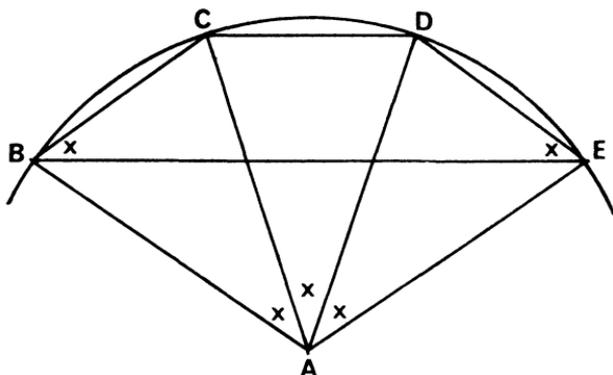### 82. The formula sin 3x = 3 sin x − 4 sin³ x.

Draw a circle with centre A and with unit radius. Let chords BC, CD, and DE each subtend an angle x at the centre. By circle geometry the angles CBE and DEB are each equal to x.

Area $BCDE = \frac{1}{2}(CD + BE)BC \sin x$

$$= \tfrac{1}{2}(CD + CD + 2BC \cos x)CD \sin x = CD^2((1 + \cos x) \sin x \qquad (1)$$

By the cosine rule $\qquad CD^2 = 2 - 2 \cos x \qquad\qquad\qquad\qquad (2)$

From (1) and (2) Area $BCDE = 2(1 - \cos^2 x) \sin x = 2 \sin^3 x \qquad (3)$

By inspection, twice area ABE = twice $ABCDE$ − twice $BCDE$ so $\sin 3x = 3 \sin x - 4 \sin^3 x$.

J. W. HESSELGREAVES

CORRESPONDENCE

To the Editor of the *Mathematical Gazette*

Calendar Reform

DEAR SIR,—I heartily welcome Mr. W. F. Bushell's plea for the reform of our absurd Calendar, with its scars of Augustus's vanity and the ten-month year. In your history books you can read that Charles I was executed in 1649; but a stone in the aisle of St. George's Chapel at Windsor commemorates his burial there in 1648, a fine instance of ὕστερον πρότερον (the cart before the horse). The royal undertaker was clearly still beginning his year in March, probably on March 25th, which was chosen as Annunciation Day because it was the anniversary of the Creation of the World—so I was told by the late M. R. James, O.M., who, if not actually present at the Creation, at any rate began life nearer to it than I did.

Much rummaging in Gibbon's *Decline and Fall* has failed to verify an ancient note (surely not the product of my own disordered

imagination?) that Gibbon describes the *Persian* Calendar as " in accuracy greatly exceeding the Julian, and even approaching the Gregorian ". This Persian Calendar was instituted in the 11th century, chiefly by Omar Khayyam, better known as a poet. Many years ago, hearing that the Persian ex-Minister of Education would visit Eton, I invited him to my school-room, quoted (or misquoted) Gibbon, and (having previously " done " the Gregorian error of 26 seconds) plunged into the Persian Calendar. Seven times this provides for *three* ordinary years and a leap year, like the Julian system, but the eighth time it gives us *four* 365-day years before the leap year. This makes the Persian year $365\frac{8}{33}$ days, or 365 d., 5 hr., 49 min., $5\frac{1}{2}$ sec. which exceeds the true year by only $19\frac{1}{2}$ sec., just $\frac{3}{4}$ of the Gregorian error. Naturally my visitor was delighted, passed a unanimous vote of censure on Gibbon, and talked to the boys about Omar Khayyam till the clock struck.

In addition to cleaning up the inside of our year on the lines suggested by Mr. Bushell, we would do well to adopt a system of leap years simpler than the Gregorian, more accurate, and recurring in a period of 33 years instead of 400. Or are we, like Horace, to " hate Persian apparatus "?

Yours, W. HOPE-JONES

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To the Editor of the *Mathematical Gazette*

DEAR SIR,— I have read the article on " The Geometry of Megalithic Man " in your last issue with much interest. May I supplement one observation by the author who mentions solar alignments, by means of outlying stones or otherwise, in Caithness, Sutherland and elsewhere.

Sixty years ago I helped my father to plan some of the Prescelly circles in Pembrokeshire. Before the coming of the motor car they were little known, but their comparative abundance led my father to write of this area as the " Westminster Abbey " of Prehistoric man, a phrase that was quoted by Dr. Thomas the geologist when, soon after the First World War, he proved that the blue stones at Stonehenge came from that area. Some of these circles had outlying stones pointing to sunrise at the summer solstice, similar to the more famous case at Stonehenge. I also found other solar alignments, not only there, but elsewhere as well.

Sir Norman Lockyer examined these monuments from an astronomical standpoint, and published his book *Stonehenge* in 1906. He claimed that some alignments, by means of outlying stones, were to the rising of well known bright stars. Owing to the precession of the equinoxes, and consequently the movement of these stars in declination, such stellar alignments were no longer accurate, and the age of the monument could be calculated. This has not met with general acceptance, but *solar* alignments were certainly employed in some cases. It is interesting for instance to find that all the remaining megalithic passage graves in the Channel Islands point to sunrise on some day in the year, and other examples could be quoted.

Yours faithfully, W. F. BUSHELL