DISSIPATIONLESS FORMATION OF ELLIPTICAL GALAXIES

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ABSTRACT. Dissipationless formation mechanisms envisage elliptical galaxies as arising from the collective relaxation of an aggregate of stars. Their key ingredients are thus a set of initial conditions derived from consideration of prior evolution, and a treatment of the relaxation process. I review numerical studies of violent relaxation carried out over the last decade and purely theoretical treatments going back twice as far. Relaxation is always incomplete, and as a result the final structure of a "galaxy" depends sensitively on the initial conditions assumed. The viability of dissipationless formation thus rests on the identification of plausible stellar initial conditions which relax to the present structure. I discuss the extent to which such initial conditions are compatible with current ideas on the origin of structure in the universe.

I. INTRODUCTION

Present elliptical galaxies appear to approximate smooth equilibrium solutions of the gravitational N-body problem. However, we believe that at high redshift their material was in the form of hot uniformly distributed gas. The transition between these two states clearly involved gravitational, hydrodynamical, radiative and perhaps hydromagnetic processes. As a result, a purely stellar dynamical model for elliptical formation is useful only if star formation occurred before the final phase of relaxation. Any dissipationless model should therefore start from initial conditions in which stars could already have formed. The main idea motivating dissipationless models is the hope that gravitational evolution will cause convergence to an equilibrium distribution which depends weakly on the initial conditions. It turns out, however, that this hope is only partially fulfilled.

Numerical experiments show that the gross equilibrium properties of an isolated system of stars are established in the first two or three free fall times of its evolution. During this phase, evolution can be thought of as resulting from an interaction between the time-dependent gravitational potential and the orbits of the stars responsible for that potential. Qualitatively, one can say that the greater the inhomogeneity of the initial conditions, the broader the spectrum of potential fluctuations excited during collapse, and the greater the randomisation of stellar orbits. In his classic paper of 1967 Lynden-Bell analysed this process by assuming

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the maximum randomisation possible; in addition he coined the oxymoron "violent relaxation" to describe it. This work and extensions of it remain the main theoretical treatments of collisionless relaxation.

In the next section, I review Lynden-Bell's work highlighting the logic which underpins it, some aspects of its predictions, the contradictions to which it leads, and its connection to notions of entropy in stellar dynamics. Much of this discussion draws on the recent paper of Tremaine, Henon and Lynden-Bell (1986). I follow it by a review of the numerical evidence that, in practice, violent relaxation does not lead to full randomisation, and that elliptical galaxies retain considerably more information about their initial conditions than is required by conservation constraints alone. Some structural properties of dissipationless formation models can, however, be derived on quite general grounds. In Section IV I give a simple argument (also presented by Jaffe at this meeting) which shows that in the outermost regions of a galaxy the stellar density is expected to decline as the inverse fourth power of radius. I also give a heuristic explanation for the power-law structure found in numerical experiments involving very strong relaxation. In a final section I return to the question of initial conditions and I consider how dissipationless formation might plausibly occur in a cosmological context. The conclusion of this discussion is that current models for the evolution of structure predict initial conditions which are sufficiently inhomogeneous that the formation process is best characterised as the collision and merging of pre-existing virialised stellar systems.

II. VIOLENT RELAXATION

As formulated by Lynden-Bell (1967), violent relaxation describes the dynamical equilibration of a system of stars which is so large that the fundamental graininess of the stellar distribution can be ignored. It can thus be thought of as the relaxation of a system which is completely described by the coupled Collisionless Boltzmann and Poisson equations:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \nabla\phi \cdot \frac{\partial f}{\partial \mathbf{v}} = 0; \tag{1}$$

$$\nabla^2\phi = 4\pi G \int m f d^3v dm; \tag{2}$$

where $f(\mathbf{x}, \mathbf{v}, m, t)$ is the single particle phase space density. The first equation is simply a statement of mass conservation for each type of star, while the second asserts that the stars are themselves the source of the (Newtonian) gravitational potential in which they move. Because $-\nabla\phi(\mathbf{x})$ is the acceleration of all stars at $\mathbf{x}$, the operator in equation (1) is just the standard convective time derivative along the stellar trajectories. The Collisionless Boltzmann equation can thus be recognised as the equation of motion of an incompressible fluid in a 6-dimensional space. Following the motion of each star, f as a function of m is conserved. We can therefore drop the mass dependence of f and consider it as the total stellar mass density at each point in phase space. In addition, since surfaces of constant f are convected with the fluid, the total volume and the total mass they contain are independent of time. Thus if we define $m(f)df$ as the mass of stars in regions with phase space density in the range $(f, f + df)$, $m(f)$ is conserved as the system relaxes. Following Tremaine, Hénon and Lynden-Bell (1986; hereafter THLB) we can then define the mass and volume at phase space densities exceeding f by,

$$M(f) = \int_f^\infty m(f')df'; \quad V(f) = \int_f^\infty m(f')\frac{df'}{f'}. \tag{3}$$

Since both these functions depend on $m(f)$ alone, they remain constant during violent relaxation. A third similar quantity is

$$S = -\int_0^\infty df' m(f') \ln(f'), \tag{4}$$

which may be recognised as the analogue of the Boltzmann entropy for a collisional gas. Hence if f is used to define an entropy, violent relaxation is adiabatic.

The second important ingredient in the theory of violent relaxation is the notion that although f is conserved, the structure of its phase space distribution becomes more and more convoluted as evolution proceeds. At late times f is no longer an observable quantity; rather the "interesting" information about the structure of a system is contained in a coarse-grained distribution, f_c, obtained by smoothing f over some finite volume. As a system evolves, initially adjacent elements of phase density become separated by arbitrarily large distances. This orbital divergence is caused by differential scattering off fluctuations in the potential, and by phase-mixing as a result of the slightly different periods of neighboring orbits. (Notice that the latter process continues even after violent potential fluctuations have died down.) Thus at late times phase space elements from very different parts of the initial conditions may contribute to the same value of the coarse-grained phase space density. Only because of the implicit assumption that such smoothing has occurred can it be said that violent relaxation causes an elliptical galaxy to "forget" the details of the initial conditions from which it formed. This information loss differs from the corresponding effect in a gas. In the latter case our failure to keep track of the positional correlations between molecules results in a loss of the details of individual collisions. For a gas, an entropy defined using the fine-grained distribution function is increased by collisions.

Lynden-Bell's analysis of violent relaxation is based on a maximal mixing hypothesis. He assumes that the evolution randomises the position of elements of f completely, subject to conservation of total energy and total angular momentum. If every averaging volume contains a large number of such elements, he shows that the most probable coarse-grained distribution function satisfying the constraints is approximately

$$f_c(\mathbf{x}, \mathbf{v}) = \int_0^\infty df' A(f') \exp(-f'\beta\epsilon), \tag{5}$$

with

$$\epsilon = v^2/2 + \phi(\mathbf{x}),$$

and where the energy and mass constraints require

$$E = \int d^3x d^3v \; f_c\epsilon; \quad m(f) = A(f) \int d^3x d^3v \exp(-f\beta\epsilon). \tag{6}$$

Equation (5) is simpler than the form given by Lynden-Bell, because I have assumed that $f'\beta\epsilon >> 1$ for all significant contributions to the integral, and that the total angular momentum of the system is negligible. The first condition is equivalent to assuming that mixing is sufficiently strong for f_c to be everywhere small compared to typical values of f in the initial conditions. Numerical experiments suggest that this condition is violated for a small fraction of the mass at the centre of a violently relaxed system; however, it appears to hold for the bulk of the stars. Equation (5) then gives f_c as a superposition of Maxwellians, one for each value of f present in the initial conditions. The weighting function $A(f)$ is related to the mass of stars with initial phase density f through equation (6); their final velocity dispersion is inversely proportional to the value of f.

It is sometimes stated that as a result of violent relaxation the central regions of an elliptical galaxy should resemble an isothermal sphere. However, the distribution function of equation (5) will resemble a single temperature Maxwellian only if the initial distribution of f satisfies rather restrictive requirements, for example if all stars are initially in regions of the same phase space density so that $m(f)$ and $A(f)$ are delta functions. In general a rather wide range of initial phase space densities is to be expected; the shape of f_c at low energies, and thus the theoretical prediction for the core structure of the resulting galaxy, then depends sensitively on initial conditions. In addition, as noted above, numerical experiments suggest that in many situations the central value of f_c in the final galaxy is not much smaller than the largest values of f_c in the initial conditions. Thus mixing of phase space elements appears to be far from complete. Failure of the maximal mixing hypothesis is also apparent in the angular momentum distributions found experimentally in systems formed by violent relaxation. Lynden-Bell shows that his hypothesis predicts the most probable distribution to be in solid-body rotation. However, the rather sparse experimental data available suggest that a typical final state has a rotational velocity at each radius which is roughly proportional to the local velocity dispersion (White 1979; Farouki and Shapiro 1982; Villumsen 1982; Frenk *et al.* 1985).

Lynden-Bell's derivation of a most probable distribution involves finding a stationary point of an entropy defined using the coarse-grained phase space density,

$$S = -\int d^3x d^3v f_c \ln(f_c). \tag{7}$$

The solution (5) has the unpleasant property that although it is derived using conservation of mass and energy as constraints, it implies an infinite mass and energy for an isolated, unconfined system. In fact, it is easy to see that there is no distribution which maximises the entropy (7) for a given finite mass and energy. A small fraction of the mass of any system can always be put into an arbitrarily weakly bound outer halo which extends to arbitrarily large distances and makes an arbitrarily large contribution to the integral in (7) (see THLB, this argument is originally due to J. Binney). Thus not only is maximal mixing never realised in practice, it does not even lead to a definite prediction for finite systems (see, however Section IV below).

If we abandon the idea of defining the endpoint of violent relaxation as a maximum entropy state, we may ask whether we can, at least, say that violent relaxation always leads to an entropy increase. A proof of this statement, originally due to Tolman (1938), runs as follows:

$$\begin{aligned} S(t_f) &\equiv \left[-\int f_c \ln(f_c) d^3x d^3v \right]_{t_f} \\ &\geq \left[-\int f \ln(f) d^3x d^3v \right]_{t_f} \\ &= \left[-\int f \ln(f) d^3x d^3v \right]_{t_i} \\ &= \left[-\int f_c \ln(f_c) d^3x d^3v \right]_{t_i} \equiv S(t_i), \end{aligned} \tag{8}$$

where t_i and t_f denote the initial and final times respectively. The inequality in the second line is a result of the averaging implicit in the definition of f_c, together with the fact that for

$$C(f) = -f \ln(f), \tag{9}$$

$\overline{C(f)} > C(\overline{f})$ for any set of unequal values of f. The equality in the third line of (8) holds because, as we have already seen, an entropy defined using equation (4) is conserved during violent relaxation. The equality in the final line of (8) reflects the assumption that in the initial conditions the fine-grained phase space density is a slowly varying function of phase space coordinates so that $f = f_c$. The result that the final entropy exceeds the initial entropy is a direct consequence of this assumption. This has some peculiar corollaries. Since equations (1) and (2) are time reversible we can imagine integrating backwards in time to find the entropy at some $t_o < t_i$. The above argument then implies $S(t_o) > S(t_i)$. In addition, if t_1 and t_2 are both later than t_i, we cannot say anything about the relative sizes of $S(t_1)$ and $S(t_2)$. Thus we have not proved that violent relaxation causes S to increase with time; merely that there is, in general, less information in the coarse-grained distribution than in the fine-grained distribution. If we assume the two to be identical at some epoch, then this time is picked out as special.

As THLB note in their discussion of Tolman's proof, the analysis only requires $C(f)$ to satisfy the inequality following equation (9). Thus the proof does not prefer the particular functional form given in that equation; any convex function $C(f)$ could replace $-f \ln(f)$ in the definition (7). Once again the situation differs from that for gaseous systems where the standard proof of the H-theorem shows that equation (4) is the only possible definition for which entropy always increases. In collisionless stellar dynamics there are infinitely many H-functions corresponding to all possible choices of C. THLB show that there is actually a one-to-one correspondence between the choice of C and the distribution function $f_c(\epsilon)$ (satisfying $df_c/d\epsilon < 0$) given by Lynden-Bell's maximisation procedure. Thus, although Lynden-Bell's distribution (5) is the unique most probable state implied by his counting rules for assigning statistical weights to macrostates, Tolman's analysis of evolution does not assign it any special status. The only clear prediction would seem to be that violent relaxation should lead to isotropic distribution functions in which the phase space density is a declining function of energy. Unfortunately even this very general prediction is not well borne out by experiment. Numerical models show the outer regions of "galaxies" formed by violent relaxation to be highly anisotropic; they are composed mainly of stars on near radial orbits which have been ejected from the central regions.

It is clearly desirable to find some way to define the effect of evolution which does not make reference to any specific choice of H-function. THLB give a simple formalism which does this by utilising the functions $M(f)$ and $V(f)$ defined in equation (3). Eliminating the fine-grained phase space density leads to a new function, $M(V)$. This specifies the maximum mass which collisionless evolution could ever conceivably pack into any phase space volume of size V. If we now imagine using the coarse-grained phase space density of the system to define similar functions $M_c(f_c)$, $V_c(f_c)$ and $M_c(V)$, then, because of the averaging implicit in the definition of f_c, $M_c(V)$ must be less than $M(V)$ for all V; the two can be equal only if $f_c = f$ throughout the system. Thus if we are willing to assume that $f_c = f$ at some particular time (say the initial time), $M_c(V)$ at all other times, earlier or later, is bounded above by $M_c(V)$ at that time. This result can be expressed, following THLB, by saying that the system must always appear more mixed than at the chosen time. Thus if we are willing to assume that the initial conditions for violent relaxation are fully specified by f_c, we can restrict the class of equilibrium distributions for the final state to those which are more mixed than the initial conditions. Notice that there is still no sense in which we have picked out an arrow of time. Our assumptions promote one particular time to special status, but we have no way to order the properties of the system at any other two times. A well-known consequence of this mixing constraint (obtained in the limit $V \to 0$) is the fact that the maximum value of f_c in the final state must be less than its maximum value in the initial conditions. A version of this argument was used by Tremaine and Gunn (1979) to put upper limits on the central density of the galaxy halos which might form in a neutrino dominated universe.

Finally, I think that it is important to note that despite the formal difficulties discussed at length above, the conceptual framework of violent relaxation theory is of considerable use. In most situations the collapse and relaxation of an inhomogeneous and relatively cold system does indeed lead to an equilibrium with typical phase space densities much lower than in the initial conditions. This final state will almost always have a significantly higher coarse-grained entropy.

III. NUMERICAL STUDIES OF DISSIPATIONLESS FORMATION

A full treatment of evolution under equations (1) and (2) can, in general, be carried out only by numerical simulation, and, indeed, any large-scale N-body simulation of gravitational evolution from nonequilibrium initial conditions can be said to be a simulation of violent relaxation. Typically if the total energy of a system is negative, it settles down after a few dynamical times to an approximately steady state. A small fraction of the mass may escape to infinity. The structure of the final state depends quite strongly on the initial conditions. Its maximum phase space density cannot exceed that of the initial conditions, but is often found to be almost equal to it (Melott 1982; Farouki, Shapiro and Duncan 1983; May and van Albada 1984). The overall shape of the final state usually reflects the geometry of the initial conditions from which it relaxed (White 1976, 1979; Aarseth and Binney 1978). In particular, Aarseth and Binney showed that relaxation from sheet-like initial conditions can lead to equilibria which are considerably more flattened than observed elliptical galaxies. Finally, systematic series of experiments by White

(1979) and van Albada (1982) showed that while violent relaxation from a wide range of initial conditions leads to density profiles which are generally similar to each other and to the luminosity profiles of elliptical galaxies, it is easy to trace systematic trends in the concentration of the final state to details of the structure of the initial conditions.

The strong concentration of elliptical galaxy luminosity profiles is one of their most striking features. For any system in virial equilibrium one can define a characteristic phase space density using its mass, its gravitational radius and its velocity dispersion as,

$$f_o = 3M/(8\pi r_g^3 < v^2 >^{3/2}) = 3(-2E)^{3/2}/(\pi G^3 M^{7/2}), \tag{10}$$

where E is the total energy of the system. For the model which King (1966) fitted to the galaxy NGC 3379 the central phase space density is about 100 f_o. Thus for violent relaxation to stand a chance of reproducing the structure of this galaxy it must occur from initial conditions where a significant fraction of the mass has phase space density a hundred times that defined by the total mass and energy. If density variations in the initial conditions are of order unity, then random velocities must be much smaller than the characteristic virial values. On the other hand, if the initial conditions are highly inhomogeneous, stars in dense regions can have moderately large random velocities and still have high phase space density. Thus cold collapse and the merging of previrialised subunits provide two possible routes for dissipationless galaxy formation. It turns out that both routes can lead to systems which resemble observed ellipticals. White (1979) and van Albada (1982) give examples which have density profiles very similar to de Vaucouleurs (1948) empirical density law, and which formed by merging and from a cold collapse, respectively. However, there is nothing magic about the $r^{1/4}$-law. The early spherical models of Gott (1973) show that if too much symmetry is imposed on a collapse, randomisation processes are too weak to produce a concentrated galaxy. More recent work shows that both merging (Villumsen 1982; Farouki, Shapiro and Duncan 1983) and cold collapse (McGlynn 1984) can also lead to systems which are more concentrated than observed galaxies. In such systems the density profiles approximate power laws of slope -3 over many decades in density, a property for which I give a tentative explanation in the next section. The observed uniformity of luminosity profiles cannot, therefore, be explained by violent relaxation alone; it must also reflect some uniformity in the initial conditions.

IV. POSSIBLE EXPLANATIONS FOR OBSERVED DENSITY PROFILES

In this section I present two simple arguments which give some insight into the results of empirical studies of violent relaxation. It seems intuitively clear, at least with hindsight, that the final equilibria should be ellipsoidal, centrally concentrated systems with an extensive outer envelope. (In fact, intermediate states with less symmetry can be surprisingly long lived, see Gerhard (1983)). The asymptotic structure of the outer envelope can be derived as follows. In a system which undergoes strong violent relaxation, stellar energies in the central active region are greatly modified over a period of a couple of dynamical times. Subsequently, stars which have been scattered into weakly bound orbits move out to populate the outer envelope, and a significant number of them escape (typically between 2 and 20 percent of the total mass). Thus if we denote by $N(E)dE$ the number of stars in the final system with energies in the range $(E, E + dE)$, we expect $N(E)$ to be continuous and nonzero in the neighborhood of $E = 0$. In the outer envelope the galaxy potential is approximately Keplerian, so that the size of an orbit is inversely proportional to E. The density profile of the outer envelope follows immediately, since

$$\rho(r) \;\propto\; 1/r^2 N(GM/r)\; d(GM/r)/dr \;\propto\; r^{-4}. \tag{11}$$

Hoffman and Shaham (1985) found an analogous result for the outer structure of the clumps which form in an open universe. Aguilar and White (1986) applied this argument to the outer parts of tidally stripped galaxies, and Jaffe (this volume) has given it in the present context. Since the stars which populate the outer envelope were all ejected from the central regions, they must be on primarily radial orbits. The asymptotic behaviour of the radial and tangential velocity dispersions is then easily derived from the hydrostatic equilibrium equation and from conservation of angular momentum:

$$\sigma_r^2 = GM/3r >> \sigma_t^2 \approx GMr_i/r^2, \tag{12}$$

where r_i is the scale of the strong interaction region from which stars are ejected. The effective value of r_i is smaller for systems which form by cold collapse than for merger products. As a result the region of strong radial anisotropy encompasses a larger fraction of the mass in the former case (compare the results of White (1979) and van Albada (1982)).

The asymptotic profile just derived applies, if at all, only to the extreme outer parts of real elliptical galaxies. The slope of the $r^{1/4}$-law luminosity profile steepens to -3 at 6 effective radii; this is near the outer limit of photometry in most systems. It would be desirable to find some explanation for the power-law structure of the main body of a violently relaxed system. The following heuristic argument suggests that this structure may perhaps be understood as the maximum entropy member of a restricted class of distributions.

As a toy model for the result of violent relaxation, consider a system with a density profile of the following form:

$$\rho(r) = \begin{cases} \rho_o(r/r_c)^{-\gamma}; & r_c < r < cr_c, \\ 0; & \text{otherwise.} \end{cases} \tag{13}$$

For given values of the shape parameters γ and c, the scale factors ρ_o and r_c are determined by the total mass and energy of the system. At each radius let us estimate the coarse-grained phase space density as

$$f_c(r) = \rho(r)/(G\rho(r)r^2)^{3/2}. \tag{14}$$

If we substitute this expression into the standard definition of entropy (equation 7), the result, for fixed total mass and energy, is a function of γ and c alone. To within an additive constant it can be written as,

$$S(\gamma, c) = \ln\left[(3-\gamma)^2\left(\frac{c^{5-2\gamma}-1}{5-2\gamma} - \frac{c^{2-\gamma}-1}{2-\gamma}\right)^{3/2}(c^{3-\gamma}-1)^{-7/2}\right] - \frac{6-\gamma}{6-2\gamma} + \frac{6-\gamma}{2}\frac{c^{3-\gamma}}{c^{3-\gamma}-1}\ \ln c. \tag{15}$$

This expression turns out to be a remarkably weak function of its arguments. For $0 < \gamma < 5$ and $3 < c < 10^6$, S varies by less than 35% around the value -1.5, except for a sharp and well defined region of high values for $\gamma = 2.9 \pm 0.3, c \geq 10^4$. This peak arises because for $2.5 < \gamma < 3$ the mass of the distribution is concentrated at large radii while the binding energy is concentrated at small radii. Thus the half-mass radius can exceed the gravitational radius by a large factor and a significant fraction of the mass of the system can be at phase space densities much lower than the characteristic value of equation (10).

From this argument we see that the profile attained by systems which undergo strong violent relaxation is close to the maximum entropy solution for a system of fixed mass and energy and with a truncated power-law density profile. Note, however, that equation (15) has no true maximum, but diverges logarithmically as $c \to \infty$ for $2.5 < \gamma < 3$. This reflects the the lack of any global entropy maximum for a finite isolated system. It is amusing that equation (15) is minimised for moderately large c and for $\gamma \approx 2$. Thus, contrary to intuition, a standard bounded isothermal halo would appear unlikely to arise from violent relaxation. Finally note that the profiles of elliptical galaxies deviate systematically and consistently from single power-laws. As already remarked above, these deviations must reflect a consistent incompleteness of the relaxation process, and thus must retain information about the initial conditions from which the galaxies formed, if indeed they formed in a dissipationless manner.

V. INITIAL CONDITIONS

Under what circumstances might elliptical galaxies form the bulk of their stars before they relax to their present state? An obvious possibility is elliptical formation by the merging of pre-existing galaxies. Nobody doubts that such mergers occur—a number of clear examples of currently merging systems are known (Toomre 1977; Schweizer 1986). These systems will end up as ellipsoidal star piles, and computer simulations suggest that their density structure and kinematic properties will be quite similar to those of real ellipticals (see White 1983 for a review). Criticism of the hypothesis that this process formed all ellipticals (Toomre and Toomre 1972) has focused on whether the systematic properties of the observed elliptical population can be a result of mergers between observed spirals (e.g. Tremaine 1981). This is not a mandatory requirement, since in most cosmological models mergers occur predominantly at early times when spirals presumably differed considerably from their present day counterparts. However, recent observational work has tended to lessen the force even of arguments against formation from present-day spirals (see Schweizer 1982,1986; Dressler and Lake 1986). The data presented by Djorgovski at this conference suggest that the observed parameters of ellipticals spread over much of the range allowed by the constraints of a fixed density profile and a fixed stellar population. These latter regularities are the only ones expected on the merger hypothesis.

Let us consider whether dissipationless formation is otherwise plausible within current cosmogonies. The latter may be broadly divided into those in which galaxies form by collapse from a coherent shock front of galactic or supergalactic scale, and those in which they form by the hierarchical aggregation of material into larger and larger systems. The old "adiabatic" and "isothermal" formation pictures of the 1970's are the basis of this classification (Sunyaev and Zel'dovich 1972; Peebles

1974; White and Rees 1978). Among more recent theories, neutrino dominated universes (Bond, Efstathiou and Silk 1980: Doroshkevich *et al.* 1980) and explosive formation theories (Ostriker and Cowie 1981; Ikeuchi 1981) belong to the first category; cold dark matter universes (Blumenthal *et al.* 1984; Davis *et al.* 1985) and string-induced galaxy formation (Vilenkin 1985) belong to the second. It is important to note that in all of these theories the characteristic scale of galaxies is set by the cooling properties of the baryonic gas. Strictly dissipationless formation (except by mergers at late times) is thus implausible in any of them.

The results of Aarseth and Binney (1978) show that if stars were formed by fragmentation of a large-scale shock front, subsequent collapse could produce flattened systems with the concentration of elliptical galaxies. In fact, it could form objects which are much flatter than any known galaxy. Thus pancake theories might in principle have difficulty explaining why elliptical galaxies are so round. However, it turns out that in most situations the shocks fragment, if at all, into smaller objects than galaxies; these must then merge into the systems we see. In hierarchical clustering models each object forms from smaller objects which collapsed earlier. Galaxy formation is thus intrinsically a merger process. However the merging systems are mixtures of stars, of any pre-existing dark matter, and of gas that may later make stars. It is not clear that a dissipationless model is appropriate for this situation. However, to the extent that it is, highly inhomogeneous initial conditions are clearly implied. Among hierarchical models, the standard cold dark matter theory predicts the minimum possible level of substructure in collapsing protogalaxies. Nevertheless, the pictures of Frenk *et al.* (1985) show that even in this case large objects form primarily by merging. This bears on the question of whether the radial orbit instability discussed by Merritt at this conference could play a role in the formation of real ellipticals. In the initial conditions predicted by currently popular cosmological theories, inhomogeneities are probably too large to allow a highly radial collapse. The strongly unstable models which have so far been the main focus of investigation thus seem unlikely to be relevant. For more plausible initial conditions it will be difficult to tell if flattening results directly from asymmetries in the initial conditions or from a gentler radial orbit instability.

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DISCUSSION

Luwel: i) Severne and I derived an evolution equation for the coarse grained distribution function which has the Lynden–Bell distribution as a stationary state. We also have an H–theorem for that evolution equation. ii) What is your opinion on Shu's (1978, *Astroph. J.*, **225**, 83) result on violent relaxation?

White: i) It is interesting that you were able to obtain a result equivalent to the classical result that a Maxwellian is a stationary state of the Boltzmann equation for a collisional gas. However, I do not see how this can be applied to real systems where relaxation appears always to be incomplete and where global constraints preclude the Lynden–Bell distribution.

ii) Shu's arguments rely on examining the limit imposed on the validity of the collisionless Boltzmann equation by the discreteness of the stars. However, the definition of violent relaxation which I have been using and which was, I believe, intended by Lynden-Bell, is that it is the dynamical relaxation undergone by a system which obeys the collisionless Boltzmann equation exactly. This would seem to be a valid approximation for most situations of interest. For example, if a large galaxy forms by the merger of two, ten or even a hundred subgalactic stellar systems it seems obvious that the discrete nature of the stars will play no role in determining how the final state is related to the properties of the initial set of systems.

Gott: You mentioned that the entropy would increase to both the future and past of the initial conditions. This occurs in any system. If you find an ice cube on a stove and ask what is the most probable configuration at a later time it is a partially melted ice cube. If you assume that it is an *isolated* system then the most probable configuration at an earlier time is also a partially melted ice cube. In a truly isolated system the ice cube is just a statistical fluctuation and to find a still larger ice cube at earlier times would be still more improbable.

White: Fair comment!

Ostriker: You argue that if there are three times $t_2 > t_1 > t_i$, then an appropriately defined entropy S would have the property such that $(S_2 > S_i, S_1 > S_i)$ but that nothing could be said concerning $(S_2 >< S_1)$. But could we not redefine t_1 to be a new "initial" time with a newly defined "fine grained" distribution, so that $S_2 > S_1$ and an arrow of time is defined?

White: I agree that the choice of the initial time is, to some extent, arbitrary. The assumption that the coarse and fine–grained distributions are identical at the initial time is essentially an expression of our ignorance about the fine details of the initial distribution. However, if we consider the evolution of any specific system we can consistently apply this assumption at one time only. The structure at any other time is related to that at the chosen time by the equations of motion which may well require specific kinds of small scale structure in the distribution function. Thus only the chosen time is picked out as having lowest entropy.

Ostriker: One difference between collapse from a sheet and a three–dimensional collapse is that the effect of overall expansion is different. The result is that infall continues in a three–dimensional collapse but is cut off at finite mass for two–dimensional collapse. Will this difference produce a different final structure in any measurable way?

White: Yes, I think collapse from a sheet would lead to an asymptotic outer profile with $\rho \propto r^{-4}$ as I described. Cosmological infall in a *flat* universe leads to profiles which, in general fall off less steeply with radius.

Djorgovski: You can do a good deal better than simply comparing the radial density profiles. After all, there are correlations, which need to be preserved: take the fundamental plane of physical properties of ellipticals, discovered by Faber *et al.*, myself, and indicated by others. It is not clear to me whether dissipationless mergers will keep the galaxies on this plane or not. This is not a question for the formation epoch only—you may ask whether, if you merge two present day ellipticals, the product will stay on the plane?

White: This is an important point which is not addressed by dissipationless formation models. In such models the distribution of characteristic properties over the population of ellipticals is a result of the distribution of the initial conditions for collapse (or merging). This can only be specified in the context of a more comprehensive theory of galaxy formation. It is possible for the remnant of a merger between two present day ellipticals to lie on the "fundamental plane" you refer to. Whether it does so depends on the orbit from which the galaxies merge and on whether they are surrounded by dark halos.

Burstein: I also wish to refer back to the real world. Recent observations indicate that almost all ellipticals have line–strength gradients that extend to, or beyond, their effective radii, (i.e., they are not just due to the central peak in line–strength). What are your current thoughts on this constraint on dissipationless formation processes?

White: The line–strength gradients do provide a constraint on dissipationless models, but they are relatively easy to understand in the context of models which have at least what I would consider to be the minimum plausible amount of initial inhomogeneity. This is so because the violent relaxation process roughly preserves the ordering of stars in binding energy. (See White 1978, *Mon. Not. R. astr. Soc.*, **184**, 185; van Albada 1982, *Mon. Not. R. astr. Soc.*, **201**, 939). Thus the most bound stars in the final galaxy tend to be those that were most bound in the smaller clumps in which they originally formed. One would naturally expect these to also be the most metal rich stars.

Palmer: If I understand you correctly, the only reason why these "entropies" increase is because you coarse–grain the distribution. The actual physics of the evolution *conserves* these functions. This is in marked contrast to the $f \ln f$ entropy for collision dominated systems, where physical collisions mix up the phase space distribution.

White: It is true that the $f \ln f$ entropy defined using the fine–grained single particle distribution function always increases in collisional systems, provided that the two–particle distribution function which appears in the Boltzmann collision term can be assumed to be a simple product of single particle distributions.

Barnes: You've shown how the splashing of stars out to infinity gives a characteristic profile to the outer part of an E galaxy. How is this argument modified by the presence of heavy halos which merge before the galaxies do?

White: My argument will probably not apply in this (quite plausible) situation. In particular, if the highest stellar energy generated during violent relaxation is less than the escape energy from the halo, the final stellar distribution will be bounded.

White and Dejonghe discuss entropy.

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