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Letter to the Editor

Kinetic modulational instability of broadband dispersive Alfvén waves in plasmas

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Abstract. We consider a kinetic modulational instability of broadband (random phase) magnetic-field-aligned circularly polarized dispersive Alfvén waves in plasmas. By treating random phase Alfvén waves as quasi-particles, we consider their nonlinear interactions with ion quasi-modes within the framework of the wave-kinetic and Vlasov descriptions. A nonlinear dispersion relation governing such interactions is derived and analyzed. An explicit expression for the kinetic modulational instability growth rate is presented. Our results can be of relevance to the nonlinear propagation of incoherent Alfvén waves, which have been frequently observed in interstellar media, in the solar corona and in the solar wind, as well as in the foreshock regions of planetary bow-shocks and laboratory plasmas.

Dispersive Alfvén waves (DAWs) are of fundamental importance in astrophysical, space and laboratory plasmas [1–8]. They can be either circularly polarized electromagnetic waves, or an admixture of electrostatic and electromagnetic fields. The dispersion of low-frequency (in comparison with the ion gyrofrequency) circularly polarized DAWs comes from the ion inertia [3], while that of kinetic (inertial) Alfvén waves [9, 10] arises from the ion thermal gyroradius/ion polarization (the parallel electron inertia) effect. Large-amplitude DAWs, which can be excited by electron and proton beams, are capable of energizing charged particles and producing electron and ion heating in magnetoplasmas [11, 12].

Large-amplitude coherent non-dispersive and DAWs are subjected to a great variety of nonlinear effects [2, 3, 9, 13–22]. The latter include the three-wave decay [13–16, 22] and modulational instabilities [18–21]. However, the coherent wave–wave and wave–particle [15, 21] interactions are inappropriate when the DAWs have random phases/broadband spectra.

In this letter, we consider the kinetic modulational instability of broadband right-hand circularly polarized DAWs propagating along the external magnetic field $\hat{\mathbf{z}}B_0$ in a plasma, where $\hat{\mathbf{z}}$ is the unit vector along the z-axis in a Cartesian coordinate system and B_0 is the strength of the magnetic field. The wave magnetic field is given by $B_k = (B_x + iB_y)\exp(-i\omega_k t + ikz)$, where $B_x(B_y)$ is the $x(y)$ component of the wave magnetic field, and the wave frequency and wavenumber are related by

$$\omega_k \approx kV_{\mathrm{A}}(1 + kV_{\mathrm{A}}/2\omega_{\mathrm{ci}}), \tag{1}$$

where $V_{\mathrm{A}} = B_0/\sqrt{4\pi n_{\mathrm{i}} m_{\mathrm{i}}}$ and $\omega_{\mathrm{ci}} = eB_0/m_{\mathrm{i}}c$ are the Alfvén speed and the ion gyrofrequency, respectively, n_{i} is the ion number density, m_{i} is the ion mass, e is the magnitude of the electron charge and c is the speed of light in vacuum. We note that the wave dispersion in (1) comes from the finite frequency correction, and that $\omega \ll \omega_{\mathrm{ci}}$.

We treat the DAWs, given by (1), as quasi-particles. The latter obey the Liouville equation [23, 24]

$$\frac{\partial I_k}{\partial t} + V_{\mathrm{g}}\frac{\partial I_k}{\partial z} + F_k\frac{\partial I_k}{\partial k} = 0, \tag{2}$$

where $I_k = |B_k|^2/8\pi$ is the Alfvén wave energy density, $V_{\mathrm{g}} = \partial\omega_k/\partial k = V_{\mathrm{A0}} + kV_{\mathrm{A0}}^2/\omega_{\mathrm{ci}}$ is the group velocity of the Alfvén wavepacket, and $V_{\mathrm{A0}} = B_0/\sqrt{4\pi n_0 m_{\mathrm{i}}}$. The force exerted due to the background ion number density perturbation n_1 ($n_1 \ll n_0$, where n_0 is the unperturbed ion number density) on the Alfvén quasi-particles is

$$F_k = -\frac{\partial\omega_k}{\partial z} = \frac{kV_{\mathrm{A0}}}{2n_0}\frac{\partial n_1}{\partial z}. \tag{3}$$

The perturbed ion number density is

$$n_1 = \int du\, f_1(t, z), \tag{4}$$

which requires the knowledge of the perturbed ion distribution function $f_1(t, z)$ in the presence of the Alfvén quasi-particles. Using a hybrid approach, we start with the collisionless ion Vlasov equation

$$\frac{\partial f_1}{\partial t} + u\frac{\partial f_1}{\partial z} + \frac{e}{m_{\mathrm{i}}}\left(E_z + \frac{1}{c}\langle \mathbf{v}_k \times \mathbf{B}_k\rangle \cdot \hat{\mathbf{z}}\right)\frac{\partial f_0}{\partial u} = 0, \tag{5}$$

where E_z is the magnetic-field-aligned electric field associated with the ion quasi-mode, the angular bracket denotes the ensemble average over the Alfvén quasi-particles, u is the z component of the ion velocity, $\mathbf{v}_k$ is the ion velocity in the dispersive Alfvén wave fields and f_0 is the unperturbed ion velocity distribution function. The inertialess electron equation of motion

$$E_z + \frac{1}{c}\langle \mathbf{v}_k \times \mathbf{B}_k\rangle \cdot \hat{\mathbf{z}} = -\frac{1}{n_0 e}\frac{\partial}{\partial z}\left(\sum_k I_k + T_{\mathrm{e}} n_1\right) \tag{6}$$

determines the parallel electric field E_z. Here T_{e} is the electron temperature. The quasi-neutrality condition for the ion quasi-mode has been invoked in (6). Hence, from (5) and (6) we have

$$\frac{\partial f_1}{\partial t} + u\frac{\partial f_1}{\partial z} - \frac{1}{\rho}\frac{\partial}{\partial z}\left(\sum_k I_k + T_e n_1\right)\frac{\partial f_0}{\partial u} = 0, \tag{7}$$

where $\rho = n_0 m_i$ is the ion mass density.

Supposing that $I_k = I_{k0} + I_{k1}\exp(-i\Omega t + iKz)$, where $I_{k1} \ll I_{k0}$ and where $\Omega(K)$ is the frequency (wavenumber) of the ion quasi-mode, we obtain from (2)

$$I_{k1} = \frac{KV_{A0}}{2}\frac{k\partial I_{k0}/\partial k}{(\Omega - KV_A - kV_G)}\frac{n_1}{n_0}, \tag{8}$$

where $V_G = KV_{A0}^2/\omega_{ci}$.

Furthermore, supposing that $f_1(t,u)$ is proportional to $\exp(-i\Omega t + iKz)$, we obtain from (7)

$$f_1 = -\frac{K}{\rho}\frac{\partial f_0/\partial u}{(\Omega - Ku)}\left(\sum_k I_{k1} + T_e n_1\right). \tag{9}$$

Multiplying both sides of (9) by du and integrating over the velocity space, we obtain by using (4)

$$n_1\left(1 + \frac{KC_s^2}{n_0}\int\frac{du\,\partial f_0/\partial u}{\omega - Ku}\right) + \frac{K}{\rho}\int\frac{du\,\partial f_0/\partial u}{\Omega - Ku}\sum_k I_{k1} = 0, \tag{10}$$

where $C_s = (T_e/m_i)^{1/2}$ is the ion sound speed.

Inserting I_{k1} from (8) into (10) we thus obtain the dispersion relation

$$1 + \frac{KC_s^2}{n_0}\int\frac{du\,\partial f_0/\partial u}{\Omega - Ku} + \frac{K^2 V_{A0}}{2\rho n_0}\int\frac{du\,\partial f_0/\partial u}{\Omega - Ku}\sum_k\frac{k\,\partial I_{k0}/\partial k}{\Omega_0 - kV_G} = 0, \tag{11}$$

where $\Omega_0 = \Omega - KV_{A0}$. Assuming a Maxwellian ion distribution function

$$f_0 = \frac{n_0}{\sqrt{2\pi}V_T}\exp(-u^2/2V_T^2), \tag{12}$$

where $V_T = (T_i/m_i)^{1/2}$ is the ion thermal speed and T_i is the ion temperature, we obtain from (11)

$$1 + \frac{T_e}{T_i}W(\xi_i) + \frac{KV_{A0}W(\xi_i)}{2V_G n_0 T_i}\frac{L}{2\pi}\int\frac{dk\,k\partial I_{k0}/\partial k}{(k - \Omega_0/V_G)} = 0, \tag{13}$$

where the length of the system is L, and the W function is [25]

$$W(\xi_i) = \frac{1}{\sqrt{2\pi}}\int\frac{d\xi\,\xi\exp(-\xi^2/2)}{\xi - \xi_i}$$

with $\xi_i = \Omega/KV_T$.

We now choose

$$I_{k0} = \frac{I_{00}}{\sqrt{2\pi}k_w L}\exp[-(k - k_0)^2/2k_w^2], \tag{14}$$

where I_{00} is the maximum Alfvén wave magnetic field energy density corresponding to a mean wavenumber k_0, and k_w represents the width of the DAW spectrum. Substituting for I_{k0} from (14) into (13) we finally obtain the desired dispersion relation

$$1 + \frac{T_e}{T_i}W(\xi_i) - \frac{(\Omega - KV_{A0})KV_{A0}W(\xi_i)}{k_w^2 V_G^2}\frac{I_{00}}{8\pi n_0 T_i}W(\xi_0) = 0, \tag{15}$$

where $\xi_0 = [\Omega - (KV_{A0} + k_0 V_G)]/k_w V_G$. For $\xi_0 \gg 1$, (15) reduces to

$$[\Omega - (KV_{A0} + k_0 V_G)]^2 = -(\Omega - KV_{A0})KV_{A0} \frac{W(\xi_i)}{[1 + \sigma W(\xi_i)]} \frac{I_{00}}{8\pi n_0 T_i}, \tag{16}$$

where $\sigma = T_e/T_i$. Letting $\Omega = KV_{A0} + k_0 V_G + i\gamma \equiv \Omega_r + i\gamma$, where $\gamma \ll \Omega_r$, we obtain from (16) the growth rate

$$\gamma = (k_0 V_G K V_{A0})^{1/2} (\mathrm{Re}\, Q)^{1/2} \left(\frac{I_{00}}{8\pi n_0 T_i} \right)^{1/2}, \tag{17}$$

where $Q = W(\xi_r)/[1 + \sigma W(\xi_r)]$ with $\xi_r = \Omega_r / KV_T$. Equation (15), together with (17), is the main result of this paper.

In summary, we have considered the nonlinear propagation of a broadband magnetic-field-aligned right-hand circularly polarized DAW in an electron–ion magnetoplasma. The dynamics of the broadband DAW has been treated by a wave kinetic equation, considering DAW propagation in a slowly varying medium containing ion density ripples (ion quasi-modes). The latter, which are obtained from the ion Vlasov equation and the parallel component of the ion momentum equation, are coupled with the random phase DAW via the ponderomotive force effect. The governing equations have been linearized around the equilibrium state to obtain a new dispersion relation, which admits a kinetic modulational instability of broadband DAWs in magnetized plasmas. In conclusion, we stress that the present result is important for understanding the stimulated scattering of incoherent (random phase) DAWs of ion quasi-modes in astrophysical [7] and laboratory plasmas [22]. Specifically, the present mechanism suggests a novel nonlinear scenario for the dissipation of DAW energy in magnetized plasmas.

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