EVOLUTIONARY MODELS OF NUCLEOSYNTHESIS IN THE GALAXY

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Abstract. A model of the galaxy is constructed and evolved in which the integrated influence of stellar and supernova nucleosynthesis on the composition of the interstellar gas is traced numerically. Our detailed assumptions concerning the character of the matter released from evolving stars and supernovae are guided by the results of recent stellar evolutionary calculations and hydrodynamic studies of supernova events. Stars of main sequence mass in the range $4 \leq M \leq 8\ M_\odot$ are assumed to give rise to supernova events, leaving remnants we identify with neutron stars and pulsars and forming both the carbon-to-iron nuclei and the *r*-process heavy elements in the explosive ejection of the core material. For more massive stars, we assume the core implosion will result in the formation of a Schwarzschild singularity, that is, a black hole or 'collapsar'. The straightforward assumptions (1) that the gas content of the galaxy decreases exponentially with time to its present level of $\sim 5\%$ and (2) that the luminosity function characteristic of young clusters and the solar neighborhood is appropriate throughout galactic history, lead to the prediction that $\approx 20\%$ of the unevolved stars of approximately one solar mass ($M_\odot$) in the galaxy today should have metal compositions $Z \lesssim 0.1\ Z_\odot$. As Schmidt has argued from similar reasoning, this is quite inconsistent with current observations; an early generation dominated by more massive stars – which would by now have evolved – is suggested by this difficulty. Many of these massive stars, according to our assumptions, will end their lives as collapsed black hole remnants. It is difficult to visualize an epoch of massive star formation in the collapsing gas cloud which formed our galaxy which would enrich the gas rapidly enough to account for the level of heavy element abundances in halo population stars; we have therefore proposed a stage of star formation which is entirely pregalactic in character. We suggest that the Jeans' length-sized initial condensations in the expanding universe discussed by Peebles and Dicke may provide the appropriate setting for this first generation of stars. Guided by these considerations, and by the need for a substantial quantity of 'unseen' mass to bind our local group of galaxies, we have constructed a model of the galaxy in which this violent early phase of massive star formation produces both (1) approximately 25% of the level of heavy elements observed in the solar system and (2) an enormous unseen mass in the form of black holes. The implications of our model for other features of the galaxy, including supernova nucleosynthesis, the cosmic ray production of the light elements, and cosmochronology, are discussed in detail.

1. Introduction

The evolution of the galaxy represents a very complex and encompassing problem in astrophysics. The details of its mode of origin and early history, tied presumably to considerations of cosmology, have been clouded by the effects of the subsequent 12 billion years of galactic evolution. Even our knowledge of the current state of the galaxy is rather uncertain and incomplete. Any general theory of the evolution of the galaxy will therefore necessarily constitute a synthesis of a large number of individual theories which describe particular phenomena in galaxies – for example, theories of stellar evolution, nucleosynthesis, star formation, cosmic ray origin and the behavior

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of the interstellar medium. As many aspects of these theories are only imperfectly developed at the present time, it might be considered rather premature to attempt a grand synthesis of such theories to account for the evolution of the galaxy as a whole. However, we have found it very instructive in our own research to attempt to find a *consistent* set of assumptions involving these various theories which predicts a large number of observed properties of the galaxy (Truran *et al.*, 1965; Truran and Cameron, 1970; Cameron and Truran, 1971). Such a set of assumptions provides a set of predictions which can be investigated in further research; if any one of these assumptions should lead to inconsistencies with observation, then it is very likely that others of our assumptions must also be modified. Proceeding in this manner, we believe that a useful contribution to galactic research can be made. We must bear in mind that we are in a very real sense playing a game, the rules of which are only rather vaguely defined. It is quite conceivable that models of the galaxy based on very different physical assumptions could provide reasonable fits to the observed features.

Determinations of the concentrations of the products of stellar and supernova nucleosynthesis (Burbidge *et al.*, 1957; Cameron, 1957) in the solar system, in stellar spectra and in the interstellar medium provide certainly the most extensive and perhaps the most reliable observational tests of the consequences of our galactic evolutionary models. Many of our assumptions therefore deal with the consequences of stellar evolution with regard to the various processes of nucleosynthesis; a detailed discussion of these assumptions is presented in Section 2. Building upon these assumptions, we have traced numerically throughout galactic history the changes in the chemical composition of the galaxy that take place as a result of stellar evolution and the interchange of gas between stars and the interstellar medium. We do not explicitly consider problems associated with galactic structure.

The rather straightforward relations which govern this evolution have been defined in previous investigations (Schmidt, 1959, 1963; Salpeter, 1959; Truran *et al.*, 1965) and will not, therefore, be elaborated here. The general approach may be outlined briefly as follows. At any time t in galactic history a prescribed mass of interstellar gas is formed into stars, their distribution in mass being specified by some chosen stellar luminosity function. These stars are then allowed to burn for a lifetime $\tau(M)$ appropriate to their mass; upon reaching the endpoint of their evolution at time $t+\tau(M)$, these stars are assumed to return some fraction of their mass to the interstellar medium, enriched in various products of nucleosynthesis. In our models, which assume the galaxy to be structureless and homogenous, this enriched gas is then instantaneously mixed with the existing interstellar gas. Stars formed at time $t+\tau(M)$ will thus be composed initially of gas enriched in the products of nucleosynthesis resulting from previous stellar generations. This cycling and recycling is a continuing process in all our models; the rate at which gas is formed into stars and the prescribed luminosity function are the distinguishing features. A discussion of the chosen form of the luminosity function is presented in Section 3.

The results of our 'conventional' model of galactic evolution are described in Section 4. For this case, we have chosen what we find to be the most straightforward a priori assumptions one can make regarding the rate of star formation and the distribution of stars as a function of mass. These assumptions lead to a very profound conflict with current observations of the general level of heavy element abundances in stars. In the construction of our 'adopted' galactic model, we have modified our initial assumptions regarding both the rate of star formation throughout galactic history and the variation of the stellar luminosity function with time in order to avoid this conflict. In arriving at our final assumptions, we have been guided as well by the need to account for the substantial quantity of missing mass required to bind our local group of galaxies (Oort, 1970). A detailed discussion of these problems is presented in Section 5, together with a description of our adopted galactic model. Various consequences of this model, including its implications for nucleosynthesis and cosmochronology, are presented in Sections 6 through 9.

2. Stellar Evolution, Supernova Explosions and Nucleosynthesis

The most crucial and extensive assumptions required for our discussion of galactic evolution involve the consequences of stellar evolution for the various processes of nucleosynthesis. Our adopted stellar evolution assumptions are summarized in Figure 1, where we have indicated the compositional structure of stars of various mass at the end point of their evolution. In Table I, the precise mass fractions determined

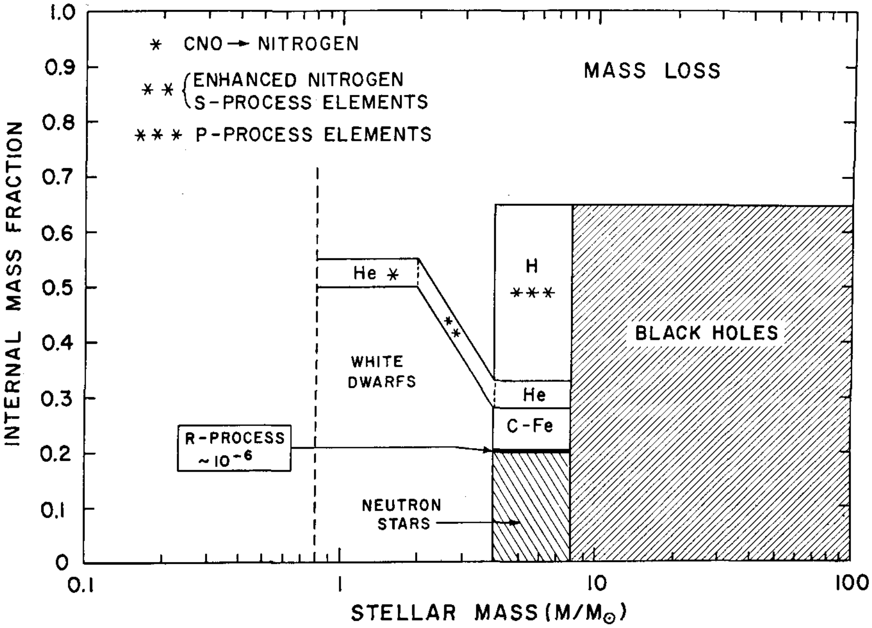

Fig. 1. The adopted compositional structures for stars in their final stages of evolution are shown. The fractional stellar masses, both in the appropriate remnant and in various nuclear burning zones, are indicated as a function of main sequence mass. The specific fractions shown are for the final consistent model of Section 5.

TABLE I

Compositional structure at end of life

Mass Range	Description	Mass Fractions		
			Consistent Model	
		Conventional Model	First Generation	Subsequent Generations
$M > 8\ M_\odot$	Black Hole Remnant	0.65	0.95	0.65
	Mass Loss	0.35	0.05	0.35
$4 \leq M \leq 8\ M_\odot$	Neutron Star Remnant	0.20	0.20	0.20
	r-Process Synthesis	1.3×10^{-6}	1.4×10^{-6}	1.4×10^{-6}
	Carbon-to-Iron Synthesis	0.075	0.08	0.08
	Helium Shell	0.05	0.05	0.05
	Hydrogen Envelope [a]	0.325	0.32	0.32
	Mass Loss	0.45	0.45	0.45
$M < 4\ M_\odot$	White Dwarf Remnant	0.50 (1.12 $M_\odot$ maximum)		0.50 (1.12 $M_\odot$ maximum)
	Helium Shell [b]	0.05		0.05
	Mass Loss	(Remainder)		(Remainder)

[a] Site of *p*-Process Synthesis: 50% of the primordial *r*-process and *s*-process nuclei are converted to *p*-process nuclei

[b] Site of *s*-Process and Nitrogen Synthesis: (a) all initial CNO-nuclei converted to nitrogen in masses $1 \leq M \leq 2\ M_\odot$; (b) for stars of mass $2 \leq M \leq 4\ M_\odot$, 6.3% of the mass is converted to nitrogen and 0.375% of the C–Fe nuclei are *s*-processed (4.7% and 0.375%, respectively, for the Conventional Model).

both for the conventional model described in Section 4 and for our final 'consistent' model described in Section 5 are presented. In this section we will elaborate various of these assumptions in terms of recent theoretical studies of stellar evolution, supernova hydrodynamics and nucleosynthesis.

A. WHITE DWARFS, NEUTRON STARS AND BLACK HOLES

The many excellent theoretical calculations of stellar evolution are rather minimally useful to us in our present study. The reason for this is that such calculations have not generally carried forward the evolution of the star to its final configuration; one is therefore forced to guess how the subsequent evolution of stars of various mass will influence the final chemical structure. Specifically, we must make reasonable estimates of the characteristics of the final stages of evolution of stars of all masses and determine both the range of stellar masses which is responsible for supernova events and those which populate the stellar graveyards – white dwarfs, neutron stars, and black holes. Our assumptions in these matters have been strongly influenced by recent studies of stellar evolution by Paczynski (1970) and Barkat (1971), by studies of supernova hydrodynamics (Arnett, 1969a; Hansen and Wheeler, 1969; Wheeler and Hansen, 1971; LeBlanc and Wilson, 1970), and finally by calculations of explosive nucleosynthesis (Truran *et al.*, 1967; Arnett, 1969b; Truran and Arnett, 1970; Arnett *et al.*, 1971).

Recent stellar evolutionary calculations by Paczynski (1970; also see Rose, 1969) indicate a convergence of the evolutionary tracks for stars in a specific range of parent masses $4 \leq M \lesssim 8\ M_\odot$. Barkat (1971) has explained this convergence and shown it to be quite general, therefore defining a rather unique pre-supernova model in this range of parent masses. Building on this model Arnett (1969a) has shown that carbon ignition taking place under degenerate conditions in the core will result in a detonation and supernova explosion which, in his view, totally disrupts the star. The question of total disruption of these cores has been considered in greater detail by Colgate (1970) and by Barkat *et al.* (1971), who suggest that rapid beta decay processes may cause part of the detonated core to reimplode to form a remnant while the remainder is expelled into space. We find this latter behavior somewhat more appealing, primarily because the results of an analysis of pulsar statistics by Gunn and Ostriker (1970) suggests that the parent stars were principally Population I objects, involving stars more massive than $\sim 4\ M_\odot$. This is also quite in agreement with the conclusion drawn by Shklovsky (1968) from a study of supernova nebular remnants that most stars more massive than approximately $5\ M_\odot$ give rise to Type II supernova events.

The range of stellar masses $4 \leq M \leq 8\ M_\odot$ has thus been adopted in our calculations as attributable to supernova events which give rise to neutron star remnants or pulsars. Rather independent of the previous considerations, there are indications from our model that this is the appropriate mass range. If we were to include stars of mass much less than $\sim 4\ M_\odot$ as stars which give rise to supernova events then, as we shall see, the supernova rate predicted for the present epoch of galactic history would be very much increased over the observed value of roughly one event per 25 years (Tammann, 1970). The determination of the upper mass cutoff for the supernova events is much less certain. However, two factors have influenced our decision in this matter. The first of these is the allowed mass of neutron star remnants. Recent theoretical calculations (see for example the review article by Cameron (1970)) indicate that neutron star masses much in excess of $\sim 2\ M_\odot$ are not realistic. Therefore, if we were to increase the upper limit from 8 to say $20\ M_\odot$, the fraction of the star which would give rise to neutron star remnants would have to be rather small – at $20\ M_\odot$, for example, being no more than 10% of the mass and perhaps less. This would seem to imply a rather large amount of explosive nucleosynthesis in the regions immediately surrounding these cores expelling a substantial quantity of heavy elements into space. Our calculations have indicated that only some 7 or 8% of the mass in the range $4 \leq M \leq 8\ M_\odot$ ejected in the form of explosive nucleosynthesis products is necessary to account for the observed solar system abundances of elements between carbon and iron – the dominant constituents of the heavy elements. If this broader mass range is assumed to correspond to the parent stars giving rise to supernova events then, independent of whether a neutron star remnant was left behind or whether the material was entirely detonated as Arnett has suggested, it is clear that only a very small fraction of the mass (less than a few percent) could possibly be expelled as heavy element enriched material. This seems a rather low mass fraction to be exposed to the range of temperature-density conditions required for explosive nucleosynthesis.

Having specified the range of stellar masses which gives rise to supernova explosions, we must now consider the fate of stars both more massive and less massive than this limited region. We have assumed that stars less massive than 4 $M_\odot$ will all ultimately give rise to white dwarf remnants. Roughly half of the mass of these stars is assumed to have been lost by mass loss mechanisms during the more stable stages of stellar evolution. The helium zone is also assumed to have been lost, perhaps by means of a nova-type event which results in the formation of planetary nebulae. The masses of the white dwarf remnants cannot exceed at any point the Chandrasekhar (1939) limit of ~ 1.44 $M_\odot$. Hence the mass fraction lert as a white dwarf star is assumed to decrease for larger masses in the manner shown in Figure 1.

The ultimate fate of stars more massive than 8 $M_\odot$, the upper limit on our supernova masses, is far more uncertain. For these stars, carbon- and oxygen-burning typically commences in a non-explosive manner. Hydrodynamic studies of the behavior of more massive cores, cores for example in excess of 3 or 4 $M_\odot$ (Colgate and White, 1966; Arnett, 1967; Wilson, 1971) indicate that these stars will not readily give rise to an explosive event. We have assumed that stars in this mass range, upon reaching the end of their evolution, will find no energy source available sufficient to halt the collapse of their core, and will therefore continue their gravitational collapse to form a Schwarzschild singularity or black hole remnant. The total mass involved in such remnants for the usual luminosity function (Limber, 1960) is rather small. However, our final model, which considers the possible production of only more massive stars during the earliest history of the galaxy, will be far more sensitive to our assumptions concerning these black hole remnants. To the extent that we cannot formally justify these assumptions, our subsequent calculations are inherently uncertain.

Detailed calculations of supernova nucleosynthesis (Arnett *et al.*, 1971) have indicated that the material expelled in these detonation events which give rise to supernovae will typically be composed of elements from carbon to iron. The production of very heavy nuclei by a rapid neutron capture process is not predicted by these calculations. An interesting possible mechanism for the production of these nuclei is suggested by recent work of LeBlanc and Wilson (1970). They have carried out a two-dimensional hydrodynamic calculation of the collapse of a star, allowing for the presence both of rotation and of magnetic fields in the interior. They found that an initially co-rotating star developed differential rotation after the collapse had taken place; consequently the rotational shear in the inner part of the resulting disk wrapped the magnetic field lines into a very tight spiral, creating an enormous magnetic energy close to the central axis of the collapsed star. The excess magnetic pressure then caused an expansion of the material along the axis which, owing to the resulting buoyancy of the material with respect to the local neighborhood, caused the ejection of jets of material along the axis of rotation. LeBlanc and Wilson have estimated that these jets contain roughly 10^{-2} $M_\odot$ of material (for an initial model of 7 $M_\odot$) which has been compressed to a very high density, so that it may be neutron rich. We find this a possible mechanism for *r*-process nucleosynthesis, as will be discussed in a subsequent section.

B. STELLAR MASS LOSS

One of the greatest uncertainties in stellar evolution theory is that concerning the magnitude and influence of mass loss. Red giant stars are observed to be losing mass at a prodigious rate, often as much as $10^{-6}\,M_\odot$ per year (Deutsch, 1969). Current stellar evolution calculations indicate that in clusters in which stars of about 1 $M_\odot$ are turning off from the main sequence, the later horizontal branch stars have masses in the vicinity of 0.5 $M_\odot$ (Iben and Rood, 1970). Consequently, mass losses of up to perhaps half the original main sequence stellar mass appear to be indicated by the current interplay of theory and observation. There is no good quantitative estimate of the total amount of mass lost from more massive stars during the course of their evolution, but such stars should certainly lose a good deal of mass in the course of their red giant phases. As they evolve more rapidly, however, the extent of mass loss may be reduced relative to lower masses. We have therefore adopted the following prescription for stellar mass loss: for all stars of mass $M > 4\,M_\odot$, the loss of 35% of the initial main sequence stellar mass is assumed, while for $M < 4\,M_\odot$ all mass above the helium-burning shell source is lost non-explosively. In the latter case we do not distinguish between stellar wind mass loss and the formation of planetary nebulae.

There is evidence that stars lose even greater amounts of mass than this, since, for stars with masses in the general vicinity of one solar mass, the pre-main sequence T Tauri phase is one in which extremely rapid rates of mass loss occur (Kuhi, 1966). However, in the luminosity functions used in this study, which have been estimated by Salpeter (1955, 1959) and Limber (1960), the distribution of stellar masses on the main sequence is given. It is therefore appropriate to ignore the mass loss which occurs prior to the main sequence, since the mass involved is promptly lost back to the interstellar medium with essentially no nuclear transformation occurring within it. There is no net effect of such mass having been incorporated, however briefly, in stars at all.

C. NUCLEOSYNTHESIS

An important test of the consistency of our model of galactic evolution with observations is provided by considerations of nucleosythesis. Guided by the results of recent calculations of stellar evolution and supernova hydrodynamics, we have previously specified the ranges of main sequence mass which give rise to white dwarfs, neutron stars (and supernova events) and collapsed remnants or black holes. We must now consider in detail the chemical composition of the matter released from these stars both by less violent mass loss mechanisms and by explosive ejection. The concentrations in the interstellar medium and in stars of the products of five distinct processes of nucleosynthesis have been traced throughout our galactic histories. These are (1) the abundances of nuclei from carbon to iron (C–Fe), assumed here to have been formed collectively by explosive charged-particle nucleosynthesis; (2) N^{14}; (3) the *s*-process elements; (4) the *r*-process elements, and (5) the *p*-process elements.

C-Fe *Synthesis*

The synthesis of elements from carbon through the iron equilibrium peak has been demonstrated to take place under explosive burning conditions resulting from the thermonuclear ignition of material in stellar cores in supernova events (Truran *et al.*, 1967; Arnett, 1969b; Truran and Arnett, 1970; Arnett *et al.*, 1971). The stellar matter must be comprised initially of roughly comparable concentrations of C^{12} and O^{16} (resulting, presumably, from a prior helium burning phase); it must then be subjected to a range of post shock temperature-density conditions $T \sim 2\text{–}6 \times 10^9$ K and $\varrho \sim 10^5\text{–}10^7$ g cm^{-3}. Although no existing hydrodynamic calculation of a supernova event predicts this precise range of conditions, the very satisfactory agreement of the abundances resulting from these explosive burning conditions with the solar system abundances (Cameron, 1968) seems suggestive. We have thus assumed that this will occur in the outer layers of the cores of those stars which give rise to neutron star remnants. This involves the additional assumption that the detonation of the cores of stars in this mass range (Arnett, 1969a; Wheeler *et al.*, 1970; Buchler *et al.*, 1971) does not result in complete disruption. The mass fraction assigned to this process in Figure 1 (8% of the total stellar mass) has provided throughout our investigations a very reasonable representation of the build-up of the carbon-to-iron elements in the galaxy, and only rather minor adjustments in our assumptions have been necessary to improve agreement with solar system abundances. (The somewhat larger fraction quoted by Cameron and Truran (1971) was the result of a programming error which misrepresented the build-up of this range of nuclei).

Nitrogen Synthesis

The nucleus N^{14} constitutes a major exception to our assumption that the C–Fe nuclei are formed by explosive nucleosynthesis. N^{14} is not formed in the helium-burning phase which produces C^{12} and O^{16}; further, even for explosive conditions which only incompletely burn the initial C^{12}, no substantial N^{14} concentration is achieved. It is known to be produced, however, when the primordial C^{12} and O^{16} in a star act as catalysts in the conversion of hydrogen to helium – the CNO-cycle, which occurs at the relatively high temperatures characteristic of hydrogen-burning shells (Caughlan and Fowler, 1962). Thus, the helium layer in a star in an advanced stage of evolution can be expected to have had its initial carbon and oxygen abundances completely converted to N^{14}. We have assumed that this 'secondary' production of N^{14}, building on the C^{12} and O^{16} formed in previous generations of stars, has taken place in the helium zones for stars of mass less than 2 $M_\odot$.

It is also possible that a considerably enhanced production of N^{14} can take place for stars which undergo helium-burning shell flashes in their advanced stages of evolution (Schwarzschild and Harm, 1965, 1967; Weigert, 1966; Rose, 1966, 1967). According to these investigators, the helium shell flashes lead to the formation of a convection zone extending through the helium zone toward the hydrogen layer. The more extensive calculations of Schwarzschild and Harm (1967) for a 1 $M_\odot$ star indicated that a significant exchange of material between the hydrogen and helium layers might result from convective mixing. We have assumed that this mechanism can result in enhanced nitrogen production for stars of mass 2–4 $M_\odot$ (the calculations of Weigert (1966) referred to a star of 5 $M_\odot$) by mixing C^{12} formed in the helium zone into the region of the hydrogen-burning shells where the carbon-nitrogen-oxygen cycles are operating. We find that a mass fraction $f_{14}=0.063$ of the helium zone must have been transmuted to N^{14} for stars in this mass range to account for the present level of N^{14} in solar system material. This implies a significant enhancement over the amount of N^{14} resulting from the conversion of only the primordial C^{12} and O^{16} in these stars.

s-Process Synthesis

There are three processes which are primarily responsible for the production of the heavy elements beyond the iron equilibrium peak ($A \gtrsim 60$–70). These are neutron capture on a slow time scale (the *s*-process of Burbidge *et al.*, (1957)), neutron capture on a rapid time scale (the *r*-process) and rapid proton capture (*p*-process). The *s*-process, which requires the maintenance of a relatively low free neutron density over a rather extensive time interval (Seeger *et al.*, 1965) is generally assumed to operate on neutron sources which are available during the more stable phases of evolution of stars. Although the precise neutron sources involved are still uncertain, we believe that the most likely site of *s*-process synthesis is provided by the convective mixing of the hydrogen and helium layers of a star following the helium-burning shell flashes discussed by Schwarzschild and Harm (1967). The admixture of protons into a helium burning zone in which a substantial C^{12} abundance has been formed can result in the production of free neutrons by the reactions

$$C^{12}(p,\gamma)\,N^{13}(e^{+}\nu)\,C^{13}$$
$$C^{13}(\alpha,n)\,O^{16}$$

as described in detail by Sanders (1967) and Cameron and Fowler (1971). This mechanism also allows the possibility that the heavy elements thus formed can subsequently be mixed to the surface of the star where they may be lost as a part of a less violent stellar mass loss process. This has the additional advantage that the heavy elements produced by the *s*-process are not asked to survive the extreme shock wave heating which one expects to be associated with ejection in a supernova event.

We have therefore assumed that the synthesis of the *s*-process elements takes place in the helium zone of stars in the mass range 2–4 $M_\odot$ which undergo these helium shell flashes. (We exclude stars of mass $<2\,M_\odot$ following the conclusion of Sanders (1967), and a private communication from Prof. Schwarzschild, that mixing is not extensive enough for these lower masses to generate a sufficient neutron flux.) The seed nuclei for this process are provided by the primordial iron-peak nuclei existing in the star. Our calculations indicate that a fraction ~ 0.004 of the initial C–Fe nuclei in the helium zones of these stars must be converted to *s*-process elements if the integrated abundance is to be consistent with solar system material at the time of formation of the sun in the galaxy. As iron peak nuclei constitute a fraction by mass ~ 0.07

of the carbon-to-iron nuclei, we find that only $\sim\frac{1}{10}$ of the iron nuclei in these zones are in fact required as *s*-process seeds. It is encouraging that this *s*-process mechanism does not need to be completely efficient in destroying the available iron nuclei.

r-Process Synthesis

The production of the *r*-process heavy elements can take place readily in material which has been compressed to nearly nuclear densities (Cameron *et al.*, 1970). We have therefore assumed that the site of *r*-process synthesis is somehow tied to the mechanism of formation of a neutron star. The amount of material indicated in Figure 1 as forming *r*-process heavy elements has been entirely adjusted to fit the observed abundances of these elements in the solar system. The amount of mass, $\sim 10^{-6}$ of the stellar mass per event from just that range of stars leading to neutron star remnants, is a very small mass fraction indeed. The predicted mass ejection for the supernova mechanism of LeBlanc and Wilson (1970) discussed previously amounts to a fractional stellar mass of $\sim 10^{-3}$, more than sufficient for our purposes. This may suggest that this mass ejection mechanism is not operative in all cases. Alternatively, a substantial fraction of this mass might be in the form of neutron-rich isotopes of lighter nuclei (e.g., Ti^{50}, Cr^{54} and Fe^{58}) which are dominant under certain higher temperature neutron-rich equilibrium conditions (see for example, Clifford and Taylor (1965)).

Among the heavy elements formed under these *r*-process conditions are some which are radioactive with rather long half-lives. These include the natural radioactivities found on earth, Th^{232}, U^{235} and U^{238}. One can estimate relative production rates for these nuclides on the basis of reasonable nucleosystematics. We have in fact defined the time of formation of the sun in our galactic models by the concordance of the Th^{232}/U^{238} and U^{235}/U^{238} ratios. A detailed consideration of problems of cosmochronology, including as well the shorter lived radioactivities I^{129} and Pu^{244}, is presented as a consequence of our adopted galactic model in a later section.

p-Process Synthesis

The *p*-process heavy elements are very much smaller in abundance than the heavy elements produced by either of the two neutron capture processes. It appears probable that they are produced by secondary processes in the passage of a supernova shock wave through the hydrogen outer layer of the pre-supernova star, raising the temperature momentarily to about 3×10^9 K. These conditions would allow the rapid addition of protons onto any *s*-process or *r*-process heavy elements which were present in the star when formed and which have survived in the hydrogen layer. The principal difference between the outer hydrogen layer of a star which is lost by mass loss prior to a supernova explosion and that which is lost during the supernova explosion is the production of the *p*-process heavy elements in the latter. We find that 50% of the primordial *s*-process and *r*-process nuclei in our assumed post-mass loss hydrogen layer of the stars which give rise to supernova events ($4 \leq M \leq 8\ M_{\odot}$) must be converted to *p*-process heavy elements to account for the solar system abundances.

Helium

In an advanced stage of evolution, a star will contain two thermonuclear burning shells due to hydrogen and helium reactions. The manufactured helium content of the star thus lies mainly between these two shells (the outer hydrogen layer will still contain the primordial stellar content of helium). Stellar evolution calculations indicate that typically 5% of the stellar mass is contained in this helium layer in a late stage of evolution (Hayashi *et al.*, 1962) and this has been assumed in the construction of Figure 1. Integrated over galactic history, we find the contribution of this helium production to the He^4 level in the interstellar gas to be rather small.

Type I Supernovae

The supernova model which emerges from these considerations may be identified with Type II supernovae. The production of *p*-process elements requires the presence of an extensive hydrogen envelope in the ejected material. However, a recent analysis of Type I supernova spectra by C. Gordon (private communication) indicates that hydrogen is very rare in the ejected envelopes of such stars. Thus our treatment of supernova explosions is incomplete.

We suggest that the following type of event may give rise to Type I supernovae. Consider a binary pair of stars, each of roughly 1 $M_\odot$. The more massive of the pair will transfer mass to its companion at the red giant stage, leaving a helium white dwarf. Later a reverse mass transfer onto the white dwarf may cause a progressive increase of mass, accompanied by hydrogen-to-helium conversion in the outer layers and a series of nova explosions. Explosive ignition of helium thermonuclear reactions can then produce a supernova explosion when the white dwarf mass approaches the Chandrasekhar limit. The ejected envelope would contain very little hydrogen.

This type of event could readily occur in elliptical galaxies, where the current rate of star formation is low. We would expect a neutron star remnant to be formed. Such events would contribute to explosive nucleosynthesis in the galaxy, and thus the mass fractions listed for the 4–8 $M_\odot$ region in Table I may be slightly too large.

D. SOLAR SYSTEM ABUNDANCES

The concentrations in the interstellar medium of the various products of nucleosynthesis should, in our models, be consistent with the solar system abundances at the time of formation of the sun in the galaxy. The adopted solar system mass fractions of elements attributed to the various processes of nucleosynthesis described above are given in Table II (Cameron, 1968). The 'carbon-to-iron' designation includes all isotopes of nuclei in the range $12 \leq Z \leq 30$.

Our estimates of the mass fractions attributed to *s*-process, *r*-process and p-process nucleosynthesis are based upon our current theoretical understanding of these processes, and may be uncertain by a factor of two or more. This is due, primarily, to uncertainties in the extent to which charged-particle explosive nucleosynthesis will build nuclei beyond the iron-peak region ($A > 60$). The mass fractions for these processes given in the table correspond to the assumption that all isotopes of mass

TABLE II

Adopted solar system abundances

Nuclei	Mass Fraction
Hydrogen	0.745
Deuterium	2.3×10^{-4}
He^3	5.0×10^{-5}
He^4	0.24
Li^6	4.4×10^{-10}
Li^7	6.5×10^{-9}
Beryllium-Boron	1.6×10^{-9}
N^{14}	9.1×10^{-4}
Carbon-to-Iron	0.0145
s-Process	4.5×10^{-7}
r-Process	2.5×10^{-7}
p-Process	7.9×10^{-9}

$A \geq 74$ are formed by neutron capture or *p*-process synthesis. If one assumes, rather, that all isotopes of mass $A \geq 68$ are properly attributed to *s*-process, *r*-process or *p*-process synthesis, the mass fractions are respectively 1.0×10^{-6}, 3.2×10^{-7} and 1.4×10^{-8}. A more thorough examination of this point by one of the authors (J.W.T.) is currently in progress. For the purposes of the numerical exercise described in this paper, these uncertainties may be ignored. It is a simple matter to scale the *r*-process and *s*-process production fractions defined previously to compensate for revised estimates of the solar system mass fractions.

3. Luminosity Function

A realistic stellar luminosity function is an essential input to our galactic evolution calculations. In a previous investigation, Truran *et al.* (1965) chose, as a representative and consistent set of relations between stellar mass, lifetime and luminosity function, those given by Limber (1960). The adopted luminosity function for stars of absolute visual magnitude brighter than +5 consisted of an equally weighted mean of three sets of values: two by Sandage (1957) and one by van den Bergh (1957). One of those given by Sandage was derived by modifying the observed luminosity function in the solar neighborhood for the effects of evolution, while the others were based on the luminosity functions of young galactic clusters. For stars of magnitude fainter than +5, the observed luminosity function for the solar neighborhood was used.

In our present calculations we have again assumed that this luminosity function, characteristic of young clusters and of the solar neighborhood, is appropriate throughout *most* of galactic history. It has also been assumed, as before, that the stellar evolutionary lifetimes of the stars are essentially the main sequence lifetimes and that these are independent of the primordial stellar composition. The adopted stellar masses and lifetimes as a function of absolute visual magnitude are those derived by Limber (1960) from the calculations of Schwarzschild and Harm (1958), Henyey *et al.* (1959) and others for stars on or near the main sequence.

Our adopted luminosity function is shown in Figure 2, where we have plotted the number of stars per logarithmic mass interval as a function of stellar mass. This number is reasonably consistent, over most the mass range of interest, with the functional dependence $M^{-1.35}$. When this assumed stellar birth function is used throughout galactic history, however, an inconsistency of our models with observation results.

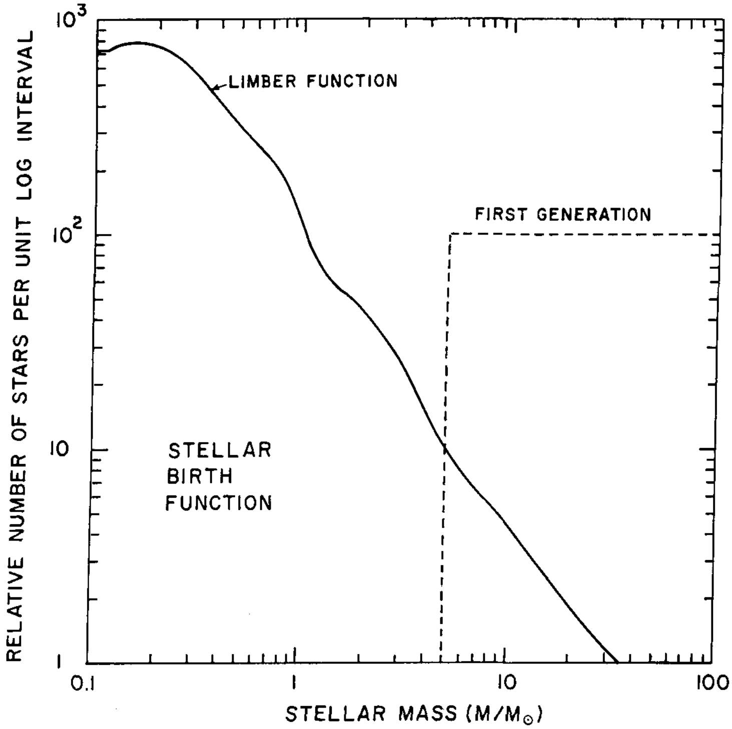

Fig. 2. The adopted luminosity function. The relative number of stars per unit logarithmic mass interval is plotted as a function of main sequence mass (Limber, 1960). The dashed line indicates the stellar birth function adopted for the proposed pregalactic phase of star formation.

Specifically, stars of relatively low mass ($M \lesssim M_{\odot}$) which accumulate without evolving off the main sequence over a galactic lifetime of approximately 12 billion years, are found to have too great a spread in their distributions of heavy element abundances. A modified stellar birth function, for which only stars more massive than 5 $M_{\odot}$ are formed with a flat logarithmic mass dependence (see Figure 2), was adopted for the early stages of galactic history in an attempt to resolve this inconsistency. A detailed examination of this problem is presented in our subsequent discussion of the 'conventional' model.

There are some indications that this Limber birth function is not universally representative of the star formation process. Spinrad (1966) has found that elliptical galaxies have a much larger content of low mass stars than do spiral galaxies. Such a situation would require a much steeper stellar birth function than that derived by Limber. A similar situation may exist in old globular clusters. We discuss several implications of this later in this paper.

4. The Conventional Model

In our initial attempts at the construction of a galactic model, the most straightforward possible assumptions were made concerning the various characteristics of the evolution – these define our 'conventional' model. The galaxy is assumed at all times to be structureless and homogeneous; any mass evolving from stars is thus assumed instantaneously to be mixed with the remaining interstellar gas. The rate of formation of stars in the galaxy is taken to be proportional to the residual mass of gas; this gives rise to an expression for the mass of gas as a function of time in the galaxy of the form $M_g(t) = M_g(0)\, e^{-t/\tau}$, where we have determined the constant 'τ' by the specification that after 12 billion years of galactic history, 5% of the initial gas remains. The stellar luminosity function chosen for this conventional model is that arrived at by Limber (1960) as representative of stars in the solar neighborhood and young galactic clusters, as has been described in Section 3. It is assumed that this luminosity function is appropriate throughout the entire history of the galaxy; no time variation of the luminosity function is taken into account for this model. The initial composition of the gas was assumed to be 77% hydrogen and 23% helium. Galactic models have also been constructed and evolved in which the influence of minor modifications of these assumptions within reasonable limits has been studied. As no significant changes in the consequences of the model follow from such variations, we will therefore present only the results of a single representative calculation.

The predictions of this conventional model regarding the various processes of nucleosynthesis are summarized in Figure 3. Here we have plotted the ratios of the products of various processes of nucleosynthesis to their solar system values (Cameron, 1968) as a function of galactic age. The time of formation of the sun, indicated by an arrow, corresponds to the time of concordance of the cosmochronological ratios Th^{232}/U^{238} and U^{235}/U^{238} in our model. This takes place at a galactic age of approximately 9.4 billion years. The arrow labeled 'NOW' is placed some 4.6 billion years after the time of formation of the sun. It is evident from this figure that a consistent set of values may be obtained for the abundances of the various products of nucleosynthesis at the time of formation of the sun in our galactic model. Some variations of the mass fractions attributed to these nucleosynthesis processes, defined in Section 2, has been necessary. The specific choices of parameters required to account for the consistency shown in this figure are these (see Table I): 7.5% of the mass of those stars which give rise to supernova explosions is required to account for the level of the carbon-to-iron elements in solar system material; a mass fraction $\sim 1.3 \times 10^{-6}$ is required in the form of *r*-process elements ejected in these same supernova events; half of the initial *s*-process and *r*-process nuclei in the ejected hydrogen envelopes of supernovae are required to have been processed to *p*-process nuclei; the

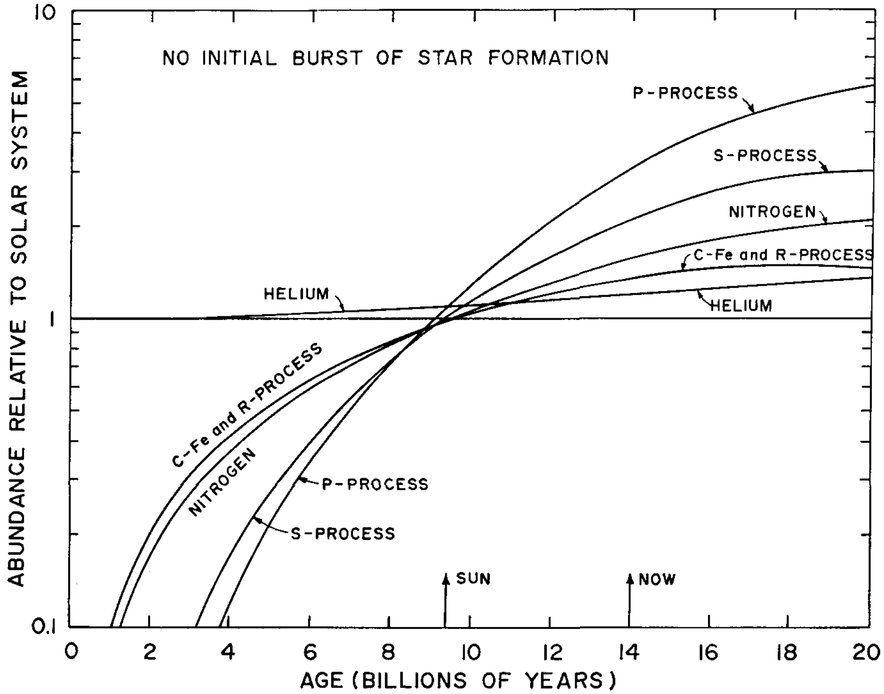

Fig. 3. The abundances of nuclei formed by the various mechanisms of nucleosynthesis, relative to their solar system values, are shown as a function of galactic age for the 'conventional' model. The time of formation of the sun is specified by the concordance of the U^{235}/U^{238} and Th^{232}/U^{238} cosmochronological ratios.

production of N^{14} in the envelopes of stars in the range 2–4 $M_\odot$ must have taken place in a fraction 0.047 of the helium zone; the production of s-process nuclei, taking place in the helium zones of these same stars, must involve the conversion of a fraction 0.00375 of the initial C–Fe nuclei to s-process products (this corresponds, approximately, to 5% of the initial iron peak nuclei in the helium zone). In the light of our considerations of the processes of nucleosynthesis, these constitute quite reasonable fractions.

While this simple model of galactic evolution provides very satisfactory agreement with the current levels of the products of nucleosynthesis in the interstellar gas for very reasonable assumptions concerning stellar and supernova nucleosynthesis, there is one point on which this model is quite in conflict with observation. Our assumption that the Limber (1960) luminosity function is appropriate throughout the entire history of the galaxy leads to the prediction that stars of relatively low mass ($M \lesssim M_\odot$), which will have accumulated without having evolved off the main sequence over a galactic lifetime of approximately 12 billion years, will have far too wide a spread in their abundances of the heavy elements. This defect has been emphasized by Schmidt (1963). This behavior is illustrated in Figure 4, where we have plotted the

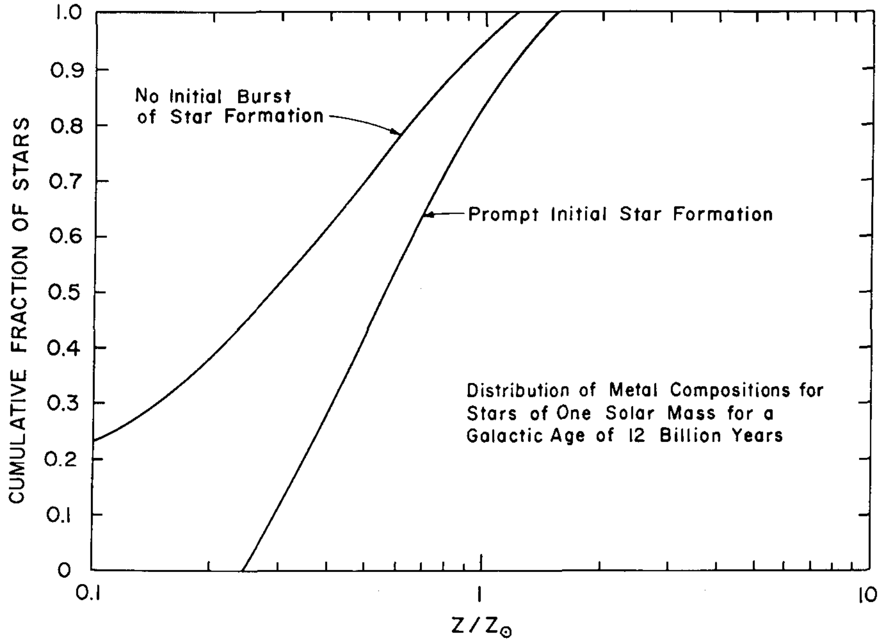

Fig. 4. The distribution of metal composition for stars of one solar mass in the galaxy is shown both for the conventional model and for our adopted model (prompt initial star formation).

cumulative fraction of stars as a function of the ratio of their metal composition to that characteristic of solar system material ($Z/Z_\odot$). These results indicate that some 23% of the stars of mass $\sim 0.9\ M_\odot$ will have a metal composition less than $\frac{1}{10}$ of the metal composition of solar system matter. This is quite contrary to observation, however, since the vast majority of stars in space have a heavy element content within perhaps a factor of three of the sun, and stars which are greatly depleted in heavy elements are in fact extremely rare. One very natural way of accounting for this characteristic of stellar abundances, suggested long ago by Schwarzschild and Spitzer (1953) and later by Schmidt (1959, 1963), is to assume that the luminosity function was drastically different during the earliest stages of galactic history. The luminosity function must evidently have differed in such a manner that nearly all stars formed with very low concentrations of heavy elements must have been sufficiently massive that they evolved and disappeared from the galactic scene long ago.

This general conclusion concerning the character of the stellar luminosity function during the very earliest stages of galactic history is consistent with current thinking about star formation (see for example the recent work of Larson and Starrfield (1971)). Star formation appears to occur when an interstellar cloud is compressed to a sufficiently high density so that it goes into gravitational collapse. During the collapse of an interstellar cloud, fragmentation is greatly assisted if strong density-dependent cooling is present; such cooling reduces the velocity of propagation of pressure waves and hence tends to isolate one part of the gas from its surroundings. Cooling is very inefficient below about 10^4 K if heavy elements are not present in the gas to form grains and ions with low lying excited states. Therefore, it is reasonable to expect that somewhat more massive stars are formed from gas of extremely low metal content. While these rather qualitative arguments make at least plausible the assumption of a time variation of the luminosity function, unfortunately no quantitative theory of star formation is available at the present time which makes detailed predictions of such variations. Our assumptions concerning the revised form of the luminosity function, shown in Figure 2, are therefore extremely uncertain. We have simply assumed that only stars of mass greater than some limit – in this case 5 $M_\odot$ – will be formed in the absence of a significant heavy element abundance. Above this limiting mass, in the absence of any theoretical justification for a particular distribution of mass, we have chosen a flat mass spectrum; that is, the number of stars formed per logarithmic mass interval is assumed to be constant. It should be noted that the same prescriptions regarding stellar evolution that gave rise to a very minor mass fraction in the form of black holes as a consequence of the Limber luminosity function now predicts that a significant fraction of the mass formed into stars will ultimately be associated with these collapsed remnants.

We emphasize that the only strict requirements that we have discovered for the first generation of stars are that very few stars of about 1 $M_\odot$ can be found and a substantial number of stars in the supernova range, $4\ M_\odot \leq M \leq 8\ M_\odot$, must be formed. Any prescribed stellar birth function for the first generation of stars which satisfied these requirements could be used to produce a reasonably satisfactory model of galactic nucleosynthesis, in the sense of the model which we discuss later. The actual specifications which we make for this first generation birth function are based on an exploration of the consequences of certain cosmological speculations which we discuss in the next section.

5. Cosmological Speculations and Galactic Nucleosynthesis

A. MASS-TO-LIGHT RATIOS

The environment of the first generation of stars that has contributed to galactic nucleosynthesis is directly related to the problem of galaxy formation, and hence to cosmology. The stars in our galactic halo already contain some content of heavier elements; hence these stars cannot be considered to constitute the first generation of stars formed in the collapsing gas cloud which might be expected to form the galaxy. Some generation of stars must precede them.

Unless extremely violent fluctuations in density occurred in the gas cloud which formed our galaxy, it is difficult to visualize a process in which a generation of stars can form from this gas, complete their stellar evolutionary phases, and inject the products of nucleosynthesis back into the collapsing gas cloud in time for the formation of the halo stars. This suggests that there may have been a stage of star formation which was entirely pregalactic in character. In our following considerations we propose such a pregalactic phase of star formation.

Before making this proposal, it is useful to review some aspects of mass-to-light ratios in galaxies and in clusters of galaxies. This subject has recently been reviewed by Burbidge and Burbidge (1971). The picture which has emerged from numerous galactic studies can be summarized by the following very approximate statements:

(1) For the solar neighborhood and for spiral galaxies in general, $M/L \sim 5$, where M and L are the mass content of the system under consideration and the total energy output from the stars contained within it, each in solar units.

(2) For elliptical galaxies, $M/L \sim 50$. This mass-to-light ratio is consistent with the large excess of low mass stars deduced by Spinrad (1966), and hence with a steeper stellar birth function than the Limber function which we have used in our calculations.

(3) For binary pairs of galaxies with typical separations of about 50 kpc, as studied by Page (1965, and references therein), the average mass-to-light ratios for spiral and elliptical galaxies are essentially the same as those stated above.

(4) For clusters of galaxies, as deduced from an application of the virial theorem, stability requires that $M/L \sim 500$. This appears to be true of all types of clusters of galaxies, from very large and compact clusters to very small and loose clusters. If this high mass-to-light ratio, which is required for gravitational stability, should not be present, then clusters of galaxies would disperse in a time shorter than the age of the universe.

Therefore it appears that something like 90% of the mass of a cluster of galaxies cannot reside in the galaxies themselves. This problem has been recognized for a long time. The explanation of the mass discrepancy which is conventionally suggested is that there is a concentration of intergalactic gas in clusters of galaxies, which provides sufficient additional mass to stabilize them (Oort, 1970).

Severe limitations can be placed on the properties of the gas assumed to be concentrated in clusters of galaxies. The temperature must lie near 10^6 K. If the temperature were significantly greater than this, then thermal bremsstrahlung from the gas should have been detectible from observations of X-rays in space. If the temperature were significantly lower than this, then enough of the gas should be present in the form of neutral hydrogen so that detectable emission of 21 cm radiation should occur, or absorption of the Lyman continuum of quasars. If the gas is in fact present with a temperature near 10^6 K, then it is reasonable to expect that it will be detected through the emission of very soft X-rays if the proposed High Energy Astrophysical Observatories are flown in space in the next few years.

If all the intergalactic gas concentrated in clusters of galaxies has a temperature in the vicinity of 10^6 K, then it is very surprising that virial theorem estimates should give essentially the same estimate of M/L for such a wide variety of types of cluster. In a dense cluster, the gas should have a strong central concentration, but in a loose cluster the gas, having lower total mass, would be much more spread out, and the dynamics of the cluster might be sufficiently altered as to indicate an apparently different M/L ratio. At the present time, this difficulty does not indicate that the intergalactic gas concentration theory should be rejected, but nevertheless it seems advisable to explore other possibilities for the form of the unseen mass in clusters of galaxies.

In the present investigation we have adjusted the stellar birth function for the first generation of stars so as to put 90% of the mass of the universe into black holes, or collapsars, at the end of the evolution of the first generation of stars. Our motivation in doing this has been to provide another possible explanation for the unseen mass in clusters of galaxies, and to see what the implications of this alternative possibility would be for the character of the first stellar generation.

B. INITIAL CONDENSATIONS IN THE EXPANDING UNIVERSE

In considering the process which leads to the formation of the first generation of stars in the universe, we have adopted a picture discussed by Peebles and Dicke (1968). These authors postulated a hot expanding universe, in which the isotropic background radiation contains almost all of the energy density, and in which they assume that the matter accompanying the radiation has a 'white noise' spectrum of density fluctuations. Under such conditions, there are far more density irregularities with dimensions comparable to the Jeans length than those with much larger length scales. After the radiation decouples from the matter, at the time of recombination of the hydrogen, all density fluctuations with a length scale equal to about the Jeans length or larger, out to some cosmological length scale, will be unstable against contraction. Those fluctuations with dimensions smaller than the Jeans length will be damped. The amount of material contained within a Jeans length depends upon the cosmological model, but Peebles and Dicke have shown that a mass in the range 10^5 to 10^6 $M_\odot$ should be contained within such a critical fluctuation. Masses of this order will then contract to form primordial gas clouds, and systems of these gas clouds will in turn contract toward common centers of gravity, presumably forming galaxies and clusters of galaxies. It is expected that the primordial gas clouds with masses $\sim 10^6$ $M_\odot$ will collapse fastest, and the irregularities on larger length scales will collapse with somewhat longer time scales.

Peebles and Dicke attempted to follow the evolution of one of these primordial gas clouds. They assumed that, following decoupling of matter and radiation, the gas cloud would expand to a maximum radius, and then collapse until it came into equilibrium in the sense of the virial theorem. They estimated that this equilibrium stage could be represented by a convecting polytrope with a central temperature of the order of 10^3 K.

Peebles and Dicke followed the evolution of such a hypothetical model through numerical integration of equations of motion. They found that the central region of the gas cloud should cool very rapidly, leading to a rapid collapse of the center. They assumed that this would form a few massive stars, and that the ultraviolet output from these stars would sufficiently heat the surrounding gas cloud to maintain it in approximate hydrostatic equilibrium, preventing it from collapsing to form stars. The gas clouds, thus stabilized, were then thought to fall together to form galaxies, with the gas being stripped away from the few primordial stars which had formed as the gas clouds ran into one another.

Some aspects of this problem have recently been examined by Hirasawa (1971). He considered the atomic and molecular processes which should take place in the early stages of condensation of matter in the expanding universe, and showed, as Peebles and Dicke had already concluded, that there would be a rapid formation of hydrogen molecules. Such hydrogen molecules are extremely efficient in cooling the collapsing gas, once reasonable densities have been obtained. Hirasawa showed that under a wide range of conditions, the temperature of the gas would be maintained at 250 K.

These results show to be rather unlikely the assumption of Peebles and Dicke that the primordial gas clouds should undergo a stage of temporary hydrostatic equilibrium in the form of a convecting polytrope. The temperature of the gas cannot rise to the central temperature required by Peebles and Dicke. This should have two major effects upon their picture of the evolution of the gas coluds. In the absence of a condition of temporary hydrostatic equilibrium, the gas clouds should not have an opportunity to attain a high degree of spherical symmetry. They should not achieve a high degree of central density concentration. Under these conditions, it appears much more plausible to us to assume that the collapsing primordial gas clouds will fragment into stars.

The collapse of the primordial gas clouds will take place with interior temperatures of 250 K. This is much higher than the internal temperatures believed to exist in the interiors of collapsing interstellar clouds which form stars at the present time, where the temperature may only be 5 or 10 K. Under these circumstances, the speed of sound is considerably greater in collapsing primordial clouds than in collapsing interstellar clouds, so that it is probable that density fluctuations are somewhat more efficiently smoothed out, and the fragmentation of the gas into stars cannot take place so extensively. On this basis we postulate that the primordial gas clouds will collapse into clusters of massive stars. We cannot estimate the shape of the stellar birth function from such crude considerations, and hence we have arbitrarily taken it to be flat.

This leads to the following physical picture of the formation of galaxies. We propose that the bulk of the gas in the primordial gas clouds will be formed into massive stars. Most of the stars will evolve rather quickly, and the great bulk of them, with masses greater than about 8 $M_\odot$, will collapse to form black holes, or collapsars. After evolving off the main sequence, the massive stars should be expected to form 'yellow giants', rather than red giants, since, lacking metals, their surface opacities will tend to be rather low, and the outer convective envelopes should not grow to nearly the same size as those of normal stars today. Since, empirically, the rate of mass loss in the red giant phase of stellar evolution appears to increase rapidly as the surface temperature decreases, we would expect that the amount of mass loss taking place from the first generation of massive stars would be much less than that assumed in our estimates for normal stars.

Subject to these considerations, it is simple to postulate conditions for which 90% of the mass of the stars formed in the first generation will be left as black holes. The evolution of the first generation can be expected to have finished within the time scale in which the clusters of massive stars fall together toward common centers of gravity. The lower mass stars, in the supernova range, take longer to evolve, but even they should probably have finished their evolution before completion of the fall of the clusters of massive stars toward common centers of gravity. Hence the clusters of black holes, with a small admixture of neutron stars and perhaps white dwarfs, should arrive in the vicinity of common centers of gravity accompanied by gas clouds containing the gas which did not form into stars together with the gas ejected by stellar wind processes and supernovae. This gas will be stripped away from the clusters of massive black holes in the vicinity of the common centers of mass; the clusters will proceed onwards and recede to distances from the common centers of mass comparable to those from which they originated. Since these distances will be comparable to the separations between galaxies in a cluster, and since the newly-forming galaxies will themselves be in relative motion, the fluctuating tidal fields within the clusters of galaxies can be expected to strip away the clusters of black holes from the individual galaxies, putting them into independent orbits within the clusters of galaxies. This is the dynamical basis for our assumption that the products of the first generation of stars will be associated with clusters of galaxies, but that only a small fraction of their total mass is likely to be associated with individual galaxies, and that in the form of a very extended halo of clusters of black holes.

The acquisition of angular momentum by galaxies has been considered initially by Peebles (1969), and more recently by Oort (1970), and again by Peebles (1971). In the original picture discussed by Peebles, tidal interactions between the non-spherical distributions of mass which will form galaxies lead to torques between them, resulting in the acquisition of angular momentum by the forming galaxies. In Peebles' initial estimate, the amount of angular momentum which would be acquired by a galaxy appeared to be too small by nearly an order of magnitude. Oort (1970) estimated that the deficiency would be very much larger than this. However, more recently, Peebles (1971) has criticized Oort's estimate, and has carried out a numerical calculation which confirms his earlier estimate that the angular momentum acquired by the galaxies would be less than the observed amounts by a factor of 5 or 6.

We wish to point out that the picture of galaxy formation proposed here alleviates this angular momentum problem. The mass which goes into galaxies is perhaps 10% of the mass which is everywhere distributed in space at the time of fragmentation to form galaxies, and consequently the additional mass associated with the irregular gas distributions increases the torques by about an order of magnitude. This would lead to the acquisition of angular momentum by the forming galaxies of the correct order of magnitude, according to Peebles' calculations.

The massive stars formed in the first generation have some interesting properties in their own right, since they are composed of hydrogen and helium and contain no content of metals. The evolution of these stars has recently been investigated by Ezer and Cameron (1971). The absence of carbon, nitrogen, and oxygen in these stars means that the stars are much smaller on the main sequence than is normally the case, since energy generation must either occur by proton-proton reactions, in the lower range of masses, or by combined hydrogen-helium burning for the more massive stars. Surface temperatures of these stars lie in the vicinity of 10^5 K.

This situation has some interesting implications for the observability of the first generation of stars, either in the form of the individual clusters, or in the form of galaxies. If the first generation of stars formed at $\frac{1}{10}$ of the present universal age, then the light from them would be redshifted to a present color temperature of about 10^4 K. In this case, the primordial clusters of massive stars should exist as faint objects with color temperatures not very different from those of galaxies. On the other hand, if the primordial clusters of stars formed at $\frac{1}{3}$ of the present age of the universe, then the color temperature of the light radiated from them should presently appear to be about 3×10^4 K. In this case, they would appear as faint blue objects. It is possible that they exist among the very large numbers of faint blue objects present in intergalactic space. Only extensive spectroscopic observations can investigate this question. However, it should be noted that our picture is entirely inconsistent with the previous suggestion of Partridge and Peebles (1967) that the early galaxies should have their light redshifted strongly into the infrared.

C. A CONSISTENT MODEL OF GALACTIC EVOLUTION

We have made a particular choice of the form of the first generation luminosity function which leaves 90% of the initial mass from which our galaxy was formed in black hole remnants. Specifically, we have assumed an initial mass $1.6 \times 10^{12}\ M_\odot$ for the cloud from which our galaxy condensed. Of this mass, 95% participates in a first generation of stars ranging in mass from 5 to 645 $M_\odot$, with a flat mass spectrum as shown in Figure 2. For stars of mass $M > 8\ M_\odot$, which we assume to give rise to black holes, 95% of the mass is incorporated into these remnants, and only 5% constitutes mass loss. This prescription gives rise to the following characteristics at the onset of the second generation of star formation: (a) $1.6 \times 10^{11}\ M_\odot$ remains in the form of gas, (b) supernova events have formed 25% of the solar system abundances of both C–Fe nuclei and the products of *r*-process synthesis, (c) the primordial D^2 and He^3 abundances, to be discussed later, have been reduced, by destruction in the gas cycled through this first generation, to levels which are 1.8 and 0.6, respectively, times their solar system values and (d) $1.44 \times 10^{12}\ M_\odot$ is in black hole remnants. We emphasize that our set of assumptions constitutes but one possible choice; we could have resolved the inconsistency with observations concerning metal abundances in low mass stars with assumptions which were far less extreme. Our intent is simply to demonstrate that a plausible model of the galaxy can be constructed which is consistent both with observations of stellar abundances and with the production of an enormous mass in black holes.

Proceeding on the basis of these assumptions, we have constructed and evolved a 'consistent' model of our galaxy. The initial stellar generation, which leaves 90% of the total mass $1.6 \times 10^{12}\ M_\odot$ in the form of black holes distributed uniformly in globular-like clusters, provides the following conditions at the onset of the second stellar generation: the C–Fe elements and the *r*-process abundances are present in the gas at a level 0.25 their solar system values, the D^2 and He^3 abundances have been reduced substantially from their primordial values (see Section 7), and the available gas mass

$1.6 \times 10^{11}\ M_{\odot}$ is that assumed for the mass of the galaxy in our conventional model. We then continue to evolve the galaxy precisely as was done for the conventional model. The Limber (1960) luminosity function is adopted and the gas content as a function of time is prescribed by $M_g(t) = M_g(0)\, e^{-t/\tau}$, where τ is chosen to reduce the gas content to 5% after 12 billion years.

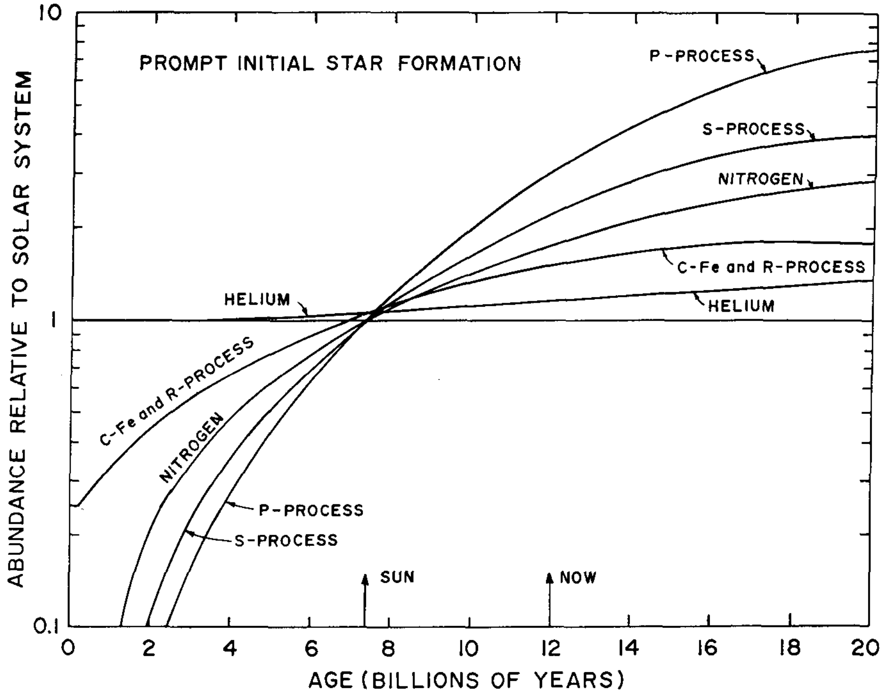

Fig. 5. The abundances of nuclei formed by the various mechanisms of nucleosynthesis, relative to their solar system values, are shown as a function of galactic age for our adopted galactic model.

The ratios of the products of the various processes of nucleosynthesis to their solar system values are plotted as a function of galactic age in Figure 5. The time of formation of the sun, indicated by an arrow, is again specified by the concordance of the cosmochronological ratios Th^{232}/U^{238} and U^{235}/U^{238} (see Section 6). This takes place at a galactic age of approximately 7.4 billion years. At this time, the abundances of the products of nucleosynthesis shown in this figure are all quite consistent with solar system values (Table II). The specific choices of parameters required to provide this consistency are as follows (see Table I): 8% of the mass of those stars which give rise to supernova explosions is required to account for the level of the carbon-to-iron elements in solar system material; a mass fraction $\sim 1.4 \times 10^{-6}$ is required in the form of *r*-process elements ejected in these same supernova events; half of the initial *s*-process and *r*-process nuclei in the ejected hydrogen envelopes (32% of the mass) of supernovae are required to have been processed to *p*-process nuclei; the production of N^{14} in the envelopes of stars in the range 2–4 $M_\odot$ must have taken place in a fraction 0.063 of the helium zone; and the production of *s*-process nuclei, taking place in the helium zones of these same stars, must involve the conversion of a fraction 0.00375 of the initial C–Fe nuclei to *s*-process products. These fractions are similar to those required for the conventional model, and again are quite compatible with considerations of nucleosynthesis (Section 2).

The consistency of this model with other observations is also satisfactory. The rate of supernovae now in the galaxy is predicted in this model to be one event per 25 years; Tammann (1970) gives one event per 23 years for the observed supernova rate in galaxies like our own. Furthermore, the inconsistency of the predictions of our conventional model with observations of metal abundances in low mass stars has now been resolved. This is indicated in Figure 4, where we have also plotted the cumulative fraction of stars as a function of metal abundance for our current assumptions, which

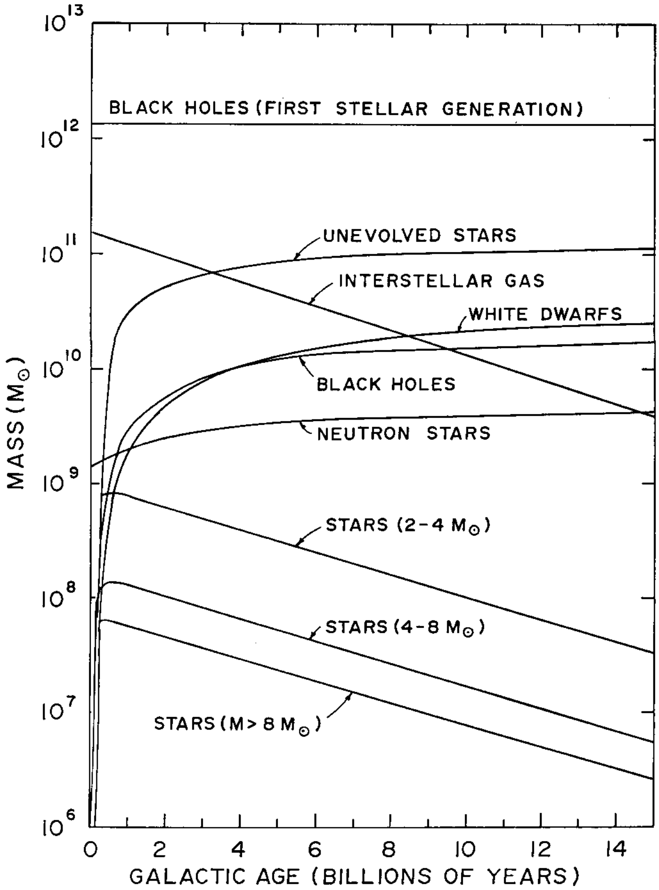

Fig. 6. The masses in the form of stars, gas and remnants are plotted as a function of galactic age. The pregalactic black holes are plotted separately.

include an initial burst of 'prompt' star formation. The masses in the form of gas, stars and remnants are plotted as a function of galactic age in Figure 6.

It should be recognized that the assumption that the second generation of stars, which includes all of the stars now observable in our galaxy, should commence with 25% of the solar content of metals, is necessarily unrealistic. Yet, in some average sense, this picture appears to be qualitatively true, since stars with $\frac{1}{10}$ of the solar content of metals or less are actually exceedingly rare.

Within the context of our cosmological speculations, we interpret this situation in the following way. There will be inhomogeneities in the mixing of the products of explosive nucleosynthesis with the remainder of the gas ejected by stellar winds from massive stars in the evolving primordial clusters which we have postulated. These inhomogeneities need not be completely removed when the gas is stripped off the original clusters to form galaxies. In this way some portion of the second generation of stars, now constituting the galactic halo, may be formed with very low metal contents.

It is also possible that in the case of an elliptical galaxy, where there seems likely to have been a much greater degree of spherical symmetry in the infall of material, the evolution of stars in the supernova range may have required a time comparable to that required for the clusters of massive stars to fall close to the common centers of gravity. It is conceivable that such a process may even lead to a very large enhancement of the metal content in the gas which will be deposited near the centers of elliptical galaxies. This is one possible way of accounting for the apparent tendency toward large supermetallicity in the central regions of elliptical glaxies (Spinrad and Taylor, 1969).

In the following sections, the implications of this 'consistent' galactic model for cosmochronology, light element synthesis and other problems are discussed in some detail.

6. Cosmochronology

The products of *r*-process nucleosynthesis include a number of nuclei which are radioactive with rather long half lives. Among these are the natural radioactivities found on the earth, Th^{232}, U^{235} and U^{238}. As mentioned earlier, we have defined the time of formation of the sun in the galaxy by the concordance of the cosmochronological ratios (Th^{232}/U^{238}) and (U^{235}/U^{238}) with the primordial ratios inferred for solar system material.

Problems of cosmochronology have recently been considered in great detail by a number of authors (Wasserburg *et al.*, 1969; Dicke, 1969; Hohenberg, 1969; Schramm and Wasserburg, 1970; Fowler, 1971). Of particular concern in several of these papers is the implication of recent measurements of the levels of I^{129} and Pu^{244} in meteorites. We have also chosen to keep track, throughout our galactic histories, of the abundances of the shorter lived radioactivities I^{129} and Pu^{244}. While the extremely long-lived radioactive isotopes yield information concerning the mean age of the elements, the abundances of short-lived and intermediate-lived isotopes, relative to stable isotopes, yield information on the rate of *r*-process nucleosynthesis throughout galactic history (see for example the article by Schramm and Wasserburg (1970)).

We wish to consider the implications of our galactic models for problems of cosmochronology. In order not to introduce any new estimates of relative production ratios and other parameters which influence the cosmochronology, we have chosen as our input data that given by Fowler (1971). This data is summarized in Table III, where we have given for each of the nuclei Th^{232}, U^{235},U^{238}, I^{129} and Pu^{244} the mean lifetime, the production ratio relative to U^{238} and the abundance in primordial solar system material. There are significant uncertainties associated with various of these parameters which can influence our resulting cosmochronology. The relative production ratios given by Fowler (1971) are based on considerations of the detailed nuclear physics associated with the formation of these isotopes in *r*-process nucleosynthesis. It should be kept in mind that under typical *r*-process conditions, the production of these nuclei will take place in the neutron-rich regions well off the valley of beta stability, where the properties of the nuclei involved are not well determined (Seeger *et al.*, 1965; Cameron *et al.*, 1970). Mass formula predictions (see for example Truran *et al.* (1970) and Seeger (1970)) of the properties of these nuclei are generally required in order to obtain these estimates of relative production ratios. In a recent consideration of this particular aspect of the problem, Seeger and Schramm (1970) have determined *r*-process production ratios of cosmochronological importance from the predictions of a number of different mass formulae. While the average production ratios they obtain for the cases of (U^{235}/U^{238}), (Th^{232}/U^{238}) and (Pu^{244}/U^{238}) are crudely consistent with those quoted by Fowler (1971) and used in our investigation, the range allowed on the basis of their calculation is rather broad. This behavior is summarized in Table IV, where we have given the range of values allowed by their models as well as their quoted average values. These large uncertainties are particularly important with regard to the cosmochronological ratio Pu^{244}/U^{238} and its implication for the recent rate of *r*-process nucleosynthesis, hence supernovae explosions, in our galaxy.

TABLE III

Cosmochronological parameters

Nucleus	Mean Lifetime	Relative Production	Relative Abundance
Th^{232}	2.004×10^{10}	1.65 ± 0.15[a]	2.48 ± 0.15[a]
U^{238}	6.506×10^{9}	1.00	1.00
U^{235}	1.029×10^{9}	1.42 ± 0.19[a]	0.313 ± 0.026[a]
Pu^{244}	1.179×10^{8}	0.90 ± 0.10[a]	0.01270 ± 0.0026[b]
I^{129}	2.45×10^{7}	1.52 ± 0.50[a, c]	$(1.07 \pm 0.04) \times 10^{-4}$[a]

[a] Fowler (1971)
[b] Podosek (1970)
[c] relative to stable I^{127}

The required rate of *r*-process nucleosynthesis in the galaxy immediately prior to the time of formation of the sun is particularly sensitive to the abundance ratio

TABLE IV

Production ratios

	Fowler (1971)	Seeger and Schramm (1970)	Range
Th^{232}/U^{238}	1.65 ± 0.15	1.96 ± 0.25	1.52–2.43
U^{235}/U^{238}	1.42 ± 0.19	1.89 ± 0.36	1.61–2.54
Pu^{244}/U^{238}	0.90 ± 0.10	0.96 ± 0.21	0.52–1.35

Pu^{244}/U^{238}. Two rather conflicting determinations of this ratio have been used, and the choice one makes will determine to a large extent the assumptions required for models of galactic nucleosynthesis. Wasserburg, Huneke and Burnett (1969) have determined a value $Pu^{244}/U^{238} = 0.033 \pm 0.006$ at the onset of xenon retention from an analysis of the whitlockite in the St. Severin meteorite. Both Hohenberg (1969) and Wasserburg, Schramm and Huneke (1969) have pointed out that the presence of this high level of Pu^{244} in primordial solar system material implies the need for a last minute 'spike' of *r*-process synthesis superimposed on any continuous nucleosynthesis model. This would have the effect of enriching the solar system with *r*-process material immediately prior to the time of formation of the sun in our galaxy. More recently Podosek (1970) has determined a somewhat lower value of the Pu^{244}/U^{238} ratio for whole St. Severin material. His value, $Pu^{244}/U^{238} = 0.0127 \pm 0.0026$, is in fact sufficiently low that it implies no need for sudden synthesis, and the last few hundred million years of continuous *r*-process synthesis will suffice to explain this level of Pu^{244} in the solar system. In our models, where nucleosynthesis is in fact assumed to take place continuously over the last few hundred million years, this value for the unfractionated solar system material allows our model to be consistent with cosmochronology. The identification of extinct Pu^{244} has been confirmed by Alexander *et al.* (1971), who have measured the fission xenon yield from a laboratory sample of Pu^{244}.

The ratios Th^{232}/U^{238}, U^{235}/U^{238}, Pu^{244}/U^{238} and I^{129}/I^{127} relative to their accepted primordial solar system values are plotted as a function of galactic age in Figure 7 for our adopted galactic evolutionary model. The dashed lines in this figure trace the levels of these abundance ratios throughout the remainder of galactic history, following the time of formation of the sun. In this model we find that a concordance of the Th^{232}/U^{238} and U^{235}/U^{238} ratios takes place at a galactic age of approximately 7.4 billion years. This, as we have noted previously, is taken to correspond to the time of formation of the sun in the galaxy. The decays of the shorter lived radioactivities Pu^{244} and I^{129} are also indicated in this figure. Both of these radioactivities produce xenon isotopes, the former as a result of a fission process. The relative amounts of these decay products measured in meteorites indicate the amount of I^{129} or Pu^{244} present in the meteorites at the time their parent bodies had cooled sufficiently to prevent any further diffusion of xenon away from the production site. It has been apparent for some time that solar system material became isolated from sources of galactic nucleosynthesis before the formation of the solar system and cooling of the meteorite parent bodies. Thus we do not expect the ratios of these

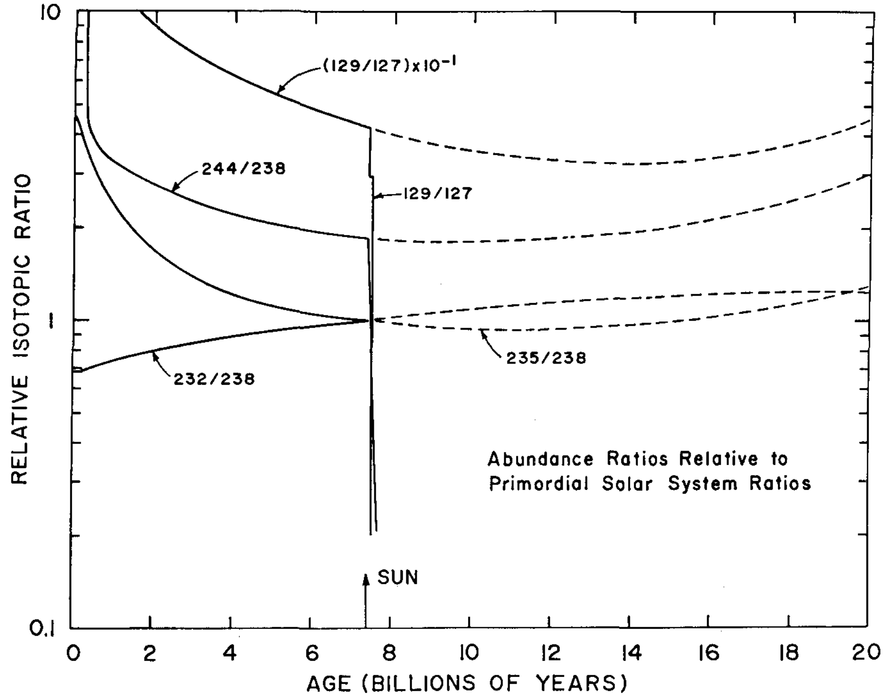

Fig. 7. Radioactive ratios of cosmochronological importance, relative to the primordial solar system ratios, are shown as a function of galactic age. The solid lines show the production termination at time of formation of the solar system; the dashed lines show the subsequent ratios in the interstellar medium.

shorter lived radioactivities to the amounts found on meteorites to be unity in this figure at the time of formation of the sun; instead, the excess amounts of the radioactivities produced by galactic nucleosynthesis allows an estimate of the total isolation time of material which is to form the solar system before such formation actually occurred and the meteorite parent bodies cooled. The excess abundances of these two isotopes indicated in the figure should thus be consistent with a single such isolation time. Indeed, the over-abundances shown in this figure indicate an isolation time of roughly 80 million years. This may correspond to the last time prior to the cooling of the meteorite parent bodies that fresh radioactive debris from Type II supernovae, assumed to have produced these radioactivities, mixed into the interstellar cloud that was later to collapse to form the sun and other stars.

These decays are shown in greater detail in Figure 8, where the time scale has been considerably expanded. Again, while we have not achieved an absolute concordance, the consistency of our results with an isolation time of approximately 80–100 million years is apparent. Uncertainties in production ratios and in the rate of mixing of radioactive supernova debris into the interstellar medium can certainly account for the discrepancies evident in this figure.

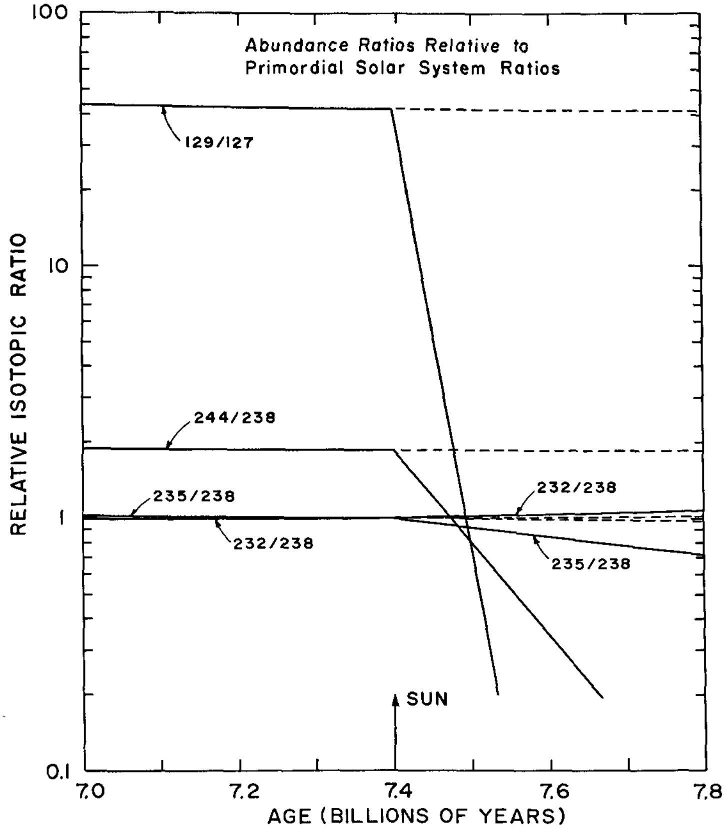

Fig. 8. The region of concordance of the cosmochronological ratios is shown. An isolation time of approximately 100 million years is consistent with both the I^{129}/I^{127} and the Pu^{244}/U^{238} abundance ratios.

7. Formation of the Light Elements

The abundances of the light elements – deuterium, He^3 and the isotopes of lithium, beryllium and boron – have historically provided a challenge to theories of nucleosynthesis. A variety of mechanisms have in fact been proposed to explain their existence, including spallation in stellar surfaces (Fowler *et al.*, 1955), spallation during the early history of the solar system (Fowler *et al.*, 1962), and cosmological nucleosynthesis (Wagoner *et al.*, 1967). A review of these theories together with a discussion of the associated difficulties has been presented by Mitler (1967).

The production of these light elements by means of the interaction of galactic cosmic rays with the interstellar medium has recently received considerable attention. In a survey of problems associated with cosmic ray production, Reeves *et al.* (1969) concluded that the abundances of deuterium and He^3 could not be satisfactorily explained by means of this mechanism. The production of lithium, beryllium and boron seemed far more likely, but they concluded further that at present no satisfactory explanation of the lithium isotopic ratio Li^7/Li^6 in meteorites is available. More recently Mitler (1970) has considered this entire problem in somewhat greater detail. Consistent with the results of Reeves *et al.*, he finds that the abundances of deuterium and He^3 produced by cosmic ray interactions are not sufficient to account for the solar system values. He has found also that, when allowance is made both for the interchange of material between stars and the interstellar medium and for variations in the intensity of cosmic radiation over galactic history, one can explain the lithium, beryllium and boron abundances observed in the solar system by this mechanism. In order to explain the isotopic ratio Li^7/Li^6, however, it is necessary in his view to invoke cosmological production of Li^7. Our present numerical study enables us to trace in detail the concentrations of these various elements over the entire course of galactic history, with a detailed treatment of the interchange of matter between stars and the interstellar medium. In the following sections we will consider separately the implications of our model for the formation of: (a) deuterium and He^3, (b) Li^6 and the isotopes of beryllium and boron, and (c) Li^7.

A. DEUTERIUM AND He^3

The many difficulties associated with the production of deuterium and He^3 in stars and supernovae, or by means of cosmic ray interactions with the interstellar medium, point strongly to the conclusion that these elements are in fact cosmological in their origin. Wagoner *et al.* (1967) have found that significant quantities of deuterium and of He^3 can be produced in the universal fireball. They point out that reasonable agreement with solar system abundances for these nuclei, together with He^4 and Li^7, may be obtained if the present temperature is 3 K and if the present density is approximately 2×10^{-31} g cm^{-3}.

We wish to determine the precise amounts of deuterium and He^3 wihch are consistent both with reasonable assumptions concerning cosmological synthesis and with a consistent model of galactic evolution. It must be kept in mind that neither deuterium nor He^3 will be formed in substantial quantities in stars over the lifetime of the galaxy; rather, any initial concentrations of deuterium and He^3 in the interstellar medium resulting from cosmological synthesis will gradually tend to be depleted as these elements are destroyed in the envelopes of stars. For temperatures in excess of $\sim 10^6$ K, any initial concentration of deuterium present in stellar envelopes will be destroyed by thermonuclear reactions to form He^3. Similarly, any initial He^3 concentration will be destroyed for temperatures in excess of $\sim 10^7$ K. In order to estimate the degree of destruction of these elements in stars, a detailed examination of the models of Iben (1965; 1966a, b, c; 1967a, b), and those of Ezer and Cameron (1967) has been conducted. We have concluded that any initial concentration of deuterium will effectively be completely destroyed in the entire mass lost from all stars which evolve on the time scale of our model. This is consistent as well with the studies of Bodenheimer (1966) of deuterium depletion during pre-main sequence contraction.

The extent of He^3 destruction is somewhat more complex in its dependence upon the detailed temperature structure of a star. Typically, temperatures $T \gtrsim 10^7$ K will not be realized through the entire envelopes of less massive stars at any point in their evolution unless suitable circulation currents exist. Guided largely by the models of Iben, we have adopted the following prescription for He^3 depletion: for masses $M > 25\,M_\odot$, He^3 is completely destroyed; for masses $10 \leq M \leq 25 M_\odot$, He^3 survives only in the outer 10% by mass of the star; for masses $3 \leq M \leq 10\,M_\odot$, He^3 survives only in the outer 25% by mass of the star; for masses $M < 3\,M_\odot$, He^3 survives in the entire envelope structure above the hydrogen burning shell. It is important to note that this prescription ignores the possibility of He^3 production as a consequence of shell hydrogen burning; Iben's calculations suggest that some production of He^3 may be possible by this mechanism, but no very precise estimate is possible.

We need now only specify the initial deuterium and He^3 abundances in order to trace over galactic history the concentrations of these two elements. The choice of these conditions is sensitive to one important consequence of our assumption of an exponentially decreasing gas content of the galaxy: roughly half of the mass in the interstellar medium at the time of formation of the sun had never been processed through a star. This implies that any initial concentration of deuterium or He^3 can be depleted at most by a factor of two for such a prescription for gas depletion. While the violent initial phase of star formation that we have hypothesized will of course allow for a further dilution of these initial abundances, nevertheless the primordial galactic deuterium and He^3 abundances cannot exceed by enormous factors the current solar system values. We have assumed that 5% of the gas mass is not involved in the initial generation of star formation; it would seem unreasonable to be forced to reduce this fraction substantially.

The abundances of hydrogen, deuterium, He^3, He^4 and Li^7 resulting from element production in low density universes as calculated by Wagoner *et al.* (1967) are summarized in Table V. In this Table we give the ratios of the deuterium, He^3 and Li^7 abundances relative to their current solar system values. It is clear from these results that a lower He^4 abundance is not possible if one wishes to account satisfactorily for the observed concentrations of deuterium and He^3. For an initial He^4 abundance of 19% by mass, for example, the deuterium abundance formed in cosmological nucleosynthesis is high by roughly a factor of 13. We have found no reasonable way to reduce this factor in our model of galactic nucleosynthesis; certainly, in the absence of our

TABLE V

Cosmological nucleosynthesis

Identification	Density ($T_0 = 2.7$ K)						
	3×10^{-33}	10^{-32}	3×10^{-32}	10^{-31}	3×10^{-31}	10^{-30}	3×10^{-30}
H^1	0.95	0.89	0.81	0.76	0.75	0.74	0.73
He^4	0.032	0.098	0.19	0.24	0.25	0.26	0.27
$(D^2/H^1)/(D^2/H^1)_\odot$	53	34	13	2.7	0.40	0.054	1.2×10^{-3}
$(He^3/He^4)/(He^3/He^4)_\odot$	53	16	4.2	1.2	0.67	0.36	0.16
$(Li^7/H^1)/(Li^7/H^1)_\odot$	0.063	0.44	0.50	0.080	0.033	0.70	3.4

initial generation of star formation such a large reduction would be impossible. Our final choices for the primordial concentrations of deuterium and He^3 in the galactic material are as follows: hydrogen (0.77), He^4 (0.23), deuterium (8.2×10^{-4}) and He^3 (7.1×10^{-5}) by mass, consistent with a present mean density $\lesssim 10^{-31}$ g cm^{-3}.

There is an apparent discrepancy between this indicated value of the mean density of matter in the universe and our cosmological speculation that the observed matter in galaxies may represent only 10% of the mean density, with the remainder in clusters of black holes. This discrepancy might be resolved if the Hubble constant for the rate of galactic recession could be lowered somewhat further, a continuation of a historical trend. This would increase the age of the universe and suggest that our galaxy had formed at about half of the universal age, for consistency with the cosmochronology of our model of galactic evolution.

The results of our calculations of the evolution of the He^3 and deuterium abundances over galactic history are summarized in Figure 9. As this graph illustrates, roughly a factor of two reduction in the abundances of deuterium and He^3 results from the first generation of star formation discussed in the previous sections. A further factor of two reduction occurs in the galactic evolution model, which brings the deuterium level into agreement with the solar system value at the time of formation of the sun – roughly 7.4 billion years in our model. Two different prescriptions have been

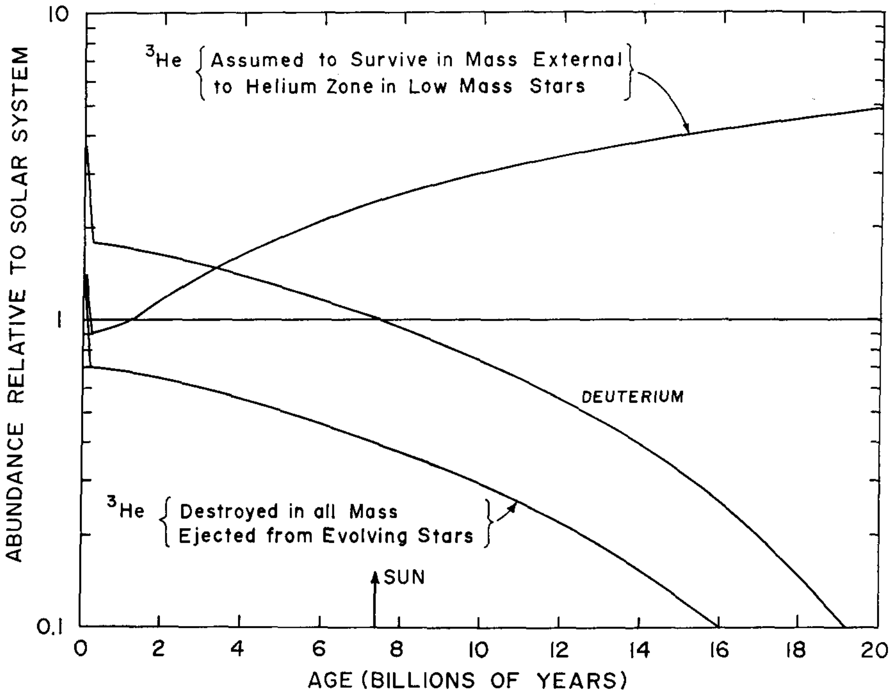

Fig. 9. The abundances of D^2 and He^3 in the interstellar medium, relative to solar system matter, are plotted as a function of galactic age. The two curves for He^3 correspond to limiting assumptions concerning its destruction in stellar envelopes.

chosen for the variation of the He^3 abundances. In the first of these it is assumed that He^3 is destroyed in all mass ejected from evolving stars, this assumption being exactly that chosen for deuterium and requiring the existence of circulation currents in stellar atmospheres. In the second case we have assumed He^3 to survive as dictated by our prescription built upon an analysis of Iben's models, and to be formed from the destruction of deuterium. As can be seen, the proper handling of the He^3 problem very likely falls somewhere between these two extremes. We believe that a more realistic treatment of this problem must be based on detailed quantitative predictions of stellar evolutionary models, including a consideration of the possible production of He^3 in hydrogen burning shells during the late phases of evolution of more massive stars, and of the possible existence of circulation currents in stellar atmospheres which pass the He^3 through regions with a temperature high enough to destroy it.

B. PRODUCTION OF Li^6, BERYLLIUM AND BORON

As discussed previously, Mitler (1970) has found that the abundances of lithium, beryllium and boron produced by the interaction of cosmic rays with the interstellar medium are ample to explain the solar system abundances, when allowance is made for the interchange of gas between stars and the interstellar medium and for the variation of cosmic ray intensity with time. His estimates of the degree of interchange of material between stars and gas and of the variation of the cosmic ray intensity were based on rather crude models of galactic evolution. Our galactic evolutionary calculations enable us to trace numerically in time the detailed abundances of these constituents.

The approach taken in our calculations proceeds as follows. Mitler (1970) has provided us (Table VI) with production rates (P_i) of the various isotopes of lithium, beryllium and boron in units g^{-1} s^{-1}. These production rates involve integrations of the

TABLE VI

Production rates[a] of the isotopes of lithium, beryllium and boron by spallation of C, N, O and Ne nuclei by hydrogen and helium

	Production (g^{-1} s^{-1}) for Target Nuclei			
Nucleus	Carbon	Nitrogen	Oxygen	Neon
Li^6 (H)	21.4×10^{-6}	9.9×10^{-6}	39.3×10^{-6}	1.26×10^{-6}
Li^6 (He)	5.39	1.59	9.53	1.23
Li^7 (H)	54.3	15.4	57.0	1.80
Li^7 (He)	8.63	2.43	14.54	1.87
Be^9 (H)	11.3	1.58	7.36	0.24
Be^9 (He)	2.39	0.96	5.75	0.74
B^{10} (H)	47.6	2.90	43.8	1.40
B^{10} (He)	2.70	1.14	6.85	0.88
B^{11} (H)	170.7	23.9	85.0	2.70
B^{11} (He)	3.65	1.13	6.78	0.87

[a] Mitler (1970).

production cross sections over the incident cosmic ray spectrum. Mitler evaluated these using an estimate for the demodulation of the cosmic ray spectrum measured at the earth to obtain the spectrum in interstellar space. We believe that he may have overestimated the low energy portion of the spectrum in interstellar space, and hence that he may have overestimated the production rates of the beryllium and boron.

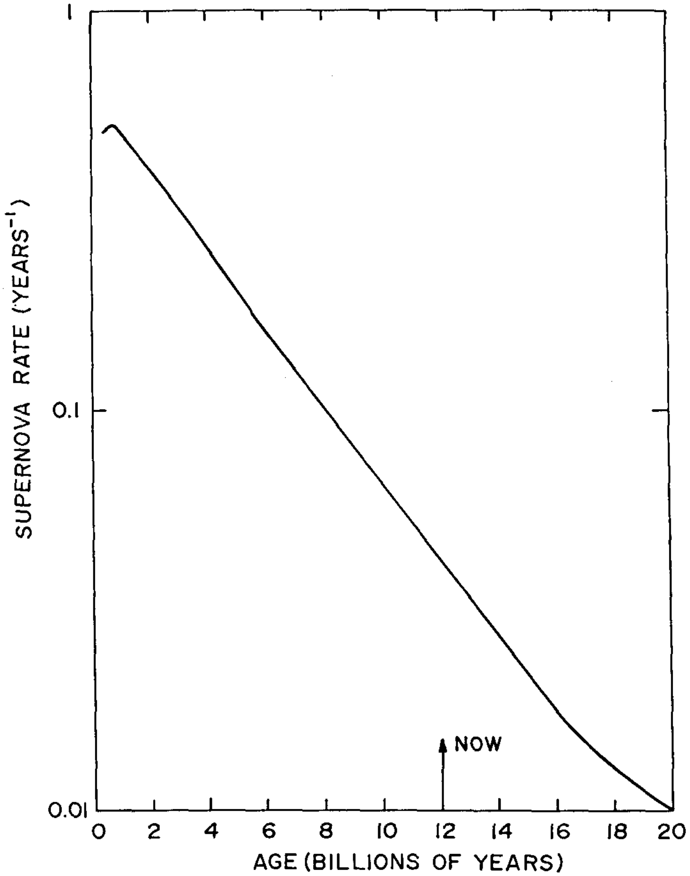

Fig. 10. The rate of occurrence of supernovae (events per year) in the galaxy is plotted as a function of galactic age.

The total production per second in the galaxy today is simply given by Mitlers' production rate times the current mass in the form of interstellar gas $P_i M_g$(NOW) It is a straightforward matter to relate the past rate of production to the parameters appropriate to today and to the parameters of our evolutionary calculations. The rate of production is proportional to the rate of supernova events (Figure 10), assuming that these are the site of cosmic ray acceleration, and to the composition of the interstellar gas, which determines the number of target nuclei available. The production rate at any time t in the past is therefore equal to the production rate now times the ratio of the supernova rates times the ratio of the composition X_i of the interstellar gas

$$P_i(t) = P_i(\text{NOW}) \left(\frac{\text{Supernova Rate at time } t}{\text{Supernova Rate NOW}} \right) \left(\frac{X_i(t)}{X_i(\text{NOW})} \right)$$

This expression neglects effects due to the stopping of cosmic rays and variations in their rate of excape from the galaxy due to the larger gas content in the past.

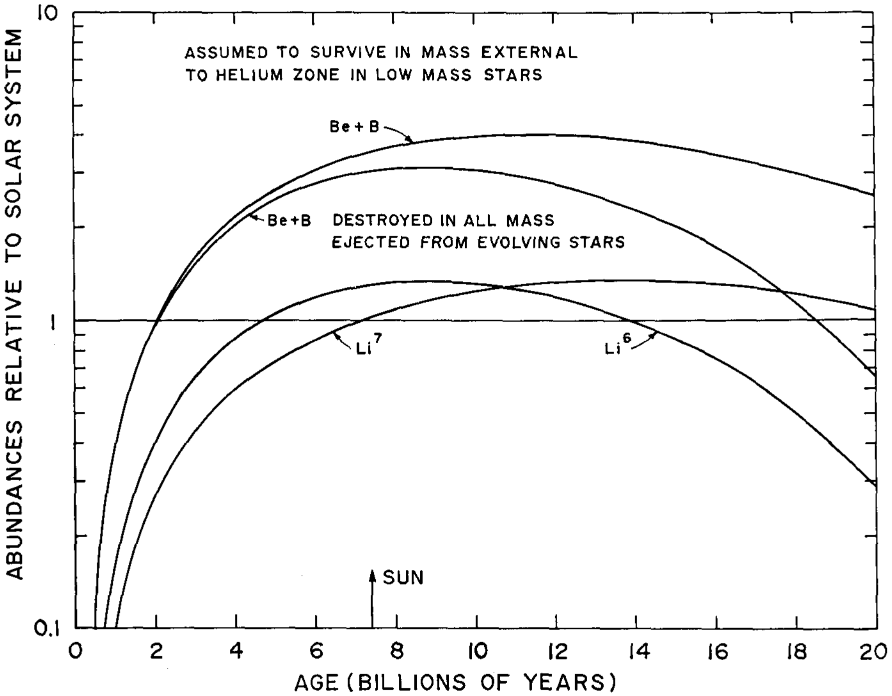

Fig. 11. The abundances of Li^6, Li^7 and the isotopes of beryllium and boron in the interstellar medium, relative to solar system matter, are plotted as a function of galactic age. The two curves for Be + B correspond to limiting assumptions concerning its destruction in stellar envelopes.

Proceeding in this manner, we have traced in time over galactic history both the abundance of Li^6 and the total abundance of the beryllium and boron isotopes. We have made this distinction between Li^6 and beryllium and boron in order to take properly into account the varying degrees of destruction of these isotopes in stellar envelopes. The abundance of Li^6 (and of Li^7) will typically be destroyed, as is deuterium, in all mass ejected form evolving stars. The appropriate prescription for the destruction of beryllium and boron in stellar envelopes, while uncertain, is much more like that adopted for He^3 in the previous section. We have therefore calculated our evolution for two different assumptions for the beryllium and boron abundances, completely analogous to our treatment of He^3. The results of these calculations are shown in Figure 11. The two curves for beryllium and boron correspond to the two assumptions: (a) beryllium and boron are destroyed in all mass ejected from evolving stars and (b) beryllium and boron nuclei survive in the mass external to the helium zone in certain low mass stars. The resulting abundance of Li^6 in these calculations is in quite good agreement with the solar system abundance at the time of formation of the sun in our galactic calculations. The abundances of beryllium and boron are rather high, however, even when it is assumed that these elements are destroyed in all mass ejected from all stars. This behavior is a consequence of the fact that the production ratios for beryllium and boron relative to lithium are somewhat higher as calculated by Mitler (1970), than the accepted values based on solar system abundances. Specifically, Mitler finds a production ratio for Li^6 relative to beryllium and boron of approximately 0.2, whereas the chondritic values are in fact closer to the value 0.5. These discrepancies in the production ratios can very likely be attributed to uncertainties in the appropriate reaction cross sections and in the low energy demodulated interstellar cosmic ray flux. We find, consistent with the conclusions of Mitler (1970) and Reeves *et al.* (1969), that the production of lithium, beryllium and boron over galactic history by cosmic ray interactions with the interstellar medium seems very probable.

C. Li^7

The production of Li^7 by means of cosmic ray interactions is difficult, if Mitler's calculated production ratios are correct. He finds, specifically, a production ratio of Li^7 relative to $Li^6 \sim 1.7$, in conflict with an observed chondritic value of 12.2. This discrepancy is not so easily explained in terms of uncertainties in the spallation cross sections for the production of these isotopes. Mitler has chosen to conclude, on the basis of these difficulties, that the production of Li^7 may be substantially cosmological in origin. This conclusion is not consistent, however, with our assumed primordial deuterium and He^3 abundances. For the conditions that we find appropriate, the abundance of Li^7 produced cosmologically will be low by roughly a factor of 10 relative to solar system values. There seems no obvious way to avoid this difficulty with the cosmological production.

There is, however, an alternative mechanism for the production of Li^7 which we find more appealing. Cameron and Fowler (1971) have recently considered the possible production of lithium in red giant stars. The question here is whether helium-burning shell flashes that can take place in advanced stages of stellar evolution may, in some instances, give rise to the complete convective mixing of the outer envelope down to the position of the helium-burning shell. This behavior has been analyzed in some detail by Schwarzschild and Harm (1967), and the processes which may occur following the mixing of hydrogen into the helium layer have been considered by Sanders (1967). Cameron and Fowler concluded that, if complete mixing does occur, the He^3 present in the envelope will be converted to Be^7 and the subsequent delayed electron capture to form Li^7 may give rise to a significant lithium abundance in the surfaces of these stars. They anticipate on this basis that the Li^7/Li^6 ratio in these stars may be extremely large, typically greater than 100. The abundance of He^3 in these envelopes will typically be that due to He^3 production during shell hydrogen burning. As the precise amount of He^3 formed by this mechanism is uncertain, so is the amount of Li^7 which will result from the subsequent evolution. We have assumed some arbitrary mass fraction of the helium zone to have been converted to He^3 and then to Li^7, and have, by tracing the Li^7 abundance over the history of the galaxy, determined that mass fraction which is consistent with the present level of Li^7 in solar system material. The variation of the Li^7 abundance with time is also shown in Figure 11. The cosmic ray produced Li^7 has also been included. Our calculations indicate that a mass fraction 5×10^{-7} of the helium zone must be converted to Li^7 in stars in the range 2 to 4 $M_\odot$, where we believe this mixing occurs, to explain the current solar system value for Li^7. This seems to us to be a reasonable fraction and one which is conservatively consistent with the level of He^3 concentration predicted by the stellar evolutionary calculations of Iben.

8. Supermetallicity and All That

The fractions of the mass released by evolving stars which are attributed to various products of nucleosynthesis (Section 2) have been adjusted in our study to assure the consistency of the abundances in the interstellar gas with solar system abundances at the time of formation of the sun in the galaxy. It is of interest to examine the implications of these prescriptions for the current levels of abundances in the interstellar medium and in young stars. The calculations illustrated in Figure 5 predict the following overabundance factors relative to hydrogen after 12 billion years of galactic history (NOW): elements in the C–Fe group and the products of *r*-process nucleosynthesis are overabundant by a factor ~ 1.5, N^{14} is overabundant by a factor ~ 1.75, the *s*-process elements are overabundant by a factor ~ 2.25 and the *p*-process elements are overabundant by a factor ~ 3. The enhanced factors for the *s*-process and *p*-process products reflect the secondary and tertiary characters, respectively, of their modes of production; that is, the seed iron-peak nuclei required for *s*-process synthesis require a prior stellar generation for their formation, and the *s*-process nuclei themselves provide the seeds for *p*-process synthesis in a subsequent generation.

As carbon-to-iron nuclei comprise the bulk of the mass of heavy elements, the overabundance factor $\gtrsim 1.5$ realized for these nuclei in our calculations should be characteristic of young stars formed in our galaxy today. The magnitude of this effect is unfortunately rather small; it is uncertain whether spectral studies of stars appreciably younger than the sun would reveal such an averaged overabundance factor. Recent observations indicate, however, that individual stars and certain galactic clusters can shown substantial overabundances of metals. Spinrad and Taylor (1969) have found, for example, that evolved K stars with abundances of Ca, Mg and Na as high as ~ 4 times the solar values exist in substantial numbers; the cool evolved stars in the galactic clusters M 67 and NGC 188 were also found to be overabundant in these elements. More recently, Spinrad *et al.* (1970) have observed that the hotter unevolved stars in these clusters are also ‘super-metal-rich’. The conclusion they reach on the basis of these and other investigations is that uniform enrichment has probably played no significant role in the history of the disk of our galaxy.

Any considerations of the influence of local inhomogeneities on stellar abundances and galactic evolution are clearly beyond the scope of this paper. It is appropriate, however, to ask what might be the consequences of gross inhomogeneities in a model of the galaxy. Studies of both the cores of giant elliptical galaxies and the nuclei of spiral galaxies (Spinrad, 1966; McClure and van den Bergh, 1968) reveal an average super-metal-rich abundance character. We have suggested previously that such gross differences in metal abundances could result from large-scale metal enrichment near the centers of elliptical galaxies due to the delayed evolution of the supernovae relative to the massive stars in the first stellar generation.

There are other consequences of our model of nucleosynthesis in the galaxy which are related to the secondary characters of the modes of production of certain nuclei. The production of *s*-process nuclei, for example, is dependent upon the initial abundance of iron-peak nuclei which act as seeds. Our particular model is somewhat unrealistic in its treatment of this problem: we have formed no stars in the mass range 2–4 $M_\odot$, which we assume to be the site of *s*-process synthesis, until the abundance of the C–Fe nuclei is $\sim 25\%$ of the solar system value. It is nevertheless worth noting that the *s*-process products are predicted to be underabundant relative to iron by a factor ~ 5 in stars for which the C–Fe elements themselves are only a factor of two underabundant relative to the sun. In a more realistic treatment of the time variation of the stellar luminosity function, a few low mass stars of low metal content would properly be formed with perhaps even more pronounced relative deficiencies of *s*-process nuclei. Observations of extremely metal-poor halo-population stars reveal, in a number of instances, such relative deficiencies of *s*-process products, e.g.: the star +39°4926 studied by Kodaira *et al.* (1970), deficient in iron by a factor $\gtrsim 100$, shows a strontium deficiency relative to iron of approximately a factor of 20; the star HD 122563 studied by Wallerstein *et al.* (1963) and more recently by Pagel (1968), deficient in iron by a factor ~ 1000, shows a barium deficiency relative to iron in the range 10–100. While these deficiencies seem real and perhaps age correlated (Pagel, 1968), only further spectral analyses of metal deficient stars can provide the answers. To the extent that N^{14} formation constitutes a secondary process, the abundance of N^{14} relative to C–Fe nuclei might be expected to exhibit a similar behavior. However, we have found that most of the N^{14} in our galactic model is formed by a primary process.

As the formation of nuclei by these secondary processes is delayed until earlier stellar generations provide the seed C–Fe nuclei, the curves in our Figure 5 which trace the ratios of the abundances of these products to their solar system values have characteristically steeper slopes. This is true both before and subsequent to the time of formation of the sun, accounting perhaps for the relative deficiencies of *s*-process nuclei in metal-poor stars. Stars younger than the sun, which are predicted to have somewhat larger metal abundances, are also predicted to have *enhanced* abundances relative to iron of nuclei produced by secondary processes.

9. A Luminosity Function for Globular Clusters

Recent analyses of the distribution of stars in globular clusters (see, for example, Simoda and Kimura, 1968) point toward the conclusion that the initial luminosity functions for these clusters typically fall off more steeply with increasing stellar mass than is observed for galactic clusters or for the solar neighborhood (Limber, 1960). A similarly modified initial luminosity function may also be appropriate to elliptical galaxies, where it might account at least in part for the high mass-to-light ratios characteristic of these galaxies.

An indirect measure of the slope of the initial luminosity functions of globular clusters may be obtained from other considerations. An excess of brightness at the center of M 15 has recently been reported by King (1967, 1968), who suggested that this could be explained by an excess of mass at the center of the cluster in the form of gas or of an unusual concentration of binaries. Wyller (1970) has suggested that this mass excess could be explained, rather, by the existence of a black hole at the cluster center with 1% of the cluster mass. An upper limit on the mass of concentrations of black holes of about 1% of the cluster mass for several globular clusters is imposed by the fact that any larger mass would visibly influence the velocity distribution of the stars – an effect which has not been observed.

According to our stellar evolutionary assumptions, all stars of mass $M > 8\ M_\odot$ will end their lives in collapse to black hole remnants involving 65% of the initial main sequence mass. For a single generation of stars formed according to the stellar birth function compiled by Limber (1960) and shown in Figure 2, the fraction of the total mass which evolves to black holes is ~ 0.12. A steeper luminosity function dependence on stellar mass will clearly reduce this fraction. We wish to determine the approximate slope of the luminosity function which is consistent with there being 1% of the mass of a globular cluster in the form of black holes. Following Salpeter (1955) we consider the quantity $\xi(M)$, the number of stars of mass M per unit interval in d ln M. For the luminosity function compiled by Limber (1960), $\xi(M)$ is quite well approximated by $\xi(M) = \text{constant}\ M^{-1.35}$.

For the purposes of the present exploration, we have assumed that a mass dependence of the form $\xi(M) = \text{constant}\ M^{-n}$ is appropriate over the mass range $0.1 \leq M \leq$ $\leq 60\ M_\odot$. The resulting black hole mass fraction is somewhat sensitive to the assumed upper mass limit; the value of $60\ M_\odot$ we have adopted is consistent with recent investigations by Ziebarth (1970) and by Larson and Starrfield (1971). For the adopted mass limits, we have determined the mass fraction in the form of black holes resulting from a single generation of stars as a function of the mass exponent n. We have assumed, further, that all gas released by evolving stars has been lost from the cluster as a consequence of its passage through the galactic plane.

The results of these calculations are summarized in Table VII. The mass exponent n which gives rise to a mass fraction of one percent in the form of black holes lies in the range $n \sim 1.9$. For this case, the mass fraction in the form of unevolved stars (stars of lifetime greater than 12 billion years) is 0.94, and the remaining 5% of the mass is

TABLE VII

Fractional mass in black holes as a function of the assumed stellar birth function

	Fractional Mass			
Mass Exponent n	Black Holes	Neutron Stars	White Dwarfs	Unevolved Stars
1.35	0.099	0.017	0.109	0.77
1.50	0.057	0.011	0.089	0.84
1.75	0.021	0.0056	0.059	0.91
2.00	0.0074	0.0026	0.037	0.95
2.25	0.0026	0.0012	0.0227	0.97

in the form of neutron star and white dwarf remnants. Detailed analyses of the distributions of stars in globular clusters will ultimately define for us the appropriate luminosity functions. It seems suggestive, however, that the luminosity function we have estimated is at least consistent with current observations in spite of the many inherent uncertainties.

10. Conclusions

A number of the assumptions defining our ‘consistent’ model of galactic evolution are extremely speculative in nature. Future theoretical and observational findings, including, for example, the detection of intergalactic gas clouds of temperatures near 10^6 K in the form of a substantial soft X-ray flux, could force the abandonment of various aspects of the picture we have adopted. There are, however, a number of rather general conclusions which can be drawn from our investigations which are independent of these speculations.

(1) *Luminosity function.* The luminosity funtion or ‘stellar birth function’ characteristic of young galactic clusters and the solar neighborhood *cannot* have been appropriate during the earliest stages of galactic history. If this function *were* universal, one would expect to observe many stars of low mass (which have not evolved over the lifetime of the galaxy) with metal abundances in the range $Z \lesssim 0.1\ Z_\odot$. Further theoretical studies of the mechanism of star formation are required to determine the time variation of the form of the luminosity function throughout galactic history.

(2) *Cosmochronology.* The assumption of an exponentially decreasing gas content of the galaxy provides a consistent cosmochronology with regard to the radioactivities I^{129}, Pu^{244}, Th^{232}, U^{235} and U^{238}, provided that the stars in which these nuclei are produced are reasonably short lived ($\tau \lesssim 5 \times 10^8$ years or $M > 2\ M_\odot$). Our model gives a concordance at 7.4×10^9 years, defining the time of formation of the sun in the galaxy. An isolation time – the time between the isolation of the solar system material from sources of galactic nucleosynthesis and the cooling of the meteoritic parent bodies – of approximately 80 million years is consistent with both the Pu^{244} and the I^{129} abundance levels. No last minute spike of *r*-process synthesis is required to account for the Pu^{244}, assuming the lower value of the Pu^{244}/U^{238} ratio determined by Podosek (1970).

(3) *Primordial galactic abundances.* The composition of the gas cloud from which our galaxy condensed was taken to be hydrogen (0.77), He^4(0.23), deuterium ($8.2\times\times 10^{-4}$) and He^3(7.1×10^{-5}) by mass. These abundances are consistent with the calculations of cosmological nucleosynthesis of Wagoner, Fowler and Hoyle (1967) for a universe of present mean density $\sim 10^{-31}$ g cm^{-3}. No significant production of He^4 takes place in our models for reasonable assumptions concerning the evolution of normal stars. The initial D^2 abundance is high by a factor ~2.7 relative to solar system material; the destruction of D^2 in the envelopes of stars of all masses reduces its abundance by this factor prior to the time of formation of the sun. It is important to note that one does not have great latitude in choosing a cosmological abundance distribution, if the resulting deuterium abundance is to be consistent with the solar system value (see Table V). For a universe of present mean density 3×10^{-32} g cm^{-3}, the initial D^2/H^1 ratio is high by a factor ~13; dilution of the interstellar gas by matter which has passed through stars is insufficient to reduce this by more than a factor ~2–3. For a universe of present mean density 3×10^{-31} g cm^{-3}, the initial D^2/H^1 ratio is low by a factor ~2; no mechanism for deuterium production in the galaxy is known at present which could provide the required build-up.

(4) *Lithium, beryllium and boron.* The production of Li^6 and the isotopes of beryllium and boron by the interaction of cosmic ray protons and alpha-particles with the abundances of C^{12}, N^{14}, O^{16} and Ne^{20} in the interstellar medium is found to be consistent with the details of our galactic model. In our treatment, the production rates calculated by Mitler (1970) are adjusted throughout galactic history by the predictions of our model for the rate of supernova events (assumed to be the source of the cosmic ray flux) and the composition of the interstellar medium. The production of Li^7 by cosmic ray interactions is insufficient to account for the Li^7/Li^6 ratio ($Li^7/Li^6=12.2$) observed in chondrites. We have assumed that Li^7 is produced as a consequence of convective mixing in the helium-burning shell flashes discussed by Scharzschild and Harm (1967); only a small fraction of the He^3 formed during shell hydrogen burning must be burned to Be^7 to account for this Li^7 production.

(5) *Mass loss.* The magnitude and influence of stellar mass loss is extremely uncertain. We can obtain a crude limit, from our model, on the extent of mass loss in stars which give rise to supernovae, assuming that p-process synthesis takes place in the passage of the supernova shock through the residual hydrogen envelope. For stars of mass in the range $4\leq M\leq 8\ M_\odot$, 45% of the initial main sequence mass is assumed to constitute mass loss; this leaves approximately 32% of the mass in a hydrogen layer above the hydrogen burning shell. We have found that half of all initial s-process and r-process nuclei in this region must be converted to p-process products, in order to account for the abundances of the p-nuclei in solar system material. If we assume this process to be totally efficient (a rather extreme assumption), then the *complete* conversion of heavy elements to p-process nuclei must take place in only 15% of the mass of these stars. The *total allowed mass loss* consistent with this picture is therefore $\lesssim$60% of the initial main sequence mass for stars in this mass range.

The mass loss restrictions imposed by these p-process conditions might be considerably relaxed, however, if some enhancement of the abundances of the seed *s*-process nuclei relative to hydrogen were possible. Such enhancement indeed is known to occur in *S* and carbon red giant stars; the operation of the *s*-process and subsequent mixing which may account for these stars has been discussed by Sanders (1967) and by Cameron and Fowler (1971). We have found that this mechanism operating in stars of mass 2–4 $M_\odot$ can account for the solar system abundance of *s*-process nuclei, if a mass fraction ~0.006 of the helium zone is processed in this manner. The same mass processed first to *s*-process and then to *p*-process nuclei in stars of mass 4–8 $M_\odot$ could easily account for the solar system *p*-process concentration. These arguments would still require, however, that a few percent of a solar mass, overlying the helium layer, must be raised to a temperature in excess of $\sim 2\times10^9$ K.

(6) *Supernova rate.* The current supernova rate predicted by our galactic model is one per 25 years, in agreement with the value of one in 23 years arrived at by Tammann (1970). This number can be adjusted somewhat by varying the rate of depletion of interstellar gas, by revising our estimate of the total mass of gas in the galaxy following the initial pregalactic phase of star formation, or by choosing a slightly different mass range as that giving rise to supernovae. It is not possible, however, to reduce the lower limit on the masses of supernovae significantly below 4 $M_\odot$ without restricting the allowed mass range considerably. We have found, for example, that the assumption that supernovae correspond to stars in the mass range $2 \leq M \leq 8$ $M_\odot$ leads to a supernova rate considerably higher than the Tammann value.

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Note added in proof. Our current thinking concerning the site of nucleosynthesis includes the view that supernovae corresponding to stars in the mass range $4 \leq M \leq 8$ $M_\odot$ eject mainly iron-peak elements. We wish to thank Dr. W. D. Arnett for quite correctly pointing out that Paczynski's (1970) models for stars in this mass range predict temperature and density conditions for the regions immediately above the core which are inconsistent with the formation of intermediate mass nuclei ($20 \lesssim A \lesssim 46$). The products of explosive carbon and oxygen burning and of incomplete silicon burning, on this picture, result from the explosions of more massive stars. We expect to carry out calculations of galactic histories subject to such revised assumptions. We do not anticipate, however, that the more general qualitative conclusions drawn in this paper will be invalidated.

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