

BOOK REVIEWS

BENSON, D. J., *Representations and cohomology*, Vol. 1: Basic representation theory of finite groups and associative algebras (Cambridge Studies in Advanced Mathematics 30, Cambridge University Press 1991), pp. xii+224, 0 521 36134 6, £25.

This is the first of two volumes on representation theory and cohomology of groups. As such, a good part of the book consists of preparatory material. In particular the first three chapters give a rapid introduction to the necessary background in rings and modules, homological algebra and modules over group algebras. It is necessary to have a reasonable background in algebra (the author suggests the three-volume work of Jacobson), as the pace is rather brisk.

The main part of this first volume is Chapter 4 which gives an introduction to the Auslander–Reiten style representation theory of finite-dimensional algebras: quivers are introduced and those of finite representation type or tame representation type are classified via Dynkin and Euclidean diagrams; almost split sequences are discussed and the important theorems of Roiter, Reidtmann and Webb are proved.

Chapter 5 introduces representation rings and various induction theorems are proved. Chapter 6 studies the Block Theory of a group algebra using the Auslander–Reiten theory from Chapter 4. The classical case of blocks with cyclic defect group is treated and also the more modern work of Erdmann analysing tame blocks is presented.

I found this to be a good book to read. Much of the material is quite technical and consequently the proofs may require careful reading. However, there is sufficient commentary around the proofs so that the reader is able to keep a clear idea of where the treatment is heading.

T. H. LENAGAN

BASTON, R. J. and EASTWOOD, M. G., *The Penrose transform: its interaction with representation theory* (Oxford Mathematical Monographs, Clarendon Press, Oxford 1989), pp. 232, 019 853565 1, £25.

One of the early successes of Penrose's "Twistor programme" to reformulate fundamental physics on Minkowski space in terms of the holomorphic geometry of "twistor space", $\mathbb{P} = \mathbb{C}\mathbb{P}^3$, was the description of solutions of the massless field equations (an important series of equations on Minkowski space including the wave equation and the source-free Maxwell equations) in terms of contour integrals of homogeneous holomorphic functions on regions of \mathbb{P} . This process, which has since become known as the "Penrose transform", can be carried out in a far more general geometrical setting. Said very briefly, the Penrose transform as considered here is a machine, analogous to the Radon or Fourier transforms, which relates sheaf cohomology groups on one space to kernels and cokernels of differential operators on another—where the spaces involved are quotients of complex semi-simple Lie groups by parabolic subgroups. I will return to this case shortly, but it is helpful first to consider the transform in greater generality.

The basic setting in which the Penrose transform exists is that one has a "correspondence" between two complex manifolds X , Z via an intermediate complex manifold Y

$$Z \xleftarrow{\pi} Y \xrightarrow{\sigma} X$$