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Null-homotopic knots have Property R

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Abstract

We prove that if K is a nontrivial null-homotopic knot in a closed oriented 3–manfiold Y such that $Y - K$ does not have an $S^1 \times S^2$ summand, then the zero surgery on K does not have an $S^1 \times S^2$ summand. This generalises a result of Hom and Lidman, who proved the case when Y is an irreducible rational homology sphere.

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1. *Introduction*

Given a null-homologous knot K in a 3–manifold Y and a slope $p/q \in \mathbb{Q} \cup \{\infty\}$, let $Y_{p/q}(K)$ be the p/q–surgery on K. Gabai's famous Property R Theorem [6] asserts, among others, that if K is a nontrivial knot in S^3, then $S^3_0(K)$ is irreducible. In particular, $S^3_0(K)$ does not have an $S^1 \times S^2$ summand.

In recently years, many generalisations of this theorem have been proved using Heegaard Floer homology. See, for example, the overview in [14]. Hom and Lidman [9] proved two generalisations of Property R. One result they proved is, if K is a nontrivial null-homotopic knot in an irreducible rational homology sphere Y, then $Y_0(K)$ does not have an $S^1 \times S^2$ summand. The aim of this paper to remove the restrictions on the ambient manifold.

THEOREM 1·1. *Let Y be a closed, oriented, connected* 3*–manifold, and $K \subset Y$ be a nontrivial null-homotopic knot such that $Y - K$ does not have an $S^1 \times S^2$ summand, then $Y_0(K)$ does not have an $S^1 \times S^2$ summand.*

In Gabai's work [6], it is proved that $S^3_0(K)$ remembers the information of K about the genus and fiberedness. Motivated by this result, a concept "Property G" was introduced in [13] as a generalisation of Property R. Known results on Property G are summarised in [14]. We will not give the complete definition of Property G here. Instead, we just state the explicit result for genus–1 null-homotopic knot.

COROLLARY 1·2. *Let $K \subset Y$ be a genus–*1 *null-homotopic knot, then K has Property G. That is, if F is a genus–*1 *Seifert surface bounded by K, and $\widehat{F} \subset Y_0(K)$ is the torus obtained by capping off ∂F with a disk, then $[\widehat{F}] \in H_2(Y_0(K))$ is not represented by a sphere. Moreover, if $Y_0(K)$ is a torus bundle over S^1 with fiber $\widehat{F}$, then K is a fibered knot with fiber F.*

Corollary 1·2 answers the genus–1 case of a question of Boileau [10, problem 1·80C]. There are easy counterexamples to the original question of Boileau, so one should modify the question to add the condition on the fiber of the zero surgery. See [12] for more details.

The strategy of the proof of Theorem 1·1 is as follows. If $b_1(Y) > 0$, the theorem easily follows from a result of Lackenby [11] and Gabai [5]. If $b_1(Y) = 0$, we use results about degree-one maps and a result in [4] to show that if $Y_0(K) = Z\#(S^1 \times S^2)$ then $\pi_1(Z) \cong \pi_1(Y)$. Theorem 1·1 then follows from work of Hom and Lidman [9].

We will use the following notation. If N is a submanifold of a manifold M, let $\nu(N)$ be a closed tubular neighbourhood of N, and let $\nu^\circ(N)$ be the interior of $\nu(N)$. If X, Y are two spaces, $f: X \to Y$ is a continuous map, let $f_*: \pi_1(X) \to \pi_1(Y)$ be the induced map. We will always suppress the base point in the notation when we talk about fundamental groups.

This paper is organised as follows. In Section 2, we prove general results about degree-one maps with certain properties on the induced homomorphisms on π_1. In Section 3, we prove that if the zero surgery on a knot in Y is $Z\#(S^1 \times S^2)$, then $\pi_1(Z) \cong \pi_1(Y)$. In Section 4, we use work of Lackenby [11] and Hom–Lidman [9] to prove Theorem 1·1. Corollary 1·2 is also proved as an application of this theorem.

2. *Degree-one maps which induce surface-group injective homomorphisms*

In this section, we will prove results about degree-one maps which induce surface-group injective homomorphisms on π_1.

A group Γ is a *surface group* if it is isomorphic to the fundamental group of a closed orientable surface. Let $\varphi: G \to H$ be a group homomorphism. We say φ is *surface-group injective*, if the restriction of φ to every surface subgroup of G is injective.

LEMMA 2·1. *Let $\varphi: G_1 * G_2 \to H$ be a group homomorphism. If both $\varphi|_{G_1}$ and $\varphi|_{G_2}$ are surface-group injective, then φ is also surface-group injective.*

Proof. Let Γ be a surface subgroup of $G_1 * G_2$. By the Kurosh Subgroup Theorem [8, theorem 8·3], Γ is the free product of a free group and conjugates of subgroups of G_i, $i = 1, 2$. Since Γ is not a nontrivial free product, it is conjugated to a subgroup of G_i for some i. Since $\varphi|_{G_i}$ is surface-group injective, φ is also injective on Γ.

The importance of the concept of surface-group injective maps is illustrated by the next lemma.

LEMMA 2·2. *Let X, Y be closed, oriented, connected 3–manifolds, $f: X \to Y$ be a surjective map such that f_* is surface-group injective. Let $S \subset Y$ be a separating 2–sphere, and assume that $R = f^{-1}(S)$ is a closed, oriented, connected surface. Then there exists a separating 2–sphere $E \subset X$ so that R is obtained by adding tubes to E.*

Proof. Let $\iota: R \to X$ be the inclusion map. Since $f(R) = S$ is a sphere,

$$\iota_*(\pi_1(R)) \subset \ker f_*.$$

If R is not a sphere, since f_* is surface-group injective, R must be compressible. Let R' be the surface obtained by compressing R, then R can be obtained from R' by adding a tube. Let R'_1 be a component of R', let $\iota': R'_1 \to X$ be the inclusion map, then

$$\iota'_*(\pi_1(R'_1)) = \iota_*(\pi_1(R'_1 \cap R)) \subset \iota_*(\pi_1(R)) \subset \ker f_*.$$

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If R' does not consist of spheres, since f_* is surface-group injective, R' must be compressible. So R' can be obtained from another surface R'' by adding a tube.

Continue with the above process, we conclude that R can be obtained from some spheres by adding tubes. We can rearrange the order of the tubes, so that some tubes connecting different spheres are added first to get a single sphere E, then R is obtained by adding other tubes to E.

Let $y_1, y_2 \in Y$ be two points separated by S. Since f is surjective, both $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are non-empty. These two sets are clearly separated by R, so R is separating. The process of compressing a surface does not change the homology class of the surface, hence E is also separating.

In the rest of this section, let Y be a 3–manifold which has no $S^1 \times S^2$ summand, $S_1, S_2, \ldots, S_n$ be a collection of disjoint 2–spheres in Y satisfying the following conditions: $Y \setminus (\cup_{i=1}^n S_i)$ has $n+1$ components whose closures are $\check{Y}_1, \check{Y}_2, \ldots, \check{Y}_n, \check{Y}_{n+1}$, where $\check{Y}_{n+1}$ is S^3 with n open balls removed, and a closed irreducible manifold $Y_i \neq S^3$ can be obtained from $\check{Y}_i$ by capping off $\partial \check{Y}_i = S_i$ with a ball B_i, $1 \leqslant i \leqslant n$. Then

$$Y = \#_{i=1}^n Y_i.$$

When Y is irreducible, it is understood that $n = 0$.

PROPOSITION 2·3. *Let X be a closed, oriented, connected 3–manifold, and $f : X \to Y$ be a degree-one map such that f_* is surface-group injective. Then there exists a degree-one map $g : X \to Y$ satisfying $g_* = f_*$, and each $E_i = g^{-1}(S_i)$ is a 2–sphere.*

Proof. We induct on n. When $n = 0$, there is nothing to prove. So we assume $n > 0$ and the result is proved for $n - 1$.

Using [17, theorem 1·1], we may assume $R_1 = f^{-1}(S_1)$ is a connected surface. Let $\check{Y}_0 = Y \setminus \check{Y}_1$, and let Y_0 be obtained by capping off $\partial \check{Y}_0$ with a ball B_0. Let $U_i = f^{-1}(\check{Y}_i)$, $i = 0, 1$. Then $\partial U_1 = \partial U_0 = R_1$.

By Lemma 2·2, there exists a separating 2–sphere $E_1 \subset X$, so that R_1 is obtained by adding tubes to E_1. Now E_1 splits X into two parts $\check{X}_1, \check{X}_0$, so that $\check{X}_i$ can be obtained from U_i by adding 1–handles and digging tunnels, $i = 0, 1$. Let X_i be the closed manifold obtained by capping off $\partial \check{X}_i$ with a ball, $i = 0, 1$.

We claim that each map $f|_{U_i} : U_i \to \check{Y}_i$ can be extended to a degree-one map $f_i : X_i \to Y_i$. In fact, the manifold X_i can be obtained from U_i by gluing a 3–manifold V_i which is obtained from B^3 by digging tunnels and adding 1–handles. Since B_i is a ball which is contractible, we can extend $f|_{U_i} : U_i \to \check{Y}_i$ to a map $f_i : X_i \to Y_i$ by sending V_i to B_i. The degree of f_i is 1 since the degree of $f|_{U_i}$ is 1.

Since $\deg f_i = 1$, after a homotopy supported in V_i, we may assume there exists a ball $B'_i \subset \mathrm{int}(B_i)$, such that f_i sends $B^\star_i = f_i^{-1}(B'_i)$ homeomorphically onto B'_i. Now we can glue $X_i \setminus \mathrm{int}(B^\star_i)$, $i = 0, 1$, together along their boundary, to get back

$$X = (X_0 \setminus \mathrm{int}(B^\star_0)) \cup_{S^2} (X_1 \setminus \mathrm{int}(B^\star_1)),$$

and define a map

$$f_0 \# f_1 : X \to Y = (Y_0 \setminus \mathrm{int}(B'_0)) \cup_{S^2} (Y_1 \setminus \mathrm{int}(B'_1))$$

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by gluing the restrictions of f_0, f_1. We rename

$$S_1 = \partial(Y_0 \setminus \mathrm{int}(B_0')), E_1 = \partial(X_0 \setminus \mathrm{int}(B_0^\star)),$$

then $E_1 = (f_0\#f_1)^{-1}(S_1)$.

We claim that $(f_0\#f_1)_* = f_*$. Let $D \subset E_1$ be a disk such that all tubes in S_1 are added to the interior of D, and let $D^c = \overline{E_1 \setminus D}$. Let $V \subset X$ be the handlebody obtained by adding the 1–handles bounded by the tubes to $\nu(D)$. Since $\pi_1(V)$ is a quotient of $\pi_1(\partial V)$, the map $X \setminus V \to X$ induces a surjective map on π_1. To prove $(f_0\#f_1)_* = f_*$, we only need to prove that $(f_0\#f_1)_*(\alpha) = f_*(\alpha)$ when α is a homotopy class represented by a loop in $X \setminus V$. We observe that $f_0\#f_1 = f$ on $X \setminus (V \cup \nu(D^c))$, and $\pi_1(X \setminus V)$ is the free product of the π_1 of the two components of $X \setminus (V \cup \nu(D^c))$, so $(f_0\#f_1)_*(\alpha) = f_*(\alpha)$.

By the induction hypothesis, there exists a map $g_0 : X_0 \to Y_0$, such that $g_1^{-1}(S_i)$ is a sphere for $2 \leqslant i \leqslant n$, and $(g_1)_* = (f_0)_*$. We can define a map $g = g_0\#f_1$ in a similar way as $f_0\#f_1$, then $g^{-1}(S_i)$ is a sphere for $1 \leqslant i \leqslant n$, and

$$g_* = (g_0\#f_1)_* = (f_0\#f_1)_* = f_*.$$

This finishes the induction step.

3. *Zero surgery on a null-homotopic knot*

The aim of this section is to prove the following proposition.

PROPOSITION 3·1. *Let Y be a closed, connected, oriented 3–manifold which does not have an $S^1 \times S^2$ summand, and $K \subset Y$ be a null-homotopic knot. If $Y_0(K) = Z\#(S^1 \times S^2)$, then $\pi_1(Z) \cong \pi_1(Y)$.*

We first prove a general result about surgery on null-homotopic knots.

LEMMA 3·2. *Let Y be a closed, oriented, connected 3–manifold, and $K \subset Y$ be a null-homotopic knot. Let V be the 2–handle cobordism from Y to $Y_m(K)$ for some integer m. Then there exists a retraction $p : V \to Y$, so that $p|_{Y_m(K)} : Y_m(K) \to Y$ is a degree-one map.*

Proof. The cobordism V deformation retracts to the space V' obtained from Y by adding a 2–cell e^2 along K. Since K is null-homotopic, the identity map on Y can be extended over e^2. Hence we have a retraction $V' \to Y$, which implies the existence of the retraction $p : V \to Y$. The degree of the restriction $p|_{Y_m(K)} : Y_m(K) \to Y$ is 1 since it induces an isomorphism on H_3.

The existence of the above degree-one map is a well-known result. See [3, Proposition 3·2] and [7].

In the rest of this section, let V be the 2–handle cobordism from Y to $X = Y_0(K)$, $p : V \to Y$ be the retraction in Lemma 3·2, and $f = p|_{Y_0(K)}$. Moreover, we assume $Y_0(K) = Z\#(S^1 \times S^2)$. Let $\check{Z}$ be the submanifold of $Y_0(K)$ which is Z with a ball removed. Since $Y_0(K) = Z\#(S^1 \times S^2)$, we can add a 3–handle to V to get a cobordism $W : Y \to Z$.

LEMMA 3·3. *The restriction of f_* to $\pi_1(\check{Z})$ is injective.*

Proof. Consider the following commutative diagram, where all maps except p are inclusions:

$$\begin{tikzcd}
Y \arrow[dr, bend left, "\iota_Y"] & Y_0(K) \arrow[d, "\iota_0"] & \check{Z} \arrow[l] \arrow[r] \arrow[dl, "\iota_{\check{Z}}"] & Z \arrow[dl, "\iota_Z"] \\
& V \arrow[ul, bend left, "p"] \arrow[r] & W .
\end{tikzcd}$$

Then $f = p \circ \iota_0$.

Since the 2–handle in V is added along the null-homotopic knot K, the inclusion $\iota_Y : Y \to V$ induces an isomorphism on π_1. Since $p \circ \iota_Y = \mathrm{id}_Y$, p_* is an isomorphism.

Since W is obtained from V by adding a 3–handle, the inclusion $V \subset W$ induces an isomorphism on π_1. We have the commutative diagram

$$\begin{tikzcd}
& \pi_1(\check{Z}) \arrow[r, "\cong"] \arrow[dl, "(\iota_{\check{Z}})_*"'] & \pi_1(Z) \arrow[dl, "(\iota_Z)_*"] \\
\pi_1(V) \arrow[r, "\cong"] & \pi_1(W) . &
\end{tikzcd}$$

The manifold W (after being turned up-side-down) can be obtained from $Z \times I$ by adding a 1–handle and a 2–handle, and the 2–handle cobordism is exactly V being turned up-side-down. By [**4**, Proposition 2·1], $(\iota_Z)_*$ is injective, so $(\iota_{\check{Z}})_*$ is also injective.

Now consider the commutative diagram

$$\begin{tikzcd}
\pi_1(Y) & \pi_1(Y_0(K)) \arrow[d, "(\iota_0)_*"] & \pi_1(\check{Z}) \arrow[l] \arrow[dl, "(\iota_{\check{Z}})_*"] \\
& \pi_1(V) . \arrow[ul, "p_*"] &
\end{tikzcd}$$

The restriction of f_* to $\pi_1(\check{Z})$ is just $p_* \circ (\iota_{\check{Z}})_*$, which is injective since $(\iota_{\check{Z}})_*$ is injective and p_* is an isomorphism.

COROLLARY 3·4. *The induced map $f_* : \pi_1(Y_0(K)) \to \pi_1(Y)$ is surface-group injective.*

Proof. This follows from Lemmas 2·1 and 3·3.

Proof of Proposition 3·1. We will use the notations in Section 2. Since f_* is surface-group injective, we can apply Proposition 2·3 to get a degree-one map $g : X = Y_0(K) \to Y$ so that $g_* = f_*$ and $g^{-1}(S_i) = E_i$ is a separating sphere whenever $1 \leqslant i \leqslant n$.

Since X has an $S^1 \times S^2$ summand, by the uniqueness part of the Kneser–Milnor theorem, one component of $X \setminus (\cup_{i=1}^n E_i)$ has an $S^1 \times S^2$ summand.

If the $S^1 \times S^2$ summand is in $g^{-1}(\check{Y}_{n+1})$, then $g_*(S^1 \times \{point\})$ is null-homotopic. It follows that

$$g_*(\pi_1(X) = g_*(\pi_1(Z\#(S^1 \times S^2))) = g_*(\pi_1(Z)).$$

Since $\deg g = 1$, g_* is surjective. So $g_*|_{\pi_1(Z)}$ is surjective. Our result follows from Lemma 3·3 since $g_* = f_*$.

If $\check{X}_i = g^{-1}(\check{Y}_i)$ has an $S^1 \times S^2$ summand for some i satisfying $1 \leqslant i \leqslant n$, without loss of generality, we may assume $i = 1$. The map $g|_{\check{X}_1}$ extends to a map $g_1 : X_1 \to Y_1$. Suppose that $X_1 = Z_1\#(S^1 \times S^2)$, let $P \subset X_1$ be $\{\text{point}\} \times S^2$. Then $X_1 \setminus \nu^\circ(P)$ is homeomorphic to Z_1 with two open balls removed. Since $\pi_2(Y_1) = 0$, $(g_1)|_P$ is null-homotopic in Y_1. We can then

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extend $g_1|_{X_1\setminus\nu^\circ(P)}$ to a map $h_1 : Z_1 \to Y_1$. The new map h_1 is again a degree-one map, so $(h_1)_*$ is surjective.

Using Lemma 3·3, we see that g_* is injective on $\pi_1(X_0\#Z_1)$. (Recall that X_0 is obtained from X by replacing $\check{X}_1$ with a ball.) In particular, $(h_1)_*$ is injective, so

$$\pi_1(Z_1) \cong \pi_1(Y_1).$$

We also get that g_* is injective on $\pi_1(X_0)$. Since $g|_{\check{X}_0} : \check{X}_0 \to \check{Y}_0$ is a degree-one proper map, $(g|_{\check{X}_0})*$ is surjective. So

$$\pi_1(X_0) \cong \pi_1(Y_0).$$

Since $Z \cong X_0\#Z_1$, $Y = Y_0\#Y_1$, we have

$$\pi_1(Z) \cong \pi_1(X_0) * \pi_1(Z_1) \cong \pi_1(Y_0) * \pi_1(Y_1) \cong \pi_1(Y).$$

4. *Proof of the main theorem*

In this section, we will prove Theorem 1·1 and Corollary 1·2.

Proof of Theorem 1·1. *when* $b_1(Y) > 0$ Without loss of generality, we may assume $M = Y \setminus \nu^\circ(K)$ is irreducible. Since $b_1(Y) > 0$, there exists a closed, oriented, connected surface S in the interior of M, so that S is taut in M. Notice that for the ∞ slope on K, the core of the surgery solid torus, which is K, is null-homotopic. Using [**11**, theorem A.21], which is a stronger version of the main result in [**5**], we conclude that each 2–sphere in $Y_0(K)$ bounds a rational homology ball. Hence $Y_0(K)$ does not have an $S^1 \times S^2$ summand.

PROPOSITION 4·1. *Let* Y_1, Y_2 *be two closed, oriented, connected* 3*–manifolds. If* $\pi_1(Y_1) \cong \pi_2(Y_2)$*, then*

$$\mathrm{rank}\widehat{HF}(Y_1) = \mathrm{rank}\widehat{HF}(Y_2).$$

Proof. This is a well-known consequence of the Geometrisation Theorem. As in [**2**, theorem 2·1·3], if $\pi_1(Y_1) \cong \pi_2(Y_2)$, then there is a one-to-one correspondence between the summands of Y_1 and the summands of Y_2, such that any pair of summands in the correspondence consists of either homeomorphic manifolds, (the homemorphism does not necessarily preserve the orientation), or lens spaces with the same H_1. Our result then follows from basic properties of Heegaard Floer homology [**15, 16**].

Proof of Theorem 1·1. *when* $b_1(Y) = 0$ Without loss of generality, we may assume $Y \setminus K$ is irreducible. Assume that $Y_0(K) = Z\#(S^1 \times S^2)$. By Proposition 3·1, $\pi_1(Y) \cong \pi_1(Z)$. Using Proposition 4·1, we get $\mathrm{rank}\widehat{HF}(Y) = \mathrm{rank}\widehat{HF}(Z)$. Our conclusion follows from [**9**, theorem 1·1].

Proof of Corollary 1·2. The first statement follows from Theorem 1·1. We only need to prove the second statement. In this case, $Y_0(K)$ is a torus bundle over S^1. By Lemma 3·2, there exists a degree-one map $f : Y_0(K) \to Y$. Then f_* is surjective. We claim that Y is $S^1 \times S^2$, $\mathbb{R}P^3\#\mathbb{R}P^3$ or a spherical manifold. Then our conclusion follows from [**1**].

If Y is reducible, then either $Y = S^1 \times S^2$ or Y is a nontrivial connected sum. If $Y = S^1 \times S^2$, we are done. Now we assume Y is a nontrivial connected sum, so $\pi_1(Y) \cong A * B$ with A, B nontrivial. Let T be a fiber of $Y_0(K)$, then $f_*(\pi_1(T))$ is an abelian normal subgroup of $\pi_1(Y) = f_*(\pi_1(Y_0(K)))$. By the Kurosh Subgroup Theorem [**8**, theorem 8·3], $f_*(\pi_1(T))$ is also a free product of free groups and conjugates of subgroups of A, B. Since $f_*(\pi_1(T))$ is abelian, it must be either a subgroup of $\mathbb{Z}$ or the conjugate of a subgroup of A or B. Since $f_*(\pi_1(T))$ is a normal subgroup of $\pi_1(Y)\cong A*B$, the latter case cannot happen.

Now $f_*(\pi_1(T))$ is a subgroup of $\mathbb{Z}$. Since f_* is surjective, $\pi_1(Y)/f_*(\pi_1(T))$ is a cyclic group. If $f_*(\pi_1(T))=\{1\}$, then $\pi_1(Y)$ is cyclic, a contradiction to $\pi_1(Y)\cong A*B$. So $f_*(\pi_1(T))\cong\mathbb{Z}$.

If $\pi_1(Y)/f_*(\pi_1(T))\cong\mathbb{Z}$, then $\pi_1(Y)$ contains $\mathbb{Z}^2$ as a finite index subgroup, which is not possible since $\mathbb{Z}^2$ is not the fundamental group of any closed 3–manifold. If $\pi_1(Y)/f_*(\pi_1(T))$ is finite, then $\pi_1(Y)$ contains $\mathbb{Z}$ as a finite index subgroup. It follows that Y is finitely covered by $S^1\times S^2$, thus it must be $S^1\times S^2$ or $\mathbb{R}P^3\#\mathbb{R}P^3$.

Now we consider the case Y is irreducible. If Y is a spherical manifold, we are done. If Y is irreducible and not a spherical manifold, then f_* is injective by [18, theorem 4]. So f_* is an isomorphism, a contradiction to the fact that $b_1(Y_0(K))>b_1(Y)$.

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