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TWO R-CLOSED SPACES

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1. Introduction. Throughout this paper all hypothesized spaces are T_1. A regular space is called *R-closed* [**11**] (*regular-closed* [**7**] or, equivalently, *regular-complete* [**2**]) provided that it is a closed subset of any regular space in which it can be embedded. A regular space $(X, \mathscr{T})$ is called *minimal regular* [**2**; **4**] if there exists no regular topology on X which is strictly weaker than $\mathscr{T}$. We shall call a regular space X *strongly minimal regular* provided that each point $x \in X$ has a fundamental system of neighbourhoods $\mathscr{V}_x$ such that for every $V \in \mathscr{V}_x$, $X \backslash V$ is an R-closed space.

In §2 we note that a strongly minimal regular space is minimal regular, but we do not know if the converse holds. M. P. Berri and R. H. Sorgenfrey [**4**] proved that a minimal regular space is R-closed, and Horst Herrlich [**7**] gave an example of an R-closed space that is not minimal regular.

In [**3**] Berri asked if the product of minimal regular spaces is minimal regular. Ikenaga [**8**] partially answered his question by proving that if a minimal regular space X is compact, then for every minimal regular space Y, $X \times Y$ is minimal regular. (It is well-known and easy to prove that every compact Hausdorff space is minimal regular.) In [**11**] C. T. Scarborough and A. H. Stone considered an analogous question for R-closed spaces, and they proved that if an R-closed space X is compact, then for every R-closed space Y, $X \times Y$ is R-closed. Using the Scarborough-Stone theorem and the obvious fact that a regular space which is a finite union of R-closed spaces is R-closed, one can easily obtain a proof of the following.

THEOREM 1.1. *Let X be a compact Hausdorff space. Then for every strongly minimal regular space Y, $X \times Y$ is strongly minimal regular.*

In §3 we construct a non-minimal regular R-closed space $(S, \mathscr{S})$ and a non-compact strongly minimal regular space $(T, \mathscr{T})$ such that for every R-closed space Y, $(S, \mathscr{S}) \times Y$ is R-closed, and for every strongly minimal regular space Y, $(T, \mathscr{T}) \times Y$ is strongly minimal regular. Thus we show that the converses of the Scarborough-Stone theorem and Theorem 1.1 are false.

While the spaces obtained in §3 are sequentially compact, in §4 we prove that if the Continuum Hypothesis holds, then there exist spaces $(S, \mathscr{S})$ and $(T, \mathscr{T})$ with the above properties and such that, in addition, neither is sequentially compact, and both are first countable and separable. As far as

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the author knows, it has not been known previously if there exists a non-compact, separable, first countable R-closed space.

2. Preliminaries. In this section we state a number of definitions and known results that we will need. For the most part, the lemmas listed below are just special cases of much more general results obtained by the authors referred to.

A filter base on a topological space is called *open* (*closed*) if the sets belonging to it are open (closed) sets. A *regular filter base* is an open filter base that is equivalent with some closed filter base; i.e., an open filter base in which each set contains the closure of some member of the filter base. The set of adherent points of a filter base $\mathscr{F}$, $\bigcap \{\bar{F} | F \in \mathscr{F}\}$, will be denoted ad $\mathscr{F}$.

LEMMA 2.1 [**2**; **7**]. *A regular space X is R-closed if and only if every regular filter base on X has an adherent point.*

Using Lemma 2.1, one can obtain a proof of the next result.

LEMMA 2.2. *Every open-and-closed subset of an R-closed space is R-closed.*

LEMMA 2.3 [**2**; **4**]. *A regular space X is minimal regular provided that every regular filter base on X with a unique adherent point is convergent.*

Using Lemmas 2.1 and 2.3, one can easily check that a strongly minimal regular space is minimal regular.

A space X is called *feebly compact* [**11**] (or *lightly compact* [**1**]) if any one of the following equivalent conditions holds: each locally finite system of open subsets of X is finite; every countable open filter base on X has an adherent point.

LEMMA 2.4. (i) *An R-closed space is feebly compact.*

(ii) *The closure of an open subset of a feebly compact space is feebly compact.*

(iii) *Every continuous image of a feebly compact space is feebly compact.*

(iv) *The product of a sequentially compact space and a feebly compact space is feebly compact.*

Each of the statements in Lemma 2.4 can be found in one of [**1**] and [**11**, p. 137 and p. 143].

A space is called *$\aleph_0$-bounded* [**6**] if every countable subset has compact closure.

LEMMA 2.5. (i) *An $\aleph_0$-bounded space is feebly compact.*

(ii) [**6**] *The property $\aleph_0$-bounded is productive.*

LEMMA 2.6 [**11**, p. 141]. *Every product of first countable feebly compact spaces is feebly compact.*

LEMMA 2.7 [**10**, p. 469]. *Let X be a first countable space. For any feebly compact space Y and open subset U of $X \times Y$, $\mathrm{pr}_1(\bar{U})$ is a closed subset of X.*

3. The spaces $(S, \mathscr{S})$ **and** $(T, \mathscr{T})$**.** Several authors have put to good use an ingenious non-compact R-closed space of Tychonoff [**12**, p. 553]. In papers concerning minimal regular spaces, the same has been true (see [**4**; **7**, p. 288 and p. 293; **11**, p. 140]). For example, Berri and Sorgenfrey [**4**] used a modification of Tychonoff's space to give the first known example of a non-compact minimal regular space. The construction given here will be similar to the Berri-Sorgenfrey version; however, for building blocks we will need to use not Tychonoff planks but instead what are sometimes called "big squares," and we will need to glue them together in a different manner.

Given an ordinal a, we will denote the set of all ordinals less than a by $W(a)$. B will denote $W(\omega_1 + 1) \times W(\omega_1 + 1) \backslash \{(\omega_1, \omega_1)\}$, topologized in the usual manner. $\mathbf{Z}$ will be the set of integers, with the discrete topology.

We will say that a set V *gets into the corner of* B provided that for each $a < \omega_1$ there exists a point $(x, y) \in V \cap B$ with $a \leqq x$ and $a \leqq y$.

LEMMA 3.1. *Let X be a topological space containing B as a subspace, and suppose that* (i) *B is a closed subset of X and* (ii) *$U = \{(x, y) \in B \mid x \neq y, x < \omega_1$, and $y < \omega_1\}$ is an open subset of X. Let V be an open subset of X which gets into the corner of B, and let F be a feebly compact subset of X such that $V \subset F$. Then for each $a < \omega_1$ there exists a point $(x, x) \in B \cap F$ with $a \leqq x$.*

Proof. Let $a < \omega_1$. If there exists $a \leqq x$ with $(x, x) \in V$ then we are done. Let us suppose then that $V \cap \{(x, x) \in B \mid a \leqq x\} = \phi$. Since V is open, there is a point $(x, x) \in [U \cap V]^-$ with $a \leqq x$ (see [**5**, 8L.1.]). Let $\mathscr{V}$ be a countable fundamental system of open neighbourhoods of (x, x) in the space B, and set $\mathscr{W} = \mathscr{V} \mid U \cap V$. Then $\mathscr{W}$ is a countable open filter base on the feebly compact space F, so $F \cap \operatorname{ad} \mathscr{W} \neq \phi$. On the other hand, since B is a closed subset of X and $\bigcup \mathscr{V} \subset B$, we have $\operatorname{ad} \mathscr{W} \subset \operatorname{ad} \mathscr{V} \subset \{(x, x)\}$. Thus $(x, x) \in F$.

Description of the spaces $(T, \mathscr{T})$ *and* $(S, \mathscr{S})$. Let R be the equivalence relation on $B \times \mathbf{Z}$ defined by the rule $(x, y, i) R (v, w, j)$ if: (i) $x = v$, $y = w$, and $i = j$; (ii) $x = v$ and either (a) $y = x$, $w = \omega_1$, and $j + 1 = i$ or (b) $w = v$, $y = \omega_1$, and $i + 1 = j$; (iii) $y = w$ and either (a) $x = y$, $v = \omega_1$, and $i + l = j$ or (b) $v = w$, $x = \omega_1$, and $j + 1 = i$; (iv) $x = w$, $y = v = \omega_1$, and $i + 2 = j$; or (v) $v = y$, $w = x = \omega_1$, and $j + 2 = i$. If $\mathbf{\Delta}$ is the "diagonal" in B and $n \in \mathbf{Z}$, then R just identifies points of $\mathbf{\Delta} \times \{n\}$ with corresponding points of (right side of B) $\times \{n + 1\}$ and points of (top side of B) $\times \{n - 1\}$. Choose two new points a, b not in this set and take $T = \{a, b\} \cup (B \times \mathbf{Z})/R$. We shall continue to use the same symbols to denote the points of $(B \times \mathbf{Z})/R$.

Let $B \times \mathbf{Z}$ have the product topology and $(B \times \mathbf{Z})/R$ the resulting quotient topology. We define $\mathscr{T}$ to be the topology on T that has as a base $\{V \subset T \mid V$

is an open subset of $(B \times \mathbf{Z})/R\} \cup \{V_n | n \in \mathbf{Z}\} \cup \{W_n | n \in \mathbf{Z}\}$, where the sets V_n, W_n are defined as follows:

$$V_n = \{a\} \cup \{(x, y, n) | x < y\} \cup \{(x, y, n + 1) | x < \omega_1\} \cup B \times \{m \in \mathbf{Z} | m \geqq n + 2\};$$

$$W_n = \{b\} \cup \{(x, y, n) | y < x\} \cup \{(x, y, n - 1) | y < \omega_1\} \cup B \times \{m \in \mathbf{Z} | m \leqq n - 2\}.$$

For the space $(S, \mathscr{S})$, one may take $S = T \backslash W_3$ and $\mathscr{S} = \mathscr{T} | S$.

LEMMA 3.2. *Let $n \in \mathbf{Z}$ and let Y be one of the spaces $(T \backslash V_n, \mathscr{T} | T \backslash V_n)$ and $(T \backslash W_n, \mathscr{T} | T \backslash W_n)$. Suppose that X is a feebly compact space and $f: X \to Y$ is a continuous open surjection. Then for every regular filter base $\mathscr{R}$ on X, either* (i) $\{a, b\} \cap \cap f(\mathscr{R}) \neq \phi$ or (ii) $(Y - \{a, b\}) \cap \operatorname{ad} f(\mathscr{R}) \neq \phi$.

Proof. We consider only the case $Y = (T \backslash W_n, \mathscr{T} | T \backslash W_n)$.

Suppose (i) is false. Then, since $\mathscr{R}$ is regular, there is a set $R \in \mathscr{R}$ with $a \notin f(\bar{R})$. According to Lemma 2.4, $f(\bar{R})$ is feebly compact. Since the point a has a countable fundamental system of closed neighbourhoods, there must exist a neighbourhood of a which misses $f(\bar{R})$. Let m be the smallest integer for which there is a set in $f(\mathscr{R})$ which misses V_m.

Because $f(\mathscr{R})$ is equivalent with a filter base consisting of feebly compact sets, it follows from Lemma 3.1 that there exists a set $W \in f(\mathscr{R})$ which gets into the corner of neither $B \times \{m\}$ nor $B \times \{m + 1\}$. Thus there is an ordinal $b < \omega_1$ such that the compact set

$$K = W(b + 1) \times W(\omega_1 + 1) \times \{m - 1, m\} \cup W(\omega_1 + 1) \times W(b + 1) \times \{m - 1, m\}$$

contains the set $W \cap (V_{m-1} \backslash V_m)$. Then $f(\mathscr{R}) | K$ is a filter base, and so we have $\phi \neq K \cap \operatorname{ad} f(\mathscr{R}) \subset Y - \{a, b\}$.

THEOREM 3.3. (i) *The space $(S, \mathscr{S})$ is not minimal regular.*
(ii) *If $I \neq \phi$ then the product space $(S, \mathscr{S})^I$ is R-closed.*
(iii) *For every R-closed space Y, $(S, \mathscr{S}) \times Y$ is R-closed.*

Proof. (i). The same argument as one given by Herrlich [**7**, Example 2] can be used here. If $(S, \mathscr{S}')$ denotes the Alexandroff one-point compactification of the locally compact space $(S \backslash \{a\}, \mathscr{S} | S \backslash \{a\})$, then since $\mathscr{S} | S \backslash \{a\} = \mathscr{S}' | S \backslash \{a\}$, we must have $\mathscr{S}' \subset \mathscr{S}$. Furthermore, since B is a closed non-compact subset of $(S, \mathscr{S})$, $\mathscr{S} \neq \mathscr{S}'$.

(ii). In order to prove that a regular space is R-closed, it suffices by Zorn's lemma to prove that every maximal regular filter base on the space is convergent. Let us suppose that $\mathscr{R}$ is a maximal regular filter base on $(S, \mathscr{S})^I$. We will prove that each $\operatorname{pr}_i(\mathscr{R})$ is convergent.

Let $i \in I$. The space $(S, \mathscr{S})$ is clearly $\aleph_0$-bounded, so by Lemma 2.5 the space $(S, \mathscr{S})^I$ is feebly compact. Thus we can appeal to Lemma 3.2 and conclude that there exists a point $s \in \operatorname{ad} \operatorname{pr}_i(\mathscr{R})$. Let $\mathscr{V}$ be a fundamental system of open neighbourhoods of s. Then it follows from the continuity of pr_i that $\mathscr{P} = \{R \cap \mathrm{pr}_i^{-1}(V) | R \in \mathscr{R}$ and $V \in \mathscr{V}\}$ is also a regular filter base on $(S, \mathscr{S})^I$. By the maximality of $\mathscr{R}$, $\mathscr{P}$ must be equivalent with $\mathscr{R}$. Thus for every $V \in \mathscr{V}$, $\mathrm{pr}_i^{-1}(V)$ contains some member of $\mathscr{R}$; i.e., $\mathrm{pr}_i(\mathscr{R})$ converges to s.

(iii). Since $(S, \mathscr{S})$ is of course sequentially compact, $(S, \mathscr{S}) \times Y$ is feebly compact by Lemma 2.4, and so we can use Lemma 3.2 here.

Let $\mathscr{R}$ be a regular filter base on $(S, \mathscr{S}) \times Y$. We will prove that ad $\mathscr{R} \neq \phi$.

Case 1: *there is a point* $s \in (S \backslash \{a\}) \cap \mathrm{ad}\ \mathrm{pr}_1(\mathscr{R})$. Then s has a compact neighbourhood K. By the Scarborough-Stone Theorem, $K \times Y$ is R-closed, and since $\mathscr{R} | K \times Y$ is a regular filter base on $K \times Y$, ad $\mathscr{R} \neq \phi$.

Case 2: $a \in \cap \mathrm{pr}_1(\mathscr{R})$. Then $\mathscr{R} | \{a\} \times Y$ is a regular filter base on $\{a\} \times Y$. Since $\{a\} \times Y$ is R-closed, $\phi \neq$ ad $\mathscr{R}$.

Before stating our next result, we indicate why the space $(T, \mathscr{T})$ is strongly minimal regular.

Since the space Y in Lemma 3.2 is feebly compact, the hypothesis of Lemma 3.2 is satisfied if one takes $X = Y$ and $f =$ the identity on Y. Thus it follows from Lemma 3.2 that $(T, \mathscr{T})$ is R-closed and that if $t \in \{a, b\}$, then t has a fundamental system of neighbourhoods $\mathscr{V}_t$ such that for every $V \in \mathscr{V}_t$, $T \backslash V$ is R-closed. On the other hand, the same is also true for any point $t \in T \backslash \{a, b\}$, for each point of $T \backslash \{a, b\}$ has a fundamental system of neighbourhoods consisting of open-and-closed sets.

THEOREM 3.4. (i) *The space* $(T, \mathscr{T})$ *is not compact.*

(ii) *If* $I \neq \phi$ *then the product space* $(T, \mathscr{T})^I$ *is strongly minimal regular.*

(iii) *For every strongly minimal regular space* Y, $(T, \mathscr{T}) \times Y$ *is strongly minimal regular.*

The proof is similar to the proof of Theorem 3.3.

Remark 3.5. In order to obtain a space having just the properties of $(S, \mathscr{S})$ but possibly not those of $(T, \mathscr{T})$, one can simplify the construction above, as follows: instead of using B as a building block, use $B' = \{(x, y) \in B | y \leqq x\}$, and for any $n \in \mathbf{Z}$ and $x < \omega_1$, identify the points (x, x, n) and $(\omega_1, x, n + 1)$.

4. A separable non-compact R-closed space. The construction given in this section is also similar to the Berri-Sorgenfrey space, but for building blocks we use copies of a space, denoted B, that is due to J. Isbell and S. Mrówka [**5**, 5I], and the technique we use for glueing them together, including Lemma 4.1, resembles one due to F. B. Jones [**9**].

The space B. Let $\mathbf{N}$ denote the set of natural numbers, and let $\mathscr{M}$ be a family of infinite subsets of $\mathbf{N}$ which has the following properties: $|\mathscr{M}| = \mathfrak{c}$; if $L, M \in \mathscr{M}$ and $L \neq M$, then $|L \cap M| < \aleph_0$; for every infinite subset I of

$\mathbf{N}$, $|I \cap M| = \aleph_0$ for some $M \in \mathscr{M}$. (Use Sierpiński's lemma [**5**, 6Q.1] to obtain $|\mathscr{N}| = \mathfrak{c}$ with $\mathscr{N}$ a family of infinite subsets of $\mathbf{N}$ such that the intersection of any two is finite. Then take $\mathscr{M}$ to be any enlargement of $\mathscr{N}$ that is a maximal family of infinite subsets of $\mathbf{N}$ such that the intersection of any two is finite.) Let $D = \{p_M | M \in \mathscr{M}\}$ be a new set of distinct points, and let $B = \mathbf{N} \cup D$, topologized as follows: each point of $\mathbf{N}$ is isolated; a neighbourhood of a point p_M is any set containing p_M and all but finitely many elements of M.

The space B is a first countable, zero-dimensional, feebly compact, locally compact Hausdorff space which is not normal, and D is a closed discrete subset of cardinality $\mathfrak{c}$ (see [**5**]).

Lemma 4.1. *There exists a subset A of D such that $|A| = \mathfrak{c}$, and for every open subset U of B, if $|U \cap A| = \mathfrak{c}$, then $|\bar{U} \cap (D\backslash A)| = \mathfrak{c}$.*

Proof. According to [**5**, 12B], there exists a family $\mathscr{A}$ of subsets of D which has the following properties: $|\mathscr{A}| > \mathfrak{c}$; for each $A \in \mathscr{A}$, $|A| = \mathfrak{c}$; for all $A, E \in \mathscr{A}$ with $A \neq E$, $|A \cap E| < \mathfrak{c}$. We prove that some member of $\mathscr{A}$ has the desired property.

Suppose, on the contrary, that for each $A \in \mathscr{A}$ there exists an open set U_A such that $|U_A \cap A| = \mathfrak{c}$ but $|[U_A]^- \cap (D\backslash A)| < \mathfrak{c}$. Then consider any two sets $A, E \in \mathscr{A}$. Since $|[U_A]^- \cap (E \cap A)| \leqq |E \cap A| < \mathfrak{c}$ and $|[U_A]^- \cap (E\backslash A)| \leqq |[U_A]^- \cap (D\backslash A)| < \mathfrak{c}$, $|[U_A]^- \cap E| < \mathfrak{c}$. Thus the open set $U_E\backslash[U_A]^-$ is nonempty and hence intersects $\mathbf{N}$. In particular, $U_E \cap \mathbf{N} \neq U_A \cap \mathbf{N}$. So the mapping $A \rightarrow U_A \cap \mathbf{N}$ is a one-to-one mapping of $\mathscr{A}$ into $2^{\mathbf{N}}$, in contradiction of the fact that $|2^{\mathbf{N}}| = \mathfrak{c} < |\mathscr{A}|$.

Acknowledgment 4.2. Our proof of Lemma 4.1 (or of an obvious generalization of Lemma 4.1) is similar to the proof (due to F. B. Jones) that appears on p. 144 of the Dugundji text.

Lemma 4.3. *Let $\mathscr{V}$ be an open filter base on a feebly compact space, and suppose that* $\operatorname{ad}\mathscr{V} = \phi$. *Then for every set $V \in \mathscr{V}$, $|\bar{V}| \geqq \aleph_1$.*

Proof. $\bar{V}$ is feebly compact by Lemma 2.4. Now one can show that every open filter base on a Lindelöf feebly compact space has an adherent point. Thus if $\bar{V}$ were countable, we would have $\bar{V} \cap \operatorname{ad}\mathscr{V} \neq \phi$.

Description of the spaces $(T, \mathscr{T})$ and $(S, \mathscr{S})$. Let A be as in Lemma 4.1. Choose two disjoint subsets A_1 and A_2 of A such that $A = A_1 \cup A_2$ and $|A_1| = |A_2| = \mathfrak{c}$, and let $f_i \colon A_i \rightarrow D\backslash A$, $i = 1, 2$, be bijections.

Let R be the equivalence relation on $B \times \mathbf{Z}$ defined by the rule $(x, i)R(y, j)$ if: (i) $x = y$ and $i = j$; (ii) $x = f_2(y)$ and $i + 1 = j$; (iii) $y = f_2(x)$ and $j + 1 = i$; (iv) $x = f_1(y)$ and $j + 1 = i$; (v) $y = f_1(x)$ and $i + 1 = j$; (vi) $f_1(x) = f_2(y)$ and $i + 2 = j$; or (vii) $f_1(y) = f_2(x)$ and $j + 2 = i$. Choose two new points a, b not in this set and take $T = \{a, b\} \cup (B \times \mathbf{Z})/R$.

$(T, \mathscr{T})$ and $(S, \mathscr{S})$ are defined as in §3 except that here the sets V_n, W_n are defined as follows:

$$V_n = \{a\} \cup \{(x, n) | x \in \mathbf{N} \cup A_1\} \cup \{(x, n+1) | x \in B \backslash A_2\} \cup B \times \{m \in \mathbf{Z} | m \geqq n+2\};$$

$$W_n = \{b\} \cup \{(x, n) | x \in \mathbf{N} \cup A_2\} \cup \{(x, n-1) | x \in B \backslash A_1\} \cup B \times \{m \in \mathbf{Z} | m \leqq n-2\}.$$

THEOREM 4.4. (i) *The spaces $(S, \mathscr{S})$ and $(T, \mathscr{T})$ are first countable, separable, feebly compact, and regular. $(S, \mathscr{S})$ is not minimal regular, and neither $(S, \mathscr{S})$ nor $(T, \mathscr{T})$ is countably compact.*

(ii). *If the Continuum Hypothesis holds, then $(S, \mathscr{S})$ is R-closed and $(T, \mathscr{T})$ is strongly minimal regular.*

The proof of (i) is similar to the ones in §3. The same is true of (ii), if one uses Lemma 4.3, the properties of A, and the equation $\mathfrak{c} = \aleph_1$. Given Lemma 2.7, one can easily check that 3.3 (iii) and 3.4 (iii) hold for any first countable R-closed space $(S, \mathscr{S})$ and first countable strongly minimal regular space $(T, \mathscr{T})$. Likewise one can prove that a product of first countable R-closed (minimal regular, strongly minimal regular) spaces is R-closed (minimal regular, strongly minimal regular).

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