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Representation and Reality in Kant's Antinomy of Pure Reason

Damian Melamedoff-Vosters

University of Manitoba/New York University Shanghai, Shanghai, China
Email: damian.melamedoff@gmail.com

Abstract

In this article, I take on a classic objection to Kant's arguments in the Antinomy of Pure Reason: that the arguments are question-begging, as they draw illicit inferences from claims about representation to claims about reality. While extant attempts to vindicate Kant try to show that he does not make such inferences, I attempt to vindicate Kant's arguments in a different way: I show that, given Kant's philosophical backdrop, the inferences in question are not illicit. This is because the transcendental realists that Kant was arguing against have certain philosophical commitments about the nature of *ground* which, if true, warrant the inferences that Kant draws. This historical corrective not only allows us to better understand Kant's own thinking in the Antinomies but it also has important upshots for our understanding of Kant's transcendental idealism.

Keywords: antinomy; transcendental idealism; ground; infinity

1. Introduction

In the first Antinomy, Kant presents the following argument against the eternity of the past:

> . . . if one assumes that the world has no beginning in time, then up to every given point in time an eternity has elapsed, and hence an infinite series of states of things in the world, each following another, has passed away. But now the infinity of a series consists precisely in the fact that it can never be completed through a successive synthesis. Therefore an infinitely elapsed world-series is impossible, so a beginning of the world is a necessary condition of its existence; which was the first point to be proven. (A426/B454).[1]

Together with an argument against the claim that the world has a beginning in time, Kant takes the first Antinomy to provide an indirect proof of transcendental idealism (A506/B534). This is because only the transcendental idealist, Kant thinks, can claim that the world neither has a beginning nor is eternal.

According to an influential objection to this argument, which dates as far back as Russell (1914), Kant's quoted argument is question-begging, as it already presupposes some form of idealism. Russell puts the objection as follows:

> . . . the word 'synthesis', by suggesting the mental activity of synthesizing, introduces, more or less surreptitiously, that reference to mind by which all Kant's philosophy was infected . . . (Russell 1914: 160-1)

Norman Kemp Smith similarly presents the objection, addressed to Kant's parallel argument against the infinity of space, as follows:

> From the impossibility of traversing infinite space in thought by the successive addition of part to part, Kant here argues that 'an infinite aggregate of actual things cannot be viewed as a given whole', and consequently that the world cannot be infinitely extended in space. That is, from a *subjective impossibility of apprehension* he infers an *objective impossibility of existence.* (Kemp Smith 1918: 485)

The thought behind this objection is that Kant's use of 'synthesis' in his argument presupposes some kind of idealism, as it assumes that facts about what we can or cannot do in trying to understand the world are relevant for what the world itself is like. While Kant might be happy to accept some version of this claim, it does not seem like a claim that may be *presupposed* in an argument for transcendental idealism. An argument that assumes that we can draw inferences from 'subjective impossibility of apprehension' to 'objective impossibility of existence' is, according to this objection, unsuited to be an argument for transcendental idealism.

The goal of this article will be to provide a reconstruction of Kant's argument that (a) helps explain why Kant makes this psychological presupposition in the context of this argument and (b) shows that the argument is successful against the philosophers that Kant had in mind when making it. In doing so, I will show an important continuity between Kant and his predecessors in their views about ground-consequence relations, involving what I will label the Reason Constraint on Ground:

> **Reason Constraint on Ground:** if A is the ground of B, then it is (in principle) possible to reason from A to B, and from B to A.

As I will argue, the Reason Constraint on Ground was a commonly shared assumption between Kant and his transcendental realist opponents, and one that, given this fact about the dialectical context, Kant was entitled to use without explicitly arguing for. I will also show that taking the Reason Constraint on Ground as a background assumption allows us to give a valid and compelling reconstruction of Kant's argument in the thesis of the First Antinomy.

Before we turn to our discussion, let us first briefly look at some attempted responses on Kant's behalf. One attempt at avoiding the charge of question-begging comes from Allison (2004). Allison notes that Kant draws a distinction between 'analytic' and 'synthetic' wholes:

> A *totum syntheticum* is a whole composed of parts that are given separately (at least in thought). Not only does the concept of such a whole presuppose its distinct, pre-given parts, it is also conceived as the product of the collection (in Kant's term, 'synthesis') of these parts. (Allison 2004: 369)

Allison himself does not wish to take all talk of representation out of Kant, as he takes the notion of a *totum syntheticum* to involve the 'conceptual claim' that 'a *totum syntheticum* is possible' just in case 'a complete collection of its parts is *conceivable*' (Allison 2004: 369, italics added). This leads him to conclude that Kant does have a response to the charge of psychologism, since Kant's point 'has nothing to do with the presumed psychological impossibility of grasping or comprehending the infinite' (p. 370). Rather,

> the problem is that the rule or procedure for thinking a totum syntheticum clashes with the one for thinking an infinite quantity. The former demands precisely what the latter precludes, namely, completability (at least in principle). (Allison 2004: 370)

I find myself quite sympathetic with this general approach, but I must note that it is not at all clear to me how it avoids the psychologism charge. While this argument does not infer from the impossibility of *apprehension* to the impossibility of existence, it does infer from the impossibility of an 'intellectual procedure' to the impossibility of existence. Why would this fare any better at avoiding Russell's critique?

A perhaps more promising, if also more surprising, attempt comes from Boehm (2014). Boehm's idea is that we can further extend Allison's point, but to explicate it in terms of dependence rather than in terms of intellectual procedures. Boehm takes, as his starting point, a comment from Mendelssohn:

> [Wolff] proved that Spinoza believed that it is possible to produce, by combining together an infinite stock of finite qualities, an infinite [thing]; and then he proved the falsity of this belief so clearly, that I am quite convinced that Spinoza himself would have applauded him. (as quoted in Boehm 2014: 79-80)

As Boehm sees it, talk of a 'synthetic whole' need not make reference to anything intellectual and should instead be understood as a whole that is 'composed' by its parts (ibid.). This is contrasted with an 'analytic whole' which is not composed by its parts, but is instead prior to them.[2] Kant's argument, then, is to be understood as a version of this Wolffian objection, which simply takes it to be impossible for a *composed* whole to consist in an infinite number of parts.

Boehm's argument has the virtue of avoiding Russell's objection altogether. But it does so at the cost of giving a reading of Kant that strays very far from the text. In fact, it is clear that Kant's argument does require some commitments about representation, as can be seen from the second part of the first Antinomy: Kant's argument against the infinity of space.

> ... we can think of the magnitude of a quantum that is not given as within certain boundaries of every intuition in no other way than by the synthesis of its parts, and we can think of the totality of such a quantum only through the completed synthesis, or through the repeated addition of units to each other. Accordingly, in order to think the world that fills all space as a whole, the successive synthesis of the parts of an infinite world would have to be

regarded as completed, i.e., in the enumeration of all coexisting things, an infinite time would have to be regarded as having elapsed, which is impossible. (A428-9/B456-7)

If Kant's argument was as Boehm interprets it, we would not expect Kant to make any reference to the 'enumeration of all coexisting things' in 'infinite time'. Nor would we expect the synthesis in question to be 'successive'. We simply cannot give a reconstruction of Kant's argument that does not draw some connection between representation and reality.

What this suggests is that an attempt to vindicate Kant's argument should not be trying to shy away from Kant's commitments to these psychological claims but should rather be trying to explain why Kant takes on such commitments. An accusation of 'begging the question' is only successful, after all, if it can show that the premise in question is one that the opponents of the conclusion – in our case, the transcendental realists – will in fact deny. But this should in turn lead us to the following question: who are the transcendental realists? Is it the people Kant was in conversation with in the eighteenth century, or the people that we, the historians, are in conversation with in the twenty-first? If it is the latter, then I openly admit that Kant's argument will be unsuccessful. Insofar as the transcendental realism on offer is that of twentieth-century analytic philosophy, I agree with Russell that Kant's argument will not be compelling. But the fact that Kant's arguments are not compelling to a contemporary audience does not imply that they would be uncompelling to *Kant's* audience. What I seek to do in this article is to show that, by paying closer attention to what Kant's audience believed, we can see that the argument in the Antinomy constitutes a powerful objection to their conception of our relation to the empirical world.

I will do this by showing that Kant and the Wolffian philosophers he was objecting to share a common assumption, which I have labeled the Reason Constraint on Ground. Again:

Reason Constraint on Ground: if A is the ground of B, then it is (in principle) possible to reason from A to B, and from B to A.

This assumption tells us that no ground-consequence relations can be obtained in the world unless it is possible for a reasoner to infer from ground to consequence and from consequence to ground. The specific details of the Reason Constraint on Ground, as well as a more thorough discussion of who counts as a possible reasoner, will be discussed in more detail after some historical context has been given. For now, it is simply worth remarking that, should someone accept the Reason Constraint on Ground, they would be in a position to draw inferences from claims about which kinds of reasoning activities are possible, to conclusions about what kind of ground-consequence relations are possible.

This article proceeds as follows. In section 2, I show that Wolff has a clear commitment to the Reason Constraint on Ground, as do two of his most influential (and influential to Kant) followers, Baumgarten and Meier. This means that the Reason Constraint on Ground is, excusing the pun, *common ground* between Kant and his contemporaries. In section 3, I interpret Kant's earliest published version of the argument of the first Antinomy, given in his Inaugural Dissertation of 1770. Since

Kant had not yet fully transitioned to the vocabulary that he employs in the *Critique of Pure Reason*, this text allows us to more easily see the way in which the Wolffian constraint is deployed in Kant's argument. After reconstructing Kant's argument, I turn in section 4 to the implications it has, in the *Critique*, for his idealism about space and time. I conclude, in section 5, with what I take to be some significant upshots for the interpretation of Kant.

   This approach constitutes a limited defense of Kant's argument. It is a defense, in that it shows that, given the assumptions that Kant shared with his interlocutors, the argument is convincing. It is limited, in that the argument will be unconvincing to anyone who does not share, as an assumption, the Reason Constraint on Ground. This may be disappointing to the historian who, more boldly than I, seeks to persuade a contemporary audience that Kant's arguments are still defensible. But it should be a significant contribution to the project of the historian who, rather than defend Kant, seeks primarily to understand him.

2. German rationalist metaphysics

Baumgarten's *Metaphysica*, Kant's lecture source through most of his career, defines metaphysics as 'the science of the first principles in human cognition [*cognitione*]' (§1).[3] This is an interesting definition of metaphysics for at least two reasons. First, it is anthropocentric: metaphysics is introduced as the study of something from the human perspective.[4] But secondly, and more importantly, metaphysics is the study of the principles by which things are *cognizable.* From the very first sentence of the *Metaphysica*, Baumgarten commits himself to a close relationship between the objects of metaphysical study on the one hand and our powers of representation on the other.

   This is not unique to Baumgarten. Although not every German philosopher in the eighteenth century explicitly defined metaphysics in terms of cognition, many metaphysical notions were often introduced in explicitly representational terms.[5,6] This is no less the case when it comes to the philosophical notion of a *ground*, a notion that, as we will see, is key to understanding Kant's argument in the Antinomies.[7] Wolff introduces into German philosophy the technical notion of ground (*Grund*), which he claims is a translation of the French *raison* (and the Latin *ratio*[8]), in his *German Metaphysics.* He defines a *ground* as follows:

> If a thing A contains in itself something from which one can understand [*verstehen*] why B is, B may be either something in or outside of A; one calls the thing which is found in A the **ground** of B: A itself is called the **cause**, and we say that B is **grounded** in A. (*German Metaphysics* §29, author's translation)

Wolff's definition inspired several other philosophers to also define the notion of a ground in terms of our representational capacities. Baumgarten says that a *ground* is 'that from which it can be cognized [*cogniscibile est*] why something is' (*Metaphysica* §14). Gottsched also follows Wolff in defining ground as that which allows us to grasp [*Begreifen*] why something is (*Erste Gründe* §217). And Kant himself, in his *New Elucidation*, defines an antecedently determining ground as that which renders something intelligible (NE, 1: 392).[9]

It is also important here to note that it was quite common in the eighteenth century to claim that not only is it possible to understand a consequence from its ground but that it is also possible to come to cognize (though not understand) a ground from its consequence. Meier, for example, states:

> One can always infer from a consequence [*Folge*] to the ground [*Grund*], and hence also to the sufficient ground [*hinreichenden Grund*]; or, when the consequence is there, so is the ground; or the truth and possibility of its ground, and its sufficient ground, follows from the truth and possibility of the consequence. (*Metaphysik* §41, author's translation)

The young Kant is also in agreement with the Wolffian line, as he writes in the *New Elucidation*:

> There is nothing in that which is grounded which was not in the ground itself. For nothing is without a determining ground; accordingly, *there is nothing in that which is grounded which does not reveal its determining ground.* (NE, 1: 406, italics added)

According to this general Wolffian line, then, grounds are (in principle) cognizable from their consequences, and consequences are (in principle) cognizable from their grounds.[10] We can then see the way in which the Reason Constraint on Ground makes its way into the Wolffian tradition.

But in order for us to be fully warranted in attributing this principle to Wolff and his followers, we must also discuss the way in which the activity of *reasoning* is operative in this principle. The general idea, proposed by Wolff, is to identify a faculty of Reason, which allows us to draw inferences between things that are connected by means of some ground-consequence relation or other.

> The insight we have into the *nexus* [*Zusammenhang*] of truths, or the faculty which sees the connections of truths, is called Reason. (*German Metaphysics* §368, author's translation)[11]

The notion of a *nexus* is borrowed from earlier discussions in Latin texts, and in the German rationalist tradition, it is used to express a relation of necessary connection between a dependent thing and what it depends on.[12] This means that Reason is the ability to infer a ground from a consequence or a consequence from its ground.

The relationship between reason and ground is especially salient in *Metaphysica* §643, where Baumgarten states:

> That which can be cognized through some ground is called *reasonable* [*rationabile, vernünftig*], and that which can be cognized through none is called *unreasonable* [*irrationabile, unvernünftig*]. Now every possible being is doubly rational and connected (§24). Both its ground and its consequence, along with the *nexus* between these two, are conceivable in themselves. Therefore, everything possible is reasonable. Everything unreasonable, or whatever is contrary to reason, is impossible (§§7, 8). (trans. in Baumgarten et al. 2013)

I will call the inference process of deriving a ground from a consequence or a consequence from a ground, a *reason-step*. An example of a reason-step would be to find that something has an accident, noting that such accidents must have a ground of a particular kind, and inferring its ground. Nothing here requires us to think that there is only one kind of reason-step. It may be that the kind of *nexus* that holds between parts and wholes is different from the kind of *nexus* that holds between causes and their effects and that different cognitive capacities or circumstances may be required to perform the relevant reason-step. The term 'reason-step' will here signal the generic inference process of going from one thing to another that is in a *nexus* with it. The differences between types of reason-steps will become relevant later.

Before we turn to a discussion of Kant's own argument, it is worth anticipating a few misconceptions that may arise from this approach.

First, we can note that the claim that grounds are cognizable from their consequences (and *vice versa*) does not entail that everything *has* both a ground and a consequence. While Baumgarten (*Metaphysica* §24) did think that everything had both a ground and a consequence, not everyone agreed. Crusius denied that everything has a determining ground (*Entwurf* §38, *Weg* §142). The young Kant denied that everything has a consequence (NE, 1: 408-9). But both Crusius and Kant accepted the epistemic consequences of their choice. Crusius was happy to commit to some things not being cognizable *a priori*, since they may lack a determining ground, and Kant to some things not being cognizable *a posteriori*, since they may lack a consequence. But both agree that if something has a determining ground, then it is cognizable through it, and if something is a determining ground, then it is cognizable through its consequence.

Second, this principle does not require that the *only* way to know a ground is by means of reasoning. In fact, Kant himself does not think that this is how God cognizes grounds.[13] But the fact that God does not cognize by reason-steps is no threat to the truth of the Reason Constraint on Ground. This is because the constraint states that it must always be *possible* to reason to a ground or a consequence. It does not state that this is the only way to cognize ground-consequence relations, but it does state that this must be *one* of the ways to cognize any such relation.

To make clearer why this concern is irrelevant, consider the claim that a necessary condition for being a material object is that it is possible to physically touch such an object. If someone were to object to such a condition by claiming that God, since he is incorporeal, cannot touch things, such an objection would be off the mark. The fact that material objects cannot be touched by God is no reason to deny that all material objects can be touched. Similarly, the fact that grounds cannot be inferred by God is no reason to deny that all grounds can be inferred. This shows that the fact that God does not reason poses no threat to the arguments reconstructed above. As a result, it is not a violation of the Reason Constraint of Ground to say that God can know a ground without reasoning.[14]

A third concern one might have about the constraint is that only an idealist would accept it. Now, perhaps there is a definition of idealism according to which this is true. But if we understand idealism as the view that represented reality somehow depends on our representation of it, it is hard to see why this would be a commitment of the principle, as the principle itself makes no claims at all about dependence. The Reason

Constraint on Ground is certainly *rationalist* in nature, but we should not conflate rationalism with idealism. After all, the Reason Constraint on Ground could just as easily be true because our minds were created in a way that allows us to understand the world.

Consider, as another example, the purported relation between conceivability and possibility. It is rare to find a philosopher who claims that only an idealist could hold that conceivability and possibility are co-extensive. By parity of reasoning, it should also be rare to find a philosopher who thinks that only an idealist can hold that dependence and cognizability-by-reason are co-extensive.

3. Infinite grounds

It is quite clear that Kant does accept this principle in his early thought, as evidenced by the quotations from the *New Elucidation* presented above. But there is also evidence that he accepts this principle in the Critical period as well. Note, for example, Kant's account of actuality (*Wirklichkeit*):

> The postulate for cognizing the **actuality** of things requires **perception**, thus sensation of which one is conscious – not immediate perception of the object itself the existence of which is to be cognized, but still its *nexus* [*Zusammenhang*] with some actual perception in accordance with the analogies of experience, which exhibit every real *nexus* in an experience in general. (A225/B272)

At the beginning of this section, Kant makes it clear that the postulates in question are not merely for 'cognizing' actuality, but are in fact 'nothing further than definitions of the concepts of possibility, actuality, and necessity in their empirical use' (A219/B266). This means that what Kant is telling us in the passage quoted above is that for something to *be* actual (in the empirical sense), it must be the case that one can cognize it from some actual perception by repeated applications of the principles given in the Analogies of Experience.

As we can see, then, there is reason to think that the Critical Kant accepts the Reason Constraint on Ground, at least in the case of objects in space and time. And given that the transcendental realists we have looked at accept the constraint *tout court*, this should qualify as a common assumption, shared by both realist and idealist philosophers in Kant's time. Hence, if what Kant's argument presupposes is merely the Reason Constraint on Ground, then Kant cannot plausibly be accused of begging the question against his contemporaries. He merely has, as a background assumption, something that was agreed upon by all parties in this debate.

With these preliminaries aside, let us now turn to the problem that Kant identifies in the thesis of the first Antinomy. As explained above, it will be helpful, for the sake of clearly noting Kant's connection to the rationalist tradition, to begin with a look at his presentation of the argument when he was still somewhat of a rationalist: his 1770 Inaugural Dissertation. Here, Kant notes that if matter can be continuously divided, it will be impossible to successively decompose it into simples. Similarly, if matter extends infinitely, then it will be impossible to successively synthesize it into a world.

The concept of a simple will only emerge by means of analysis, and the concept of a whole will only emerge by means of synthesis, if the respective processes can be carried out in a finite and specifiable amount of time.

But in the case of a *continuous magnitude*, the *regression* from the whole to the parts, which are able to be given, and in the case of an *infinite* magnitude, the *progression* from the parts to the given whole, have in each case *no limit*. Hence, it follows that, in the one case, complete analysis, and, in the other case, complete synthesis, will be impossible. (ID §1, 2: 388)

Kant then mentions that, 'since *unrepresentable* and *impossible* are commonly treated as having the same meaning, the concepts of both the *continuous* and the *infinite* are frequently rejected' (ID §1, 2: 388). This is the key to our interpretation: since it is not possible to reason, in a finite and specifiable amount of time, to infinitely small simples and to an infinitely large world, we appear to be forced to conclude that such entities are unrepresentable, and therefore impossible.

Putting aside for now Kant's own evaluation of the argument, it is quite noteworthy that this line of reasoning crucially relies on the Reason Constraint on Ground. The problem with infinite descent to a smallest part (and infinite ascent to a largest whole) is not with infinity *per se*, but with the fact that these infinitely small/large entities are supposed to be in a part-whole *nexus* with the ordinary objects that we immediately represent. The fact that they are in such a *nexus* means that they should be cognizable by a repeated series of part-whole reason-steps. Since this would require an endless, infinitary task, Kant concludes that this is impossible. Hence, infinitely small/large entities cannot be in a part-whole *nexus* with the entities we perceive.

Before moving to the conclusions that Kant draws from this argument, I want to note that, from the perspective of the twenty-first century, one might suspect that infinitary tasks are not impossible, due to the philosophical literature surrounding the notion of a *supertask*. Since the notion of a supertask is a historical anachronism, I reserve this discussion for the Appendix to this article, where I show that the possibility of supertasks poses no threat to Kant's reasoning.

For now, we can turn to how Kant's reasoning in the Inaugural Dissertation can be represented as a *reductio* of the claim that there is an infinitely large world.

1. Suppose there is an infinitely large whole with finitely sized parts. (Assumption for *reductio*)
2. If A is a whole of which B is a part, then A can be cognized from B by some series of reason-steps that take us from parts to wholes. (From the Reason Constraint on Ground)
3. If there is an infinitely large whole with finitely sized parts, then the world cannot be cognized from its finite parts by some series of reason-steps that take us from parts to wholes. (Premise)
4. ⊥

The thought behind premise 2 is as follows: if the reason-steps that take us from part to whole can never take us from finite things to an infinite world, then an infinite world cannot be a composite of finite things. This is dictated by the Reason Constraint

on Ground. Premise 3 is justified by Kant's claim that no infinite successive task can be completed (ID §1, 2: 388).

We can similarly see how Kant's discussion in the ID gives rise to a puzzle for the claim that there are simples in continuous space. While the details are slightly different, the general idea is the same: simples that are *parts* of material objects in continuous space are banned by the Reason Constraint on Ground.[15] The idea can be explained as follows. If space is continuous, then every occupied region of space has an occupied subregion. So, the reason-step that lets us cognize the parts of an extended object will always lead us to more extended regions. At no point in the sequence of dividing will we divide something into simples. Since it is impossible by means of part-whole reason-steps to arrive at the infinitely small from the extended, we must conclude that composite extended objects are not ultimately composed of infinitely small simple parts. Following the formalization given above, we can reconstruct the argument as a *reductio* of the claim that the world is composed of infinitely small simples.[16]

1. Suppose *o* is a whole that has infinitely small parts. (Assumption for *reductio*)
2. If A is a whole of which B is a part, then A can be cognized from B by some series of reason-steps that takes us from wholes to parts. (from the Reason Constraint on Ground, and the fact that composition is a *nexus*)
3. If *o* is a whole that has infinitely small parts, then *o*'s parts cannot be cognized from *o* by some series of reason-steps that take us from wholes to parts. (Premise)
4. ⊥

Note that the argument here is structurally identical to the argument against an infinitely large world. Premise 1 is just our assumption for *reductio*. The reason to think Premise 3 is true is that the reason-step involved in cognizing the parts of a whole is that of decomposition, division or, as Kant calls it in the ID, 'analysis'.[17] But there is no way to step-wise divide an extended thing up such that some division step divides something into non-extended things. Hence, we should also accept premise 3.[18]

4. The ideality of space and time

So far, we have noted that Kant's concern, at least in the Inaugural Dissertation, stems from a distinctly psychological premise: that it is impossible for a reasoner to reason to simples and to an infinite world. It also has metaphysical upshots, as it may suggest that the notions of *infinity* and *continuity* are themselves 'absolutely impossible' (ID §1, 2: 388).[19]

To be clear, Kant does not endorse the rejection of infinity and continuity and goes as far as to say that the reasoning here is 'perverse' (ID §1, 2: 389). But this is not because it runs together representation and reality, as Kant explicitly states that 'whatever cannot be cognized by any intuition at all is simply not thinkable and is, thus, impossible' (ID §25, 2: 413). Kant is still following the Wolffian tradition of taking epistemic facts to have metaphysical import. Rather, the reason why Kant calls the rejection of infinity and continuity perverse is because it assumes that the way to reason to simples and the world involves the kind of reason-step that takes us from parts to wholes (and vice versa). Kant thinks that a principled way to reject these

arguments, without denying that space is infinite and continuous, and while continuing to accept the Reason Constraint on Ground, is to think that our cognition of simples and the world cannot be acquired by means of reason-steps from parts to wholes (and vice versa).

But, just as the original epistemic concern has metaphysical consequences, so does this epistemic solution to the puzzle. If simples and the world cannot be inferred by means of successive part-whole reason-steps, then it follows that they are not in a part-whole *nexus* with the things that we have directly represented. This is a very powerful metaphysical upshot, as it gives Kant exactly what he tries to argue for in the Inaugural Dissertation: the existence of an intelligible world of simples that underlies (but does not *compose*) the spatial world.

To explain this in more detail, it is important to highlight two of Kant's commitments at the time of his writing of the Inaugural Dissertation: his commitment to the infinity and continuity of space and his commitment to the existence of simples and a world. Kant's commitment to the infinity and continuity of space is unwavering throughout his entire career. He argues for it in the *Physical Monadology* of 1756, the Inaugural Dissertation itself, and continues to accept it in the *Critique of Pure Reason*.[20] Whether the infinity and continuity of space entails the infinity and continuity of *things* in space is not obvious, but it certainly looks like an inference that Kant is willing to license, especially given his denial of space as a 'boundless *receptacle* of possible things' (ID §15, 2: 403; see also A432-3/B460-1).

I will not here discuss whether Kant maintains a commitment to simples and a world (of things in themselves) in the *Critique of Pure Reason.* This is because that would require taking a stand on a much bigger debate about whether there is any sense in which we can make positive claims about things in themselves.[21] But at the time in which the Inaugural Dissertation is written, Kant certainly believes that there must be simples and that there must be a world, a commitment that he has maintained since the *Physical Monadology*.[22]

Due to these two commitments, and the argument we have reconstructed in the previous section, Kant is faced with an interesting situation, as he is now committed to the soundness of the following argument:

1. There are simples; there is a world.
2. Space is both continuous and infinite.
3. There are no simples in continuous space; there is no world in infinite space.
4. Therefore, there are simples and a world that are not in space.

As we have seen, the first premise of this argument is a commitment Kant has for his entire pre-Critical career. The second premise is one he carries well into the Critical philosophy. Kant's justification for the third premise has been the focus of section 3 of this article. The conclusion follows from these premises.

We can then see that Kant's epistemic discussion in the introduction to the Inaugural Dissertation contains an argument for the metaphysical view that there are two sets of things: the things in space, which can be cognized by means of part-whole reason-steps, and the things which are not in space (the simples and the world) and must be cognizable in a different way.

Kant himself puts this very clearly in the Inaugural Dissertation, where he expresses some puzzlement at the claim that a 'series that could never be completed by successive addition could nevertheless be given as a *whole*' (ID §8, 2: 392) and offers the following solution:

> Let him who wishes to extricate himself from this thorny question note that neither the successive nor the simultaneous co-ordination of several things (since both co-ordinations depend on concepts of time) belongs to a concept of a whole which derives from the *understanding* but only to the conditions of *sensitive intuition*. (ID §8, 2: 392)

Kant's thought is that the constraint that the world as a whole be cognized by a successive series of reason-steps from spatial objects is only warranted if the world is *composed* of spatial objects – things for which cognition requires a series of part-whole reason-steps from things that we have cognized through sensation. But this only rules out the possibility of a world if we also accept that 'whatever things there are ... are necessarily somewhere' (ID §15, 2: 406). Once Kant accepts the ideality of space and time, he denies that everything is somewhere and instead concludes that simples and the world are not anywhere, and they are therefore not at the end of an infinite series of (de)composition relations. This conclusion, and only this conclusion, allows him to accept that we can cognize simples and the world, but not by means of an infinite series of reason-steps.

This finally brings us back to the argument we looked at in the beginning of this article: the thesis of the first Antinomy.

> ... if one assumes that the world has no beginning in time, then up to every given point in time an eternity has elapsed, and hence an infinite series of states of things in the world, each following another, has passed away. But now the infinity of a series consists precisely in the fact that it can never be completed through a successive synthesis. Therefore an infinitely elapsed world-series is impossible, so a beginning of the world is a necessary condition of its existence; which was the first point to be proven. (A426/B454)

It is quite clear that this is the very same argument as the one we have reconstructed from the Inaugural Dissertation. The similarities should be obvious: both arguments make appeal to the impossibility of an infinite synthesis and both involve relations of dependence.[23] Furthermore, we can easily reconstruct the argument into the same form as the ones discussed in section 3 as follows:

1. Suppose *t* is an infinitely long period of time prior to the present. (Assumption for *reductio*)
2. If A is a period of time prior to B, then A can be cognized from B by some series of reason-steps that takes us from the present to the past. (from the Reason Constraint on Ground, and the fact that there is a *nexus* between the past and the present)

3. If *t* is an infinitely long period of time prior to the present, then *t* cannot be cognized from the present by some series of reason-steps that take us from the present to the past. (Premise)
4. ⊥

The first assumption is the one that Kant attempts to argue against: that the world is infinite with regard to time. The second premise comes from the Reason Constraint on Ground, together with the claim that the past and the present are in a *nexus*, a claim that Kant argues for in the introduction to the Antinomy chapter (A411/B438). The third claim is justified in the same way as the third claim in each of the preceding arguments: although one can, by means of a series of reason-steps, successively come to cognize earlier and earlier times, one can never, by means of a series of reason-steps, come to cognize an infinitely long earlier time (an 'eternity').

This argument is structurally identical to the arguments we have considered in this article. As with the totality of the world in the ID, Kant is here showing that even if we can cognize each of an infinite number of grounds, we cannot cognize their totality *as a ground*. And since the entirety of the past (the totality of all past events) is a condition for the present, this would lead us to the conclusion that there is a condition for the present that one could not possibly reason to.[24] This would amount, in the vocabulary of the Critical Kant, to saying that an infinite past violates the Reason Constraint on Ground.

The connection is even clearer for the spatial version of the thesis to the first Antinomy:

> Regarding the second point, again assume the opposite: then the world would be an infinite given whole of simultaneously existing things. Now we can think of the magnitude of a quantum that is not given as within certain boundaries of every intuition in no other way than by the synthesis of its parts, and we can think of the totality of such a quantum only through the completed synthesis, or through the repeated addition of units to each other. Accordingly, in order to think the world that fills all space as a whole, the successive synthesis of the parts of an infinite world would have to be regarded as completed, i.e., in the enumeration of all coexisting things, an infinite time would have to be regarded as having elapsed, which is impossible. (A426-8/B454-6)

Here, I do not even need to give a formal reconstruction of the argument, as it would be the exact same as my reconstruction of the argument against an infinitely large world in the Inaugural Dissertation. This is a further virtue of my reconstruction, as it shows Kant's consistency over the course of his writing. It is also a virtue that my reconstruction of Kant's 'space' and 'time' versions of the first Antinomy arguments are structurally the same, a feature that is not captured by, for example, Boehm's reading of the first Antinomy.[25]

5. Upshots

This discussion allows us to respond to the original concern that motivated this article: the accusation that Kant's argument in the Antinomies is question-begging, as it presupposes some form of idealism. Recall Kemp Smith's version of the objection:

> From the impossibility of traversing infinite space in thought by the successive addition of part to part, Kant here argues that 'an infinite aggregate of actual things cannot be viewed as a given whole', and consequently that the world cannot be infinitely extended in space. That is, from a *subjective impossibility of apprehension* he infers an *objective impossibility of existence.*

If the historical exegesis in this article is correct, the inference from the (in principle) 'subjective impossibility of apprehension' to the 'objective impossibility of existence' is exactly the kind of inference that the German rationalists were happy to accept and the kind of inference that Kant endorsed as well. This is not because they were all idealists, but because they accepted the Reason Constraint on Ground. Now, to be clear, this is not a constraint that the average twenty-first century metaphysician is likely to endorse. But we should not let the trends in contemporary metaphysics dictate how we read historical figures, especially when the historical evidence is as clear as it is here. It is crucial that we understand the philosophical commitments that Kant shared with his targets, even if we no longer share such commitments. Only after our reading of Kant is secure can we return to these divergent commitments and see if the German rationalist position has any merit.

My discussion should also lead us to reassess the relationship between Kant's views in the Inaugural Dissertation and the *Critique of Pure Reason*. Of course, I am not the first to note a connection between Kant's arguments in the Inaugural Dissertation and his arguments in the Antinomies (cf. Guyer 1987; Grier 2001). But there are some important features of my account that have serious consequences for our thinking about Kant's philosophy.

A key point of disagreement between my reconstruction and a large part of the scholarly literature concerns the metaphysical weight of Kant's arguments in the Inaugural Dissertation. Commentators on the Inaugural Dissertation have tended to think that Kant's solution to puzzles of infinity and continuity in 1770 was merely epistemic, and not metaphysical. For example, Guyer (1987: 385-400) claims that Kant, at some point in the late 1770s, changed from thinking of the Antinomies as having a purely epistemic solution to thinking about them as giving a proof of the ideality of space and time. Guyer's main argument in favour of attributing this change of mind to Kant is that Kant's reflections through most of the 1770s do not involve him explicitly claiming that the Antinomies provide one with a proof of the distinction between appearances and things in themselves.[26] But if I am correct, Kant would not have needed to make this point explicit, as there was a general consensus among the German rationalists that the (in principle) possibility of a reason-step inference was a requirement for the existence of a ground-consequence relation. Instead, we should think that the Kant of the Inaugural Dissertation already took the arguments that would become the Antinomies to constitute a proof of the ideality of space and time.

This fact has further implications when it comes to our interpretation of transcendental idealism. The fact that Kant and his predecessors took epistemic considerations to have metaphysical implications should cast doubt on interpretations of Kant's Critical philosophy according to which his transcendental idealism is primarily an epistemic, rather than metaphysical, position (Prauss 1974; Allison 2004; Bird 2006). Perhaps more contentiously, my reading should also cast doubt on any reading of the *Critique* which posits a one-one correspondence between appearances

and things in themselves (cf. Allais 2004). Were there such a correspondence, Kant's argument would apply equally well to things in themselves, thereby leaving transcendental idealism as a non-solution to the problem of the Antinomies.

More generally, this reading also has implications for our understanding of Kant's place in the history of philosophy. Many scholars take Kant's Copernican Revolution to be fundamentally characterized by the view that representation (or represent-ability) and reality are necessarily correlated, or that questions of representation are inseparable from questions of reality, captured in Kant's slogan that 'the conditions of the possibility of experience in general are at the same time conditions of the possibility of the objects of experience' (A158/B197).[27] But this is based on anachronistically attributing a distinctively twentieth-century version of realism to pre-Kantian philosophers. As I have shown, the belief in the Reason Constraint on Ground was *common ground* between Kant and the transcendental realists of his time. Kant's radical change cannot have been the slogan cited above, as this is a claim that Wolff and Baumgarten would have also accepted, though perhaps they would have put it in terms of cognition rather than experience. Rather, Kant's Copernican Revolution must involve the much more radical claim that objects of experience '*conform* to the constitution our faculty of intuition' (Bxvii, *italics* added). That is, the way the spatio-temporal world is does not merely conceptually depend on our powers of representation, but it *metaphysically* depends on our capacities. I hope that this historical corrective will lead to a more fruitful discussion of the ways in which Kant was, and was not, an innovator.

Notes

1 References to Kant are provided using the standard form of citation of the Akademie Ausgabe (Kant 1900-) and the following abbreviations: ID = Inaugural Dissertation; NE = New Elucidation (*Nova dilucidatio*); PhyM = Physical Monadology. References to the *Critique of Pure Reason* are given in the standard (A/B) format and follow the translation of Kant 1998. For other primary sources, the following abbreviations are used: Port Royal Logic = Arnauld 1851 (1662); *Metaphysica* = Baumgarten 1779; *De Usu* = Crusius 1743; *Entwurf* = Crusius 1745; *Weg* = Crusius 1747; *Erste Gründe* = Gottsched 1733; *Entdeckung* = Lange 1724; *Metaphysik* = Meier 1755; *Anmerkungen* = Wolff 1724, *German Metaphysics* = Wolff 1983. Unless otherwise specified, all translations of Baumgarten's *Metaphysica* are from Baumgarten et al (2013).

2 Boehm sometimes uses representational vocabulary, in saying that a synthetic whole is one in which the parts 'may be separated, *at least in thought*, from the whole.' (2014: 88). But it is clear from Boehm's presentation that the argument itself is not supposed to rely on any of these representational considerations.

3 The translation provided is that of Fugate and Hymers (Baumgarten et al. 2013), with one exception. Baumgarten's term *cognitio* in *Metaphysica* is footnoted to correspond to the German *Erkenntnis*, as used both by Christian Wolff in his *German Metaphysics* and by Kant throughout the *Critique of Pure Reason*. Following contemporary Kant scholarship, I re-translate 'knowledge' to 'cognition'.

4 A note on the sense in which Baumgarten's characterization is anthropocentric. Baumgarten does not believe that metaphysics is the study of how humans cognize. Rather, it is the study of the objects of human cognition *qua* objects of human cognition. As we see in what follows, many of the central notions

of metaphysics (e.g., nothing, possible, ground) are defined in terms of what it would take for an intellect such as ours to apply such concepts.

5 Another salient example, in this case from Baumgarten, is that of the definition of 'something' and 'nothing' as the 'representable' and 'unrepresentable' respectively. (*Metaphysica* §7-8)

6 Heidegger (1967: 116) proposes, and I am inclined to agree, that Baumgarten's definition of metaphysics is a consequence of Descartes' influence in philosophy: 'since Descartes, the faculty of knowledge, pure reason, has been established as that by whose guideline all definitions of what is, the thing, are to be made in rigorous proof and grounding.'

7 It is important to keep in mind that the notion of a ground in the eighteenth century was not the same as the contemporary notion of *grounding*, as discussed by Fine (2001), Schaffer (2009), and Rosen (2010). First of all, the notion of ground in the German rationalist tradition was intended to include that of an *efficient causal ground*, where contemporary discussions of grounding tend to see it as a distinctively non-causal but *metaphysical* form of dependence. Yet the eighteenth-century notion of a ground was not limited to efficient causal grounds, where just as efficient causes were taken to be grounds of their effects, so were parts taken to be grounds of the wholes that they compose. As passages below show, epistemic basing relations were also taken to be grounding relations, as one's source of cognition was characterized as the 'ground of cognition' (*Erkenntnisgrund*). In this sense, the German rationalists thought of *ground* as a diverse but generic relational predicate which included under it many different dependence relations, none of which clearly include the contemporary notion of *grounding*. The closest thing to such a notion in the period is Crusius' notion of an *existential ground* (*Existentialgrund*), which he defines as an 'inefficacious real ground', or one that 'makes something else possible or necessary in virtue of the laws of truth through its existence alone' (*Entwurf* §36, author's translation).

8 'I call **ground** what the French call *raison*, and what the Latin call *rationem*. I could not find in German a better word through which I could translate the word *raison*.' (Wolff 1724 *Ad* §29, author's translation). Baumgarten, in *Metaphysica* §14, footnotes the term *ratio* with *ein Grund*. This makes it clear that the term *ratio*, as used by the German rationalists, was intended to be synonymous with Wolff's use of the German *Grund*. Since *Grund* has a specific definition in the Wolffian philosophy, both *ratio* and *Grund* should be understood as falling under this definition.

9 As Longuenesse (2001) points out, Kant rejects this *as a definition*, but there is no indication that he ceases to accept its extensional adequacy. In fact, if my reading is correct, then Kant is committed to the extensional adequacy of this definition. In the *New Elucidation*, just a few sections before criticizing Wolff's definition, Kant says that an antecedently determining ground 'is one, in the absence of which that which is determined would not be *intelligible* (*intelligibile*)' (NE, 1: 392). Crusius, in contrast, defines *ground* in terms of production (*hervorbringen*) (*Entwurf* §34). Despite this definition, Crusius continues to accept that every sufficient ground helps us understand the *possibility* of its consequence (§38). He even agrees that every *determining* ground can give us understanding of *why* something is the case (*De Usu* III).

10 Cf. Lange's *Entdeckung* and Crusius' *De Usu* for objections to this picture. As Crusius makes clear, these philosophers thought that this relationship between ground and cognition ought to be restricted to grounds that stand in necessary connections with their consequence, rather than eliminated altogether.

11 The choice to translate every use of *Zusammenhang* as *nexus* follows the instructions on footnote 4 to proposition §14 in Baumgarten's *Metaphysica*. I also translate Wolff's and Kant's uses of *Zusammenhang* as *nexus* for the sake of consistency.

12 For Wolff, two truths are said to be in a *nexus* when one is a ground of the other (*German Metaphysics* §381). For Meier and Baumgarten, a *nexus* is the predicate that allows us to know that a ground-consequence relation obtains between two things (*Metaphysik* §27; *Metaphysica*, §14). See also Kant's metaphysics lectures: 'The connection [*nexus oder Verknüpfung*] between ground and consequence is twofold: the connection [*nexus*] of subordination and co-ordination . . .' (Met-L2/Pölitz, 28: 549, trans. in Kant 1997). I will follow Wolff in saying that two things are 'in a *nexus*' just in case one is the ground of the other.

13 Crusius presses this point in *De Usu* XLIII, and Kant concedes: '. . . I admit that the winding course of reasoning is scarcely becoming to the measurelessness of the divine understanding. Nor does the infinite understanding need to abstract universal concepts, or combine them together, or, in order to establish conclusions, to compare them.' (NE, 1: 405)

14 What *is* a violation of the Reason Constraint on Ground is the view that there are necessitating grounds that only God can know, and that are totally inaccessible to finite minds. This is a view endorsed

in the Port Royal Logic, but, to my mind, it is not endorsed by any of the German rationalists that Kant was engaging with.

15 Note that, in the Second Antinomy, Kant explicitly restricts himself to discussing simples that are related to the objects that we perceive (A437/B465).

16 I here assume, with Kant, that the mereological structure of objects is the same as the mereological structure of space. I thank David Kovacs for pointing this out. For a more detailed discussion of how one might reject this view, see Simons (2004)

17 This is not to be confused with the meaning of the term 'analysis' as applied to judgments in the Critical philosophy, though these two meanings are clearly related. Rather, this is the sense of 'analysis' used in the Port Royal Logic, p. 302.

18 I here flag a connection with the debate in recent Leibniz scholarship over whether an infinite analysis could be done in a single step (Rodriguez-Pereyra and Lodge 2011; Steward 2014). It is clear from the texts discussed above that, for Kant, a one-step infinite analysis is not possible. Further discussion of one-step inferences to infinite totalities can be found in the Appendix.

19 This makes Kant's concerns here significantly different from the Leibnizian worries that Al-Azm (1972), Allison (2004), and Boehm (2014) have attributed to Kant in their reading of the *Critique*'s Antinomy of Pure Reason. Although Leibniz himself did object to the infinite divisibility of matter and the existence of an infinitely large totality, his arguments are generally not taken to be epistemic in nature. For a discussion of the former arguments, see Adams (1994), for a discussion of the latter, see Harmer (2014).

20 In the *Physical Monadology*, 'space which bodies fill is divisible to infinity; space does not, therefore, consist of simple parts' (PhyM, 1: 478). The argument itself is too complex to present here and has been discussed in detail in Malzkorn (1998). In the Inaugural Dissertation: 'For it is only when infinite space and infinite time are given that any definite space and time can be specified by *limiting*.' (ID §15C, 2: 405) In the *Critique of Pure Reason*, see A25/B39-40, A143/B183, A169-70/B211-2. Kant also discusses the infinity and continuity of space in the Antinomies themselves, A427/B455-A443/B471.

21 For the view that we cannot meaningfully make claims about, for example, the mereological structure of things in themselves, see Allison (2004). For a far more permissive view, see Langton (1998).

22 Kant thinks that these two claims follow from the concepts of 'simple' and 'world'. He does, in the *Physical Monadology*, argue for the former of these claims (PhyM, 1: 477), an argument he returns to in the *Critique of Pure Reason* (A434-6/B462-4). Although he never argues that the latter is true, it is clear he thinks it is absurd for there to be things without there being a totality of things.

23 An early version of this argument is even mentioned in the Inaugural Dissertation, as Kant concludes the opening chapter: 'For it is hardly possible to conceive how this *never to be completed series* of the states of the universe, which succeed one another to *eternity*, can be reduced to a whole, which comprehends absolutely all of its changes. Indeed, it necessarily follows from its very infinity that the series has no *limit* [*termino*]. Accordingly, there is no series of successive things except one that is part of another series.' (ID §2, 2: 391).

24 It is not completely obvious how the Critical notion of a condition relates to the pre-Critical notion of a ground. Yet Kant seems to think that the two notions are very closely connected, as evidenced by the following reflection on what gives rise to the Antinomies: 'A. everything has parts and is itself a part. B. everything that happens is a consequence [*Folge*], (*that is, it is conditioned*), and is itself a ground. So there is no first or last. No simples, no limit in size, no first ground, no necessary being.' (R4760, 17: 711, author's translation, italics added).

25 This is a problem for the readings proposed by Allison (2004), Bird (2006), and Boehm (2014), which have no explanation for why the concept of *time* is relevant for the antinomy of space.

26 Allison (2004) and Grier (2001) agree with Guyer that Kant was thinking about transcendental idealism in purely epistemic terms in 1770, but take this to continue to be the case in the Critical philosophy.

27 Some proponents of this narrative include Allison, who describes Kant as bringing about an 'anthropocentric' shift to philosophy (2004: 34); Hanna (2006), who takes transcendental idealism to be characterized by the claim that the representable and the possible necessarily come together; Bird, who sees Kant as breaking with tradition by claiming that 'the structure of reality as we experience it is the structure of our representations' (2006: 132); Meillassoux, who sees Kant as responsible for the view that 'we only ever have access to the correlation between thinking and being, and never to either term considered apart from the other' (2008: 9).

28 Cf. Black (1951) for interesting discussion and disagreement.

29 There is also nothing logically inconsistent about performing an infinite number of reason-step inferences in an infinite amount of time. Cf. Dretske (1965).

30 The term *supertask* is due to Thompson (1954). For further reading on the introduction of a limit into calculus and its application to Zeno's paradoxes, see Bell (2017).

31 This way of thinking about Kant's argument comes from Carus (1915). Kant himself discusses this in a footnote to the Inaugural Dissertation, where he claims that it would be 'absurd' to claim that infinity is a number (2: 389). As far as I am aware, no other commentator on the Antinomies has considered this way of reading Kant's argument.

32 This is one of Russell's (1914) objections to Kant's arguments in the Antinomies. As should now be clear, Kant is not guilty of the mistake of which Russell accuses him.

33 As Eric Watkins has helpfully pointed out to me, this distinction between a set of infinite cardinality and an infinitely large member of a set is already approached by Kant in his distinction between the *collective* and *distributive* unity of the understanding (A644/B672).

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Appendix: supertasks and totalizing

From the perspective of the twenty-first century, the arguments I have reconstructed here seem to have an obvious and important flaw. In this argument, it seems that Kant justifies the third premise of each argument by assuming that no infinite process can be completed in a finite amount of time. In other words, Kant is assuming the impossibility of performing what are now known as *supertasks*: infinitary tasks that are performed in a finite amount of time.

Yet there are reasons to think that supertasks are possible. The main reason to think that supertasks are possible is that their possibility allows us to solve Zeno's paradoxes of motion.[28] The general thought behind the paradoxes is that for someone to move from one point to the next, they would have to go through infinitely many steps, each half the distance of the previous one. For example, to get from A to B, one would have to travel through middle point C, but to get from C to B, one would have to travel through middle point D, and so on to infinity. So, moving from one place to another, if space is continuous would require a supertask.

Supertasks of this kind are often thought to be unproblematic, since each step takes half the time as the previous one. So, if time is also continuous, then there is a sense in which one can complete the task of getting from A to B by performing an infinite number of tasks in a finite amount of time (cf. Benacerraf 1962).

If this line of thinking is correct, then it would seem to cause a problem for Kant's arguments as well. This is because Kant's arguments rely on the impossibility of performing an infinite number of reason-step inferences in a finite amount of time. But, the thought goes, if each reason-step takes half the time of the previous one, then there should be no problem here either. So, there is nothing logically inconsistent with performing an infinite number of reason-step inferences in a finite amount of time.[29]

Historically speaking, it is hard to know exactly how aware of this problem Kant would have been, as the notion of a supertask is nowhere to be found in Kant's time, and analysis-based solutions to Zeno paradoxes would have to await the introduction of the concept of a *limit* into calculus in the nineteenth century.[30] Yet it should still be of interest to us whether Kant's argument can be salvaged in the face of newer considerations about infinity. I believe that it can, as the notion of a supertask does nothing to

threaten the core idea in Kant's concern. Hence, here I leave the realm of strictly historical reconstruction to provide a philosophical defense of Kant's argument in the face of the supertask-based objection.

All that is necessary for Kant's argument to work is that the sense in which a supertask can be counted as completed is not sufficient for satisfying the Reason Constraint on Ground.

> **Reason Constraint on Ground:** if A is the ground of B, then it is (in principle) possible to reason from A to B, and from B to A.

We can begin by noting that, even if we grant that it is possible to count to infinity by continuously speeding up one's counting, infinity is not itself a member of the number series. Infinity is in some sense the 'result' of our supertask, since it is what we have counted to by the time the supertask is finished. But infinity is not something that was included in the counting, since even after the supertask is finished, there is no point at which we have counted from some number to infinity. But this, the claim that the result of an infinite series of steps is not itself a member of the series, is enough to raise some serious problems for the German rationalist.

One way that the worry can be understood most easily is by thinking of its parallel with finitary and infinitary mathematics.[31] Suppose that our epistemology of mathematics was analogous to the rationalist epistemology of the material world: we start with a number that we have cognized, say the number 1, and we cognize all other numbers by repeated applications of a reason-step of succession from the numbers that we have already cognized to later numbers. By reason, then, we can cognize the successor of any number. From repeated applications of the reason-step, we can cognize very large numbers. If we are able to supertask our counting, we could in principle cognize all infinitely many natural numbers.

The Kantian thought, however, is that even repeated applications of the reason-step will not allow us to cognize infinity itself. This is because, although the totality of natural numbers that are related by succession to 1 is infinitely large, infinity is not itself a successor of any natural number. So, even if we allowed the possibility of 'counting to infinity' by means of a supertask, at no point in our counting could we arrive at infinity.

The case of infinitary mathematics is revealing. The reason why infinity cannot be cognized by means of these reason-steps is not that it would take too long to do so, but rather that infinity is not a successor of any natural number. Hence, no amount of applications of the succession reason-step would allow us to cognize it. This is not itself terribly puzzling in the case of infinity, since mathematicians and philosophers have long been happy to admit that infinity is not a *successor* of any natural number.[32] But this is not an available reply in the case of an infinitely large world, since Kant has defined a world as a *composite*. Hence, unlike infinity itself, an infinitely large world would have to be in a *nexus* with its parts. Hence, the infinitude of an infinitely large world cannot be merely the *cardinality* of the infinite series of grounds, but it must be a *member* of this series as well.[33] This is what is impossible. Whether supertasks are possible is therefore irrelevant to Kant's argument, properly understood.

This discussion should also make it even clearer why 'totalizing' the way Kant does in the thesis of Kant's second Antinomy (A434-6/B462-4) is illegitimate. A totalizing move will at best lead us to conclude that there are simples, but it will not warrant the conclusion that there are simple *parts* of composite things, since that would require that simples be members in the series of parts (A524/B552). This stronger claim is warranted if and only if it is also the case that part-whole reason-steps can be used to cognize simples. The impossibility of this kind of cognition of simples is exactly what Kant emphasizes, and what furnishes him with an argument for transcendental idealism.