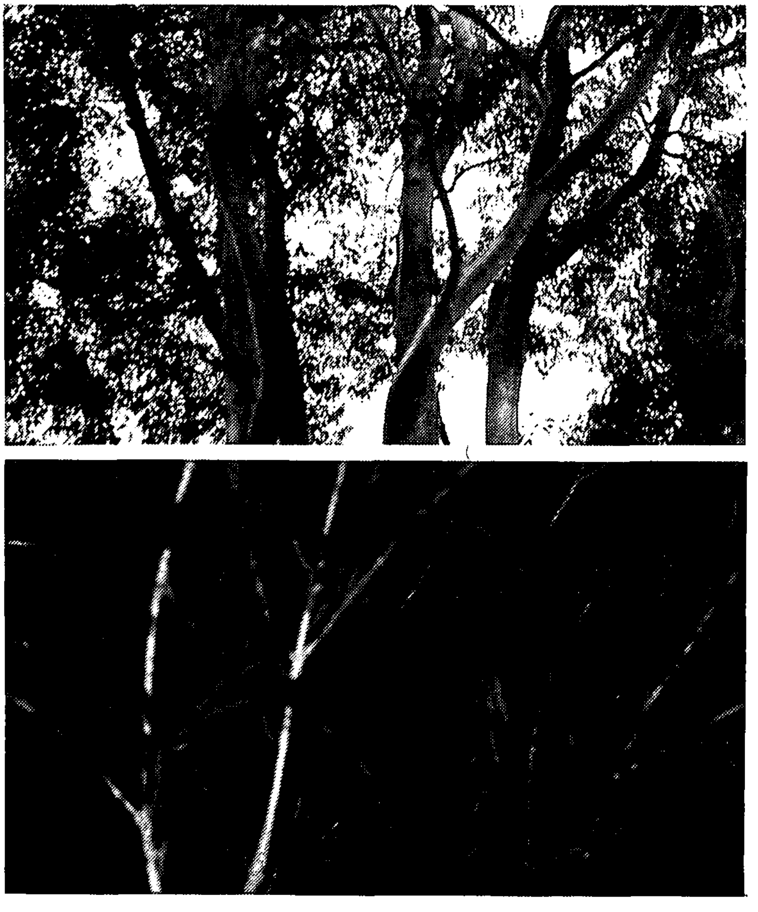
One hundred and eighty subjects in six of these classes were pre-and post-tested for perceptual sensitivity. In the remaining class, which interacted with Mandelbrot set fractal graphics, subjects were asked for subjective written responses on what they thought fractal graphics were like.

For the main-treatment groups, the number of computers per class was boosted for ease of subject access. No subjects or their teachers reported any difficulty in using the software except in a few cases when subjects attempted to save too many files for the available disk space. All participating teachers expressed enthusiasm for the project, and for the enhanced use of computer activities in their classroom.

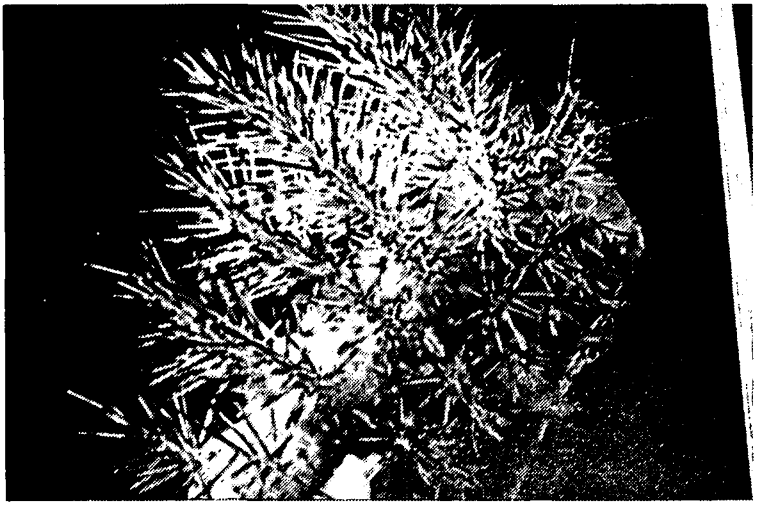
Results

The perceptual sensitivity instruments were first subjected to a principal components analysis with VARIMAX rotation. From inspection of the loaded items for interpretability, three sub-scale variables were created; *profiles* (rocks and mountain profiles), *surfaces* (skin, brick, timber surfaces), and *branching* (tree branches, hairy caterpillars). The commonality of these sub-scale items seemed to be the fractal property of similarity under scaling. The items in the *profiles* sub-scale showed jagged profiles at scales from 1m to 100m. The items in the *surfaces* sub-scale showed surface contours and pitting at scales from 0.1mm to 1cm. The items in the *branching* sub-scale showed branching at scales from 1mm to 10m. Examples of items from this sub-scale are shown in Figure 2.

Figure 2 Examples of natural form showing fractal branching



https://doi.org/10.1017/S0814062600003268 Published online by Cambridge University Press



These component structures were used to form sub-scales whose scores were used as dependent variables (DVs) in supplementary multivariate analyses.

The research question was reformulated as a null hypothesis:

Exposure to a program of fractal computer graphics will not change the environmental perceptual sensitivity of primary school children (one tailed test).

This null hypothesis was tested with a multivariate analysis of covariance (MANCOVA) design with planned contrasts between fractal treatment groups and control groups:

contrast (1) - fractal graphics vs environmental education;
contrast (2) - fractal graphics vs control.

The MANCOVA was carried out using post-test total scores on the *Perceptual sensitivity to natural form* and *Perceptual sensitivity to fractal form* as DVs, and with corresponding pret-est scores as covariates. Inspection of the within-cells regression tables showed the covariates to account for significant variance (*Perceptual sensitivity to natural form* $p < 0.11$, *Perceptual sensitivity to fractal form* $p < 0.01$).

The main effect for contrast (1) was not significant at the multivariate level. There was no significant differences in scores on the *Perceptual sensitivity to natural form* and *Perceptual sensitivity to fractal form* instruments between subjects in the fractal treatment groups and subjects in the environmental education group.

The main effect of treatment with contrast (2) was significant at the multivariate level (Wilk's $\lambda = 0.77$, mv$F = 3.65$, $p < 0.006$, mv$\eta^2 = 0.233$). Over 23 percent of the variance between treatment conditions involving fractal graphics and a control could be accounted for by differences in environmental sensitivity instrument scores.

The univariate tests showed that *Perceptual sensitivity to natural form* (F

(1,64) = 8.08, $p < 0.006$, $\eta^2 = 0.112$) was significant. Scores on the *Perceptual sensitivity to natural form* instrument could account for over 11 percent of the variance in discriminating between the fractal treatment and control groups. The adjusted means of *Perceptual sensitivity to natural form* and *Perceptual sensitivity to fractal form* for the four treatment conditions are reported in Table 1 below.

Table 1 Adjusted means of perceptual sensitivity scores for contrast (2) main effect.

	mandelfract	ifsfract	environ	control
natural form	19.37	17.28	17.85	17.20
fractal form	16.62	14.40	14.90	15.45

The treatment group exposed to Mandelbrot fractal images had higher scores on both these DVs than the control group or other treatment groups.

The null hypothesis was also tested with the same MANCOVA design using the component sub-scale scores from the *Perceptual sensitivity to natural form* instrument as DVs.

The main effect for contrast (1) was not significant at the multivariate level. The main effect for contrast (2) had multivariate significance (Wilk's λ = 0.87, mvF = 3.34, $p < 0.024$, $mv\eta^2 = 0.129$). Scores on the sub-scales derived from the components of the *Perceptual sensitivity to natural form* instrument accounted for nearly 13 percent of the variance in discriminating between the fractal treatment and control groups.

Only the DV *surfaces* showed significance at the univariate level (F (1,70) = 9.24, $p < 0.003$, $\eta^2 = 0.117$). This sub-scale related to surface features accounted for nearly 12 percent of the variance which discriminated between the fractal treatment and control groups. The adjusted means of *profiles*, *surfaces* and *branching* for the treatment conditions are reported in Table 2 below.

Table 2 Adjusted means of perceptual sensitivity sub-scales for contrast (2) main effect.

	mandelfract	ifsfract	environ	control
profiles	2.33	2.04	2.44	2.44
surfaces	2.67	2.69	2.64	2.26
branching	3.65	3.11	3.54	3.57

For *surfaces*, subjects in the two fractal treatment groups scored higher than subjects in the control group.

These findings lead to the rejection of the null hypothesis, and the acceptance of the alternative hypothesis that exposure to a program of fractal computer graphics could increase the environmental perceptual sensitivity of primary school children.

Qualitative study

The subjects were asked to write comments, prompted by the starting phrases: "I think fractals are ... because ... " and "Fractals look like ... ". The most popular completion to "I think fractals are" was "interesting". Other singular comments were "different and strange", "very complicated", "great", "very good" and "really educational". From these comments it seems that the treatment evoked positive responses.

Two comments captured the essence of fractal form: "because they keep on going and going and going and you get some very interesting shapes out of them" and "because fractals are so good we can't stop using them. In about a year fractals could take 1 week." This comment seems to refer to the increasing time to compute the plot for increasing levels of magnification. This subject continued: "fractals are good when they're printed because you can look at where you zoomed and you can find the other fractal." Some comments anticipated the second prompt: "because they ... resemble things in nature" and "because they look like natural objects found in nature, also some man made objects."

> "Fractals look like ..." typically elicited lists of objects:
> "trees, flowers, dragons, Aboriginal paintings, pathways, feathers, insects, elephants";
> "trees, zippers, planets, sea horses and other animals in the sea";
> "cactus plants, squids, braid, tinsel, branches, trees, zippers, sea horses and plants";
> "trees, peacock feathers, zips, squids, bugs, snowmen on fire";
> "leaves, root systems, flowers, star constellations, solar systems, and star fish and other creatures";
> "tree roots, electricity, sea horses, octopuses, spiders and other insects";
> "coastal strips, zippers, beetles, snowmen, lightening, dragons, elephants, aliens, fish, bones, carcasses, and black holes from outer space."

Whereas some items commonly reported, such as "zippers" and "sea horses" may have been evoked by a preceding class discussion, and are more apparent in the fractal images that this group generated, the variety of objects is of note, as is the frequency of "trees", which were not so objectively obvious in the graphics.

Other children, rather than make lists, were able to generalise. "Fractals look like all different types of weird and wonderful shapes"; "Fractals look like

https://doi.org/10.1017/S0814062600003268 Published online by Cambridge University Press

the things around you ... the way they go in big spirals"; "Fractals look like things in the earth like natural things"; and "Fractals look like things in our world."

Discussion

Given that the extraneous variables of classroom conditions seemed more likely to reduce the treatment effect rather than to produce a spurious enhancement, the naturalistic setting was a strength for the generalisability of the results.

From the MANCOVA, environmental perceptual sensitivity was higher for subjects exposed to fractal images than for subjects in the control group. This result is consistent with the findings of Voss (in Gardner, 1983), Pentland (1984), Field (1987) and Knill, Field and Kersten (1990), that the human visual system is tuned to the perception of fractal form. This finding is also consistent with the results of Peli (1990) on the contrasting imaging requirements for natural and man-made objects. That the sub-scales derived from the perceptual sensitivity instrument components could be interpreted by recourse to their fractal properties, adds support to the case for a visual predilection to fractal form. Moreover, these results show that the acuity of such perception can be improved in an educational setting. This is believed to be a new finding.

The written responses of subjects indicated a strong perceived relationship between Mandelbrot set fractals and images from the natural world. Rejeski (1982) reported that children from the earliest stage, when asked to portray what "nature" meant to them, drew a tree. Rejeski interpreted this as a recognition by children of the central function of trees within natural systems. However, science educators investigating children's science concepts have generally found that concepts relating to function are not well formed with respect to conventional science, particularly with young children, and especially with trees (Barker in Butts, 1988). Now trees display fractal form through their branching. In fact, "branching" was the first component produced from the component analysis of the perceptual sensitivity instrument. Rather than function, could children's use of the tree as an archetypal image be indicative of an unconscious recognition of the ubiquity of fractal form within natural phenomena? Such a conjecture is supported by the written responses, wherein trees were a popular choice of what fractals looked like, even though the computer graphic imagery was not directly tree-like.

Furthermore, the null result from the contrast between the groups exposed to fractal computer graphics and those exposed to natural imagery during a conventional environmental education program strengthens this interpretation. It also suggests that computer technology is unnecessary to achieve improvements in perceptual acuity to natural phenomena, providing that a rich source of visual experience, such as a rainforest, is readily available for access by students. This argument could be extended into the general area of wilderness preservation - we enjoy looking at such scenic vistas because they are what we have evolved to see best. Such a perceived relationship is the subject of a recent book featuring photography of natural scenery, *Nature's Chaos*, by J. Gleick and E. Porter (1990).

Implications for education

Winn's comment that "instructional designers have a large stake in the imagery question" (Winn, 1980, p. 130), has usually been addressed by designers of educational material with the simplification of visual stimuli by line drawings, cartoons, etc. Milne articulated the concern regarding realism:

> How do students learn from the perception of photographic imagery? ... there is lacking any substantive theory that seeks to demonstrate the degree to which the use of pictorial representations of environmental stimuli can be used to develop ... understanding (Milne, 1979, p. 328)

The results of this study may help provide an answer. The visual systems of students are tuned to perceive natural imagery more readily than other forms of imagery. Photographic realism of natural scenes should be utilised preferentially to simplified line drawings wherever possible in the classroom. Such use may even improve the acuity to such perception. This could be of particular benefit to environmental education programs.

Geake (1990b) has presented several arguments to support the introduction of aspects of Chaos Theory into school curricula: the subject area is contemporary; its study necessitates exploring the unknown; it relates directly to the natural world; there is a tangible aesthetic dimension; it is attractive to able students; and it makes good use of computers. Several of these points are relevant to these results.

Studying fractals as models of natural form could strengthen the argument that mathematics and science are relevant to students' everyday world. Such an understanding of the structure of natural systems should be of particular interest for those many students who hold a concern for the fate of our environment (Sia, 1985). It has also been suggested that the subject "fractals" is essentially interdisciplinary (Egnatoff, 1989; Geake, 1990b). A study of fractal form, then, could provide another linkage between environmental education and mathematics and the traditional sciences. Certainly the enthusiastic response of the subjects in this study to the beautiful shapes and intricate patterns of the Mandelbrot set computer graphics suggests that fractal graphics make extremely engaging teaching material.

Conclusion

There is evidence for the conjecture that humans have a predilection for the perception of fractal form in imagery of the natural environment.

Within an educational framework, there is evidence that fractal computer graphics can play a positive role in stimulating improvement in such perception in classroom settings.

Acknowledgements

I wish to thank Professor Don Fitzgerald, Centre for Research on Information Technology in Education, University of New England - Armidale, for his assistance with the planning and analysis of the study. I am indebted to the Year 6 children, their teachers and principals, of the Alstonville, Ballina, Bexhill, Casino, Casino West, Clunes, Lismore Heights and Lismore South Public Schools, for their enthusiastic participation in the study.

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