Correspondence

DEAR EDITOR,

D. V. Lindley offers a rigorous 'proof' of Stirlng's formula [1], but a more experimental demonstration may be appropriate, e.g. with physics and chemistry students (for whom the formula is important in deriving the Boltzmann distribution):

$$\ln n! = \ln a! + \sum_{a+1}^{n} \ln x \qquad (a < n)$$

$$\approx \ln a! + \int_{a+\frac{1}{2}}^{n+\frac{1}{2}} \ln x \, dx = \ln a! + \left[x \ln x - x\right]_{a+\frac{1}{2}}^{n+\frac{1}{2}}.$$

Whence $\quad n! \approx a!\left(\left(n + \tfrac{1}{2}\right)/e\right)^{n+\frac{1}{2}}\left(e/\left(a + \tfrac{1}{2}\right)\right)^{a+\frac{1}{2}} = K\left(\left(n + \tfrac{1}{2}\right)/e\right)^{n+\frac{1}{2}}$ where $K = a!\left(e/\left(a+\tfrac{1}{2}\right)\right)^{a+\frac{1}{2}}$. Now $\left(\left(n+\tfrac{1}{2}\right)/e\right)^{n+\frac{1}{2}} = \sqrt{n+\tfrac{1}{2}}\left(\tfrac{n}{e}\right)^{n}\left(1+\tfrac{1}{2n}\right)e^{-1/2}$ $\approx \sqrt{n}\left(\tfrac{n}{e}\right)^{n}$ for large n.

Investigation of the value of K^2 shows that it rapidly approaches 2π as a increases (e.g. $a = 100$, $K^2 = 6.278$; $a = 200$, $K^2 = 6.281$), and the usual formula for $n!$ immediately follows.

Reference

1. D. V. Lindley, More on Stirling's formula, *Math. Gaz.* **82** (November 1998) pp. 484-485.

Yours sincerely,

MICHAEL WARD

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DEAR EDITOR,

Tony Gardiner [1] hit the nail firmly on the head: the emperor has long been walking around naked and nobody has had the courage to remark on the fact. How can we virtually eliminate proof from the material we teach and call what is left mathematics? Surely it is proof that sets mathematics aside from other enterprises.

I would agree with his analysis that elementary Euclidean geometry remains the most effective vehicle for teaching the ideas of proof. It is accessible to the pupils *who can understand proof*, and the discipline of setting the steps of the proof out logically can have the desirable by-product of teaching pupils how to write mathematics properly. It is also a topic where that universal good luck charm of the mathematics classroom, the calculator, can be rendered impotent.

However I question Tony Gardiner's analysis that a substantial fraction of each cohort is capable of understanding proof. Certainly my experience in teaching mathematics (GCSE, A level and Oxbridge entry) in a selective school leads me to believe that it is probably no more than the fraction that sat the old O level examination. Surely proof virtually vanished from the GCSE syllabus because there is only a small fraction of the cohort who can understand proof, and the examination was designed for a much larger percentage of the cohort than the O level.

We seem to have arrived at the present situation because it was thought necessary to teach everybody mathematics, and that everybody was capable of learning mathematics. I would question both: certainly there is a requirement for most people to have a facility with number, but how many actually need mathematics beyond arithmetic and very simple algebra? Perhaps we should take a leaf from the classicists' book and provide a 'Classical Civilisation' course – call it 'Mathematics for Living' – which is designed for all the cohort, and a 'Latin' course – call it 'Mathematics' – which can then have a rigorous approach to the subject. It could cover the material in the present mathematics and additional mathematics syllabi, and provide a sound footing for the A level course. I already hear cries about disadvantaging the less able, but the present system does the opposite: it does nothing to challenge the more able, and provides a poor foundation for their further studies.

Reference

1. Tony Gardiner, The Art of Knowing, *Math. Gaz.* **82**, 495 (November 1998), pp. 354-372.

Yours sincerely,

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DEAR EDITOR,

Let $\tau(n)$ be the number of positive integers not exceeding n that are expressible as the sum of two squares. For small values of n, the ratio $\rho(n) = \tau(n)/n$ is around 0.35. For example, $\rho(50) = 0.36$, $\rho(100) = 0.35$, $\rho(150) \approx 0.37$ and $\rho(200) = 0.36$. Does this relation continue to hold for larger values of n? Perhaps a reader knows of an asymptotic formula or could test the result further using a computer.

Yours sincerely,

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DEAR EDITOR,

J. R. Goggins has pointed out a mistake (my typing error) in the article on Napoleon triangles [2]. Both entries $30 - \theta$ in family A on page 416 should be $30 - 2\theta$, the sextet being $(\theta,\ 30 - 2\theta,\ 90 + \theta;\ 2\theta,\ 30 - 2\theta,\ 30)$.

Using an improved search program, my computer has found another adventitious set – (15, 30, 51; 24, 27, 33) – bringing the total to 39.

Adventitious angles occur in other contexts. Some years ago C. E. Tripp investigated quadrangles with integral angles [4]. These are related to the sextets by transformations such as that illustrated in Figure 9 of my article. Also J. F. Rigby has drawn my attention to a paper discussing the angles associated with triples of concurrent diagonals of regular polygons [3]. These are related both to Tripp's and my adventitious angles. Rigby's results demonstrate the existence of rational adventitious sextets that are not in any