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ON THE COMPUTATIONAL ALGORITHMS FOR TIME-LAG OPTIMAL CONTROL PROBLEMS

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In this thesis we study the following two types of hereditary optimal control problems:

(i) problems governed by systems of ordinary differential equations with discrete time-delayed arguments appearing in both the state and the control variables;

(ii) problems governed by parabolic partial differential equations with Neumann boundary conditions involving time-delays.

The aim of this thesis is to devise computational algorithms for solving the optimal control problems under consideration. However, our main emphases are on the mathematical theory underlying the techniques, the convergence properties of the algorithms and the efficiency of the algorithms.

Chapters II, III, IV, and V deal with problems of the first type and Chapter VI deals with problems of the second type. Several numerical problems have been included in each of these Chapters to demonstrate the efficiency of the algorithms involved.

The class of optimal control problems considered in Chapter II consists of a nonlinear hereditary system, a nonlinear hereditary cost functional and a bounded control region. The conditional gradient technique is used to devise an iterative algorithm for solving this class of problems. The type of convergence of this algorithm is as follows:

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Let $\{u^k\}$ be a sequence of admissible controls generated by the algorithm. Then, any accumulation point of the sequence $\{u^k\}$ in the L_∞ topology, if it exists, satisfies a necessary condition for optimality.

The class of optimal control problems considered in Chapter III is obtained from that of Chapter II by adding a finite number of terminal inequality constraints. In other words, Chapter III extends the algorithm in Chapter II to the case where there is a finite number of terminal inequality constraints to be satisfied also. The algorithm is known as the feasible directions algorithm. A similar type of convergence property to that of Chapter II is obtained. The only difference is that the sequence $\{u^k\}$ of controls generated by the algorithm is now a sequence of *feasible* (rather than just admissible) controls. Since the algorithm is valid only if the initial control, u^1, is a feasible control, a method based on the control parameterization technique together with a modified Newton algorithm is given. This method can be used to construct a feasible control.

A computational scheme using the technique of control parameterization is developed for a linear time-lag system and a nonlinear time-lag system in Chapter IV and Chapter V respectively. In both cases, a nonlinear cost functional and linear control constraints are involved. However, the linear problem consists also of linear terminal equality and inequality constraints. By approximating the control variables by piecewise constant functions with the instants of switching preassigned, both the linear and nonlinear optimal control problem can be converted into a sequence of finite dimensional optimization problems. Each of these approximate problems can be solved by a standard quasi-Newton method, such as the one presented in Appendix B.

For the linear problem, we impose some convexity conditions on the cost functional. On this basis, the following convergence property is obtained:

The sequence of optimal controls of the approximate problems has a subsequence, which converges to the true optimal control of the original problem in the weak* topology of L_∞.

For the nonlinear problem, a weaker convergence result is obtained, namely:

If the sequence of optimal controls of the approximate problems has an accumulation point in the almost everywhere topology, then this accumulation point is an optimal control of the original problem.

In Section 5.5 we modify our methods for the nonlinear problem by allowing the instants of switching to be parameters, also.

In Chapter VI, we consider a class of optimal control problems involving a second order linear parabolic partial differential equation with Neumann boundary conditions. The time-delayed arguments are assumed to appear in the boundary conditions. For these problems, answers to four major questions arising in the study of optimal control theory are possible, namely:

(i) necessary conditions for optimality;
(ii) sufficient condition for optimality;
(iii) existence of an optimal control;
(iv) an efficient algorithm for constructing an optimal control.

With reference to the algorithm concerned, the following convergence result is obtained:

The sequence of controls generated by the algorithm converges to the true optimal control, both in the weak* topology of L_∞ and in the almost everywhere topology.

In Section 6.9 the finite-element Galerkin's scheme is employed. This technique converts the original distributed optimal control problem into a sequence of approximate problems involving only lumped parameter systems. A computational algorithm is then developed for each of these approximate problems. For illustration, a one-dimensional example is solved.

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