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DIAHEMITONIC MODALITY: A QUARTER-TONAL COMPOSITION SYSTEM

Simon Kaplan

Abstract: This article describes a design process for a composition system based upon the quarter-tone scale. As the semitone and quarter-tone scales share the same properties, a musical grammar based upon the former is used as a model to construct an analogous musical grammar based upon the latter. Within the scope of diatonic modal music, the organisation of pitches and durations is analysed from a scientific perspective. Several parameters are taken into consideration: scale, mode, chord, cadence, metre and rhythm. The one-to-one mapping of each facet of diatonic modality onto quarter-tonality results in diahemitonic modality.

Introduction

Microtonality is a field of musical development which has existed since the dawn of melody. It inheres in the use of intervals called microtones which lie outside the semitone scale and is exemplified in the musical literature by scales such as those of Partch and Bohlen–Pierce. The quarter-tone scale divides the octave into 24 equal intervals. Ivan Wyschnegradsky and Alois Hába pioneered the earliest quarter-tonal composition techniques in Europe, as Charles Ives did in the US, but these were only infrequently emulated at the time and a quarter-tone scale-based common language has remained largely absent from the Western musical landscape. My research is an attempt to fill this gap.

To envisage a grammar for quarter-tonal music, it is necessary to have a model from which to draw inspiration. Given the significant extent of the resources it has to offer, semitonal music must surely be this model. A word of caution, however: many pitch management systems have been designed for the semitone scale, but they are not all idiomatic. Serialism, for example, can be effective with any scale and the same may be said of set theory. Diatonic modality, however, is tailor-made for the semitone scale. It includes the diatonic scale, melodic modes, consonance-based harmony and chordal cadences and encompasses much of the history of Western music. Diatonic modal music also responds to certain metric structures and rhythmic proportions which, as I shall demonstrate, are intimately linked to the very nature of the semitone scale. Indeed, the rules which govern them are not gratuitous but derived from those which govern pitches, so it is essential to rethink the entire architectonics of music when migrating to another scale.

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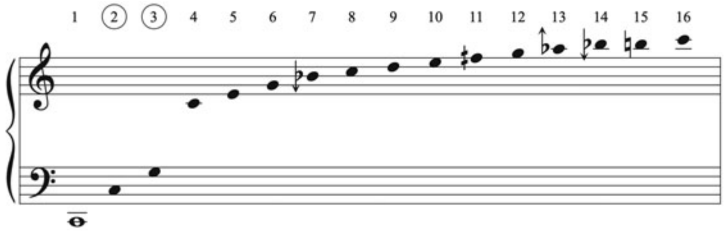

Figure 1:
Selection of the second and third harmonics within the harmonic series.

This article considers the parameters related to pitch and duration and demonstrates that the laws diatonic modality obeys are applicable to a quarter-tonal environment. I shall analyse each of the characteristics of diatonic modal music and then map them onto quarter-tonality, making possible an idiomatic quarter-tonal composition system.

Pitch

In this section, I shall focus on the intervallic relationships between notes, both on the horizontal axis of melody and on the vertical axis of harmony. I shall analyse scale, mode, chord and cadence in the semitonal context and study the possibility of analogies in the quarter-tonal one.

The Equally Tempered Scale

In an equally tempered scale, the size of each step equals the largest common factor of all its constituent intervals. The semitone and quarter-tone scales both belong to this category of scales. Moreover, both are octave-repeating, so that in both semitonal and quarter-tonal contexts, all pitches separated by one or several octaves belong to the same pitch class and, consequently, bear the same name. To devise the semitone scale, one begins with the interval between the second and third harmonics: the perfect fifth (see Figure 1).

Its root belongs to the pitch class of the fundamental. By cycling this interval until returning to that pitch class, one obtains the circle of fifths (see Figure 2).

The circle of fifths generates the semitone scale when its 12 constituent pitch classes are organised in an ascending pattern of the narrowest possible equal intervals (see Figure 3).

This is known as the cyclic principle, according to which a scale results from the cycling of an interval other than the unison once the return to the original pitch class has occurred.

The quarter-tone scale can be devised following the same principle. The interval between the eighth and eleventh harmonics is that of the major fourth (see Figure 4).

Like the perfect fifth, its root belongs to the pitch class of the fundamental. By cycling this interval until returning to that pitch class, one obtains the circle of major fourths (see Figure 5).

The circle of major fourths generates the quarter-tone scale when its 24 constituent pitch classes are organised in an ascending pattern of the narrowest possible equal intervals (see Figure 6).

The White-Key Scale

Anyone who looks at a piano keyboard will notice that the semitone scale is arranged in two rows: the white keys and the black keys. The

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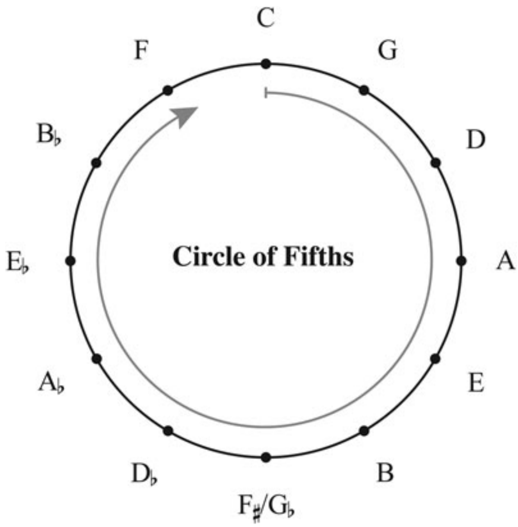

Figure 2:
Circle of fifths.

Figure 3:
Semitone scale.

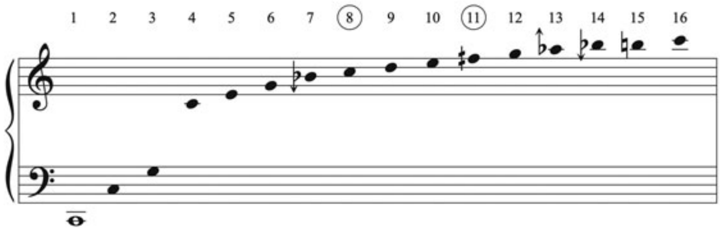

Figure 4:
Selection of the eighth and eleventh harmonics within the harmonic series.

former corresponds to the diatonic scale and the latter to the pentatonic one. To derive the diatonic scale, one begins by adding a perfect fifth, the interval at the origin of the semitone scale, above a given pitch and below its octave. This yields the interval of the tonos (ancient Greek: 'tone') (see Figure 7).

When this tone passes through the two perfect fourths it separates, the diatonic scale is formed, composed of two identical tetrachords (see Figure 8).

The first degree of the first tetrachord is the tonic and that of the second the dominant (see Figure 9).

The diatonic scale contains seven degrees. This equals half of the pitch classes contained within the semitone scale from which it is derived plus one, while its complementary black-key scale, the pentatonic, contains five degrees, half of the pitch classes minus one. Together, these two numbers of degrees form a pair of twin primes. The interval used cyclically to generate the semitone scale and its inverse correspond to these numbers when expressed in integer

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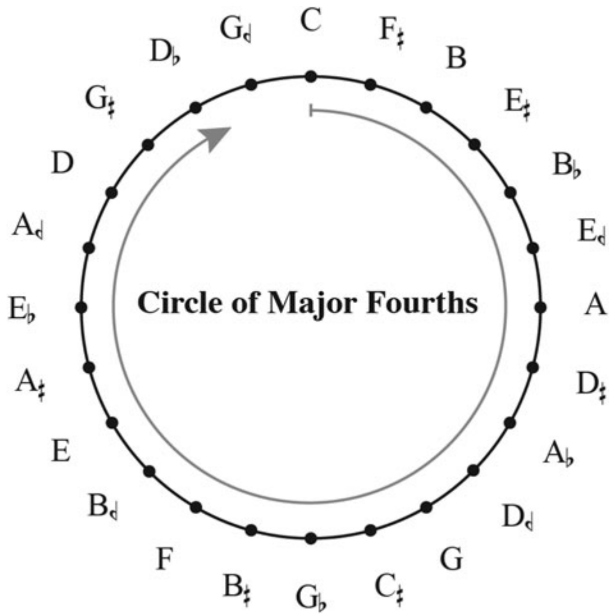

Figure 5:
Circle of major fourths.

Figure 6:
Quarter-tone scale.

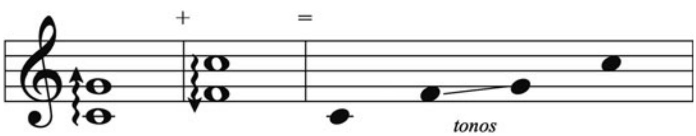

Figure 7:
Tonos.

notation: [0, 7] modulo 12 represents the perfect fifth and [0, 5] modulo 12 the perfect fourth.

A white-key scale can be derived from the quarter-tone scale in the same way. When a major fourth, the interval at the origin of the quarter-tone scale, is added above a given pitch and below its octave, the interval of the hemitonos (ancient Greek: 'semitone') appears (see Figure 10).

When this semitone passes through the two major fourths it separates, the diahemitonic[1] scale, as I term it, is formed, composed of two identical heptachords (see Figure 11).

[1] Coined from the ancient Greek diahemitonikos, meaning 'of semitones passed through (the heptachords)' (from dia, 'through', + hemisys, 'half', + tonos, 'tone', + -ikos, '-ic'), upon the model of 'diatonic'.

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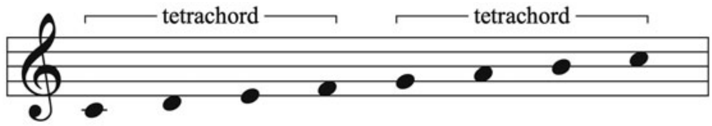

Figure 8:
Diatonic scale.

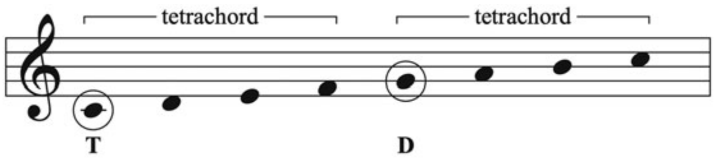

Figure 9:
Tonic and dominant in the diatonic system.

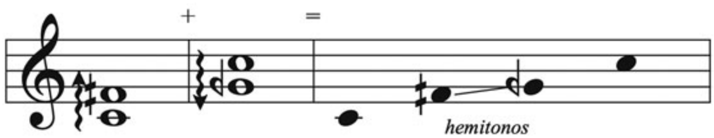

Figure 10:
Hemitonos.

Figure 11:
Diahemitonic scale.

The first degree of the first heptachord is therefore the tonic and that of the second the dominant (see Figure 12).

The diahemitonic scale contains 13 degrees. This equals half of the pitch classes contained within the quarter-tone scale from which it is derived plus one, while its complementary black-key scale, which I term the hendecatonic,[2] contains 11 degrees, half of the pitch classes minus one. Together, these two numbers of degrees form the pair of twin primes which directly follows that formed by the diatonic–pentatonic couple within the sequence of pairs of twin primes. The interval used cyclically to generate the quarter-tone scale and its inverse correspond to these numbers when expressed in integer notation: [0, 13] modulo 24 represents the minor fifth and [0, 11] modulo 24 the major fourth, which parallels the above. Wyschnegradsky described the diahemitonic scale as 'diatonicised chromaticism', the quarter-tonal analogue of the diatonic scale.[3] The mathematician, philosopher and musicologist Franck Jedrzejewski also derived this scale (starting

[2] Coined from the ancient Greek hendekatonikos, meaning 'of 11 tones' (from hendeka, '11', + tonos, 'tone', + -ikos, '-ic'), upon the model of 'pentatonic'.

[3] Ivan Wyschnegradsky, *Manual of Quarter-Tone Harmony*, tr. Rosalie Kaplan (New York: Underwolf, 2017), pp. 19–23.

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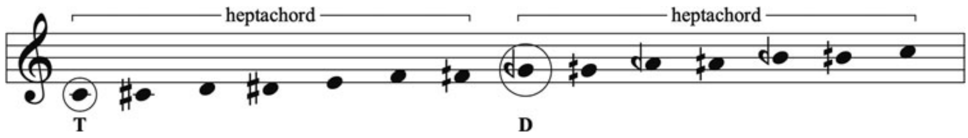

Figure 12:
Tonic and dominant in the diahemitonic system.

Table 1:
White-key scales derived algorithmically from the semi- to sixth-tone divisions of the octave

Equal Division of the Octave	White-Key Scale
12	{0, 1, 3, 5, 6, 8, 10}
18	{0, 2, 4, 5, 7, 9, 10, 12, 14, 15, 17}
24	{0, 1, 3, 5, 7, 9, 11, 12, 14, 16, 18, 20, 22}
30	{0, 1, 3, 5, 7, 8, 10, 12, 14, 15, 17, 19, 21, 22, 24, 26, 28}
36	{0, 1, 3, 5, 7, 9, 11, 13, 15, 17, 18, 20, 22, 24, 26, 28, 30, 32, 34}

from the thirteenth pitch class onwards) when running an algorithm conceived to identify the white keys within any equal division of the octave (see Table 1).[4]

The Mode

The mode is the configuration of the white-key scale once the tonic is defined. The mood of the music is dependent upon the chosen mode and some modes do indeed appear to sound brighter or darker than others, according to their intervallic pattern. Those with wide intervals may be said to sound bright and those with narrow intervals dark. I shall explain the logic following which the diatonic modes have been hierarchised from the brightest to the darkest. When the diatonic scale is left in its original state, it is configured as the C diatonic mode (see Figure 13).

When inverted, it is configured as the E diatonic mode (see Figure 14).

By virtue of their reciprocal inversion, the former must be located at the same distance from the brightest level in the hierarchy of the diatonic modes as the latter is located from the darkest. Equidistantly from these two modes lies a symmetric one, the D diatonic mode (see Figure 15).

By virtue of its symmetry, this mode must be located at the midpoint of the hierarchy of the diatonic modes. When following the circle of fifths clockwise, one establishes this hierarchy with one interval reduced by a semitone at each darker level (see Figure 16).

[4] Franck Jedrzejewski, *Dictionnaire des musiques microtonales*, new revised and expanded edition (Paris: L'Harmattan, 2014), pp. 60–62.

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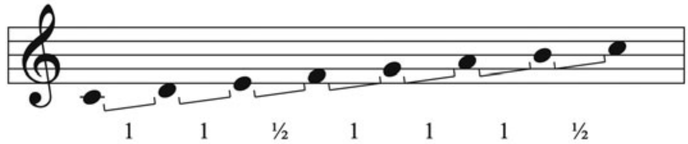

Figure 13:
C diatonic mode.

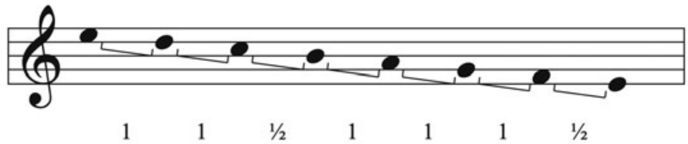

Figure 14:
E diatonic mode.

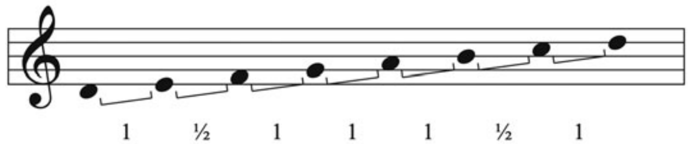

Figure 15:
D diatonic mode.

The two pitch classes involved in this intervallic reduction are separated by the interval of the tritone and the diatonic modes of which they are the tonics are located at the two ends of the hierarchy of the diatonic modes.

A hierarchy of the diahemitonic modes, from the brightest to the darkest, can be established following the same logic. When the diahemitonic scale is left in its original state, it is configured as the C diahemitonic mode (see Figure 17).

When inverted, it is configured as the F diahemitonic mode (see Figure 18).

By virtue of their reciprocal inversion, the former must be located at the same distance from the brightest level in the hierarchy of the diahemitonic modes as the latter is located from the darkest. Equidistantly from these two modes lies a symmetric one, the A-quarter-tone-flat diahemitonic mode (see Figure 19).

By virtue of its symmetry, this mode must be located at the midpoint of the hierarchy of the diahemitonic modes. When following the circle of major fourths anticlockwise, one establishes this hierarchy with one interval reduced by a quarter-tone at each darker level (see Figure 20).

The two pitch classes involved in this intervallic reduction are separated by the interval of the tritone and the diahemitonic modes of which they are the tonics are located at the two ends of the hierarchy of the diahemitonic modes, which parallels the above.

The Chord

The chord is a set of pitched notes sounded simultaneously. A variety of chord types exist, of which two are judged to be perfect: the major and the minor. The former originates in the harmonic series and

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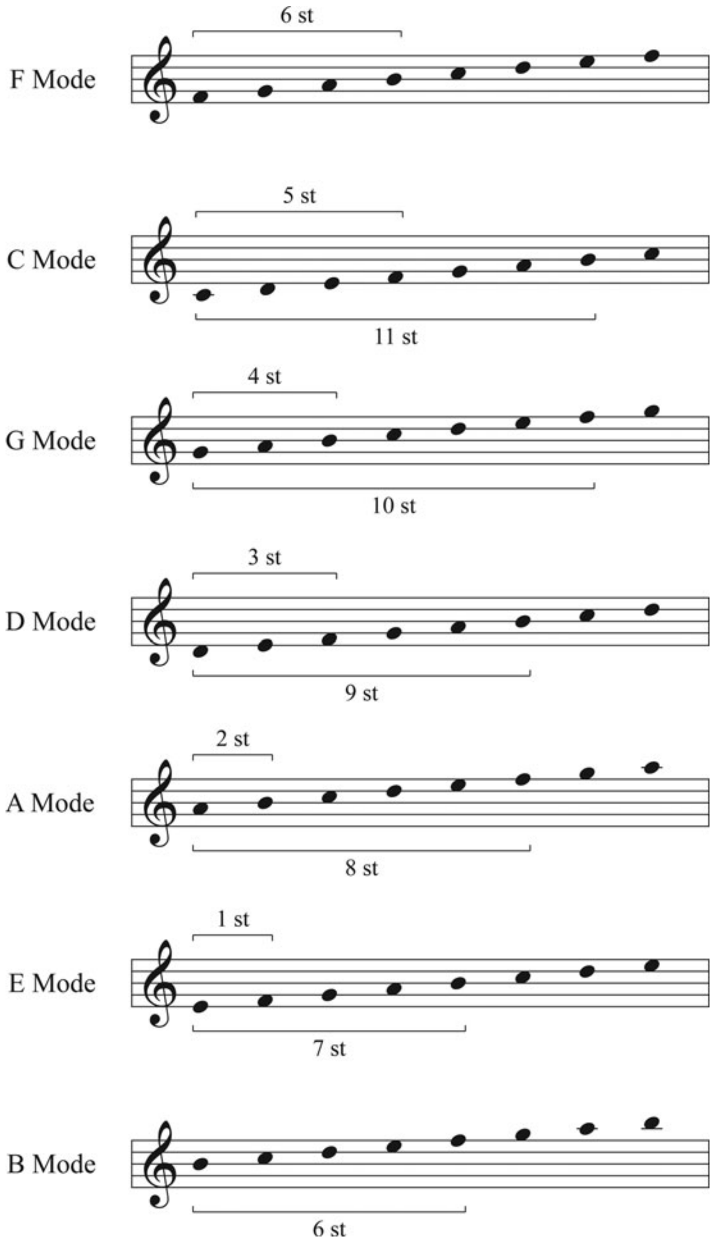

Figure 16:
Hierarchy of the diatonic modes.

the latter in the subharmonic series.[5] The fifth harmonic lies between the octave-equivalents of the second and third harmonics which form the basis of the diatonic system, enabling a ray to be traced (see Figure 21).

By tracing it, one obtains the major triad (see Figure 22).

This is known as the radial principle, according to which a perfect chord results from the tracing of a ray between two harmonics or

[5] Vincent d'Indy, *Cours de composition musicale, premier livre* (Paris: Durand, 1912), pp. 91–106.

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Figure 17:
C diahemitonic mode.

Figure 18:
F diahemitonic mode.

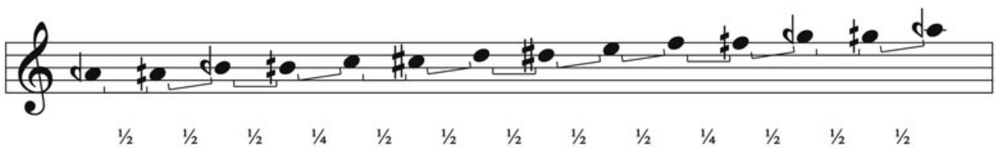

Figure 19:
A-quarter-tone-flat diahemitonic mode.

subharmonics whose frequency ratio corresponds to the interval used cyclically to generate a given scale.[6] Once the major triad is detected within the diatonic scale, two interlaced patterns appear (see Figure 23).

This enables the derivation of the triadic scale (see Figure 24).

The minor triad results from the inversion of the major triad, making it its subharmonic counterpart (see Figure 25).

Thus, in the C diatonic mode, for example, the first-, fourth- and fifth-degree triads are major, the second-, third- and sixth-degree triads are minor, and the seventh-degree triad is neither major nor minor but diminished.

Diahemitonic chords can be built following the same principle. The eighth and eleventh harmonics form the basis of the diahemitonic system and between them lie the ninth and tenth harmonics, enabling a ray to be traced (see Figure 26).

By tracing it, one obtains what I term the major tetrad (see Figure 27).

[6] Heiner Ruland, *Expanding Tonal Awareness: A Musical Exploration of the Evolution of Consciousness – from Ancient Tone Systems to New Tonalities – Guided by the Monochord*, tr. John F. Logan (Forest Row: Rudolf Steiner, 2014), pp. 44–46.

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Figure 20:
Hierarchy of the diahemitonic modes.

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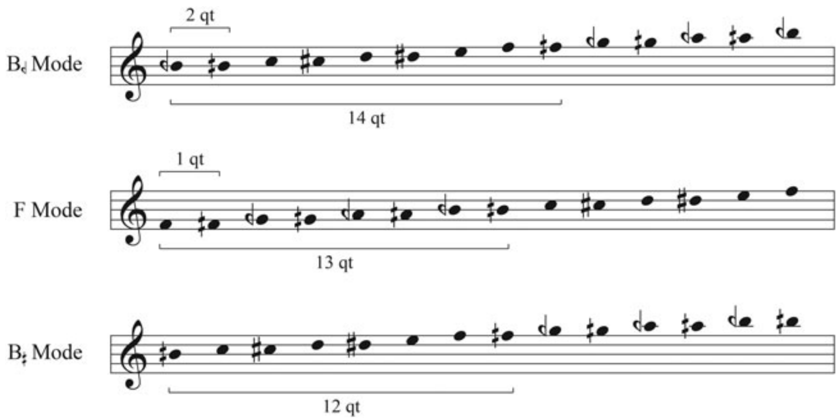

Figure 20:
Continued.

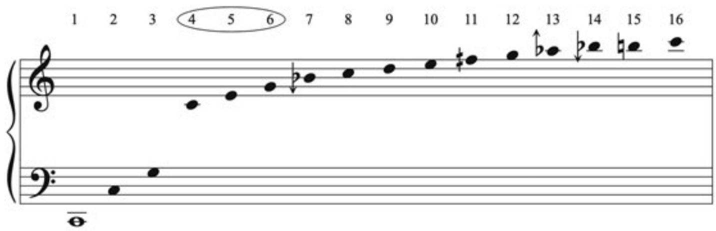

Figure 21:
Selection of the fourth to sixth harmonic within the harmonic series.

Figure 22:
Major triad.

Figure 23:
Major triad within the diatonic scale.

Once the major tetrad is detected within the diahemitonic scale, two interlaced patterns, analogous to those within the diatonic scale, appear (see Figure 28).

This enables the derivation of what I term the tetradic scale (see Figure 29).

What I term the minor tetrad results from the inversion of the major tetrad, making it its subharmonic counterpart (see Figure 30).

Thus, in the C diahemitonic mode, for example, the first-, second-, eighth- and ninth-degree tetrads are major, the fifth-, sixth-, twelfth- and thirteenth-degree tetrads are minor, the third-, fourth-, tenth- and eleventh-degree tetrads are neither major nor minor but what I

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Figure 24:
Triadic scale.

Figure 25:
Minor triad.

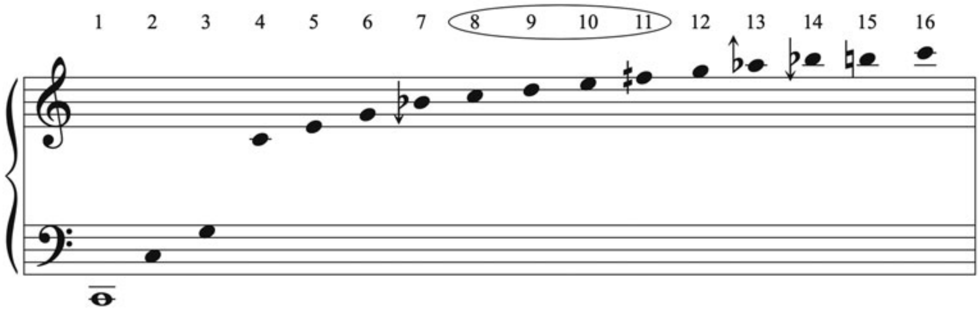

Figure 26:
Selection of the eighth to eleventh harmonic within the harmonic series.

Figure 27:
Major tetrad.

Figure 28:
Major tetrad within the diahemitonic scale.

term neutral, and the seventh-degree tetrad is neither major nor minor but augmented.

The Modal Harmonic Cadence

The modal harmonic cadence is the fall from one chord to another at the end of a phrase, punctuating the musical discourse. The two voices of this clausula vera (Latin: 'true close') resolve onto the octave

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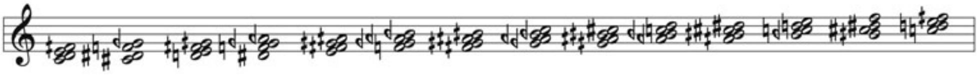

Figure 29:
Tetradic scale.

Figure 30:
Minor tetrad.

by conjunct contrary motion. A third voice is often added, resolving conjunctly upwards onto the fifth, downwards onto the third or splitting to resolve onto both by conjunct contrary motion. In this case, the diatonic modal harmonic cadence may be defined as consisting of the progression of the last-degree triad in first inversion towards the first-degree triad in root position (see Table 2).

Analogously, the diahemitonic modal harmonic cadence consists of the progression of the last-degree tetrad in first inversion towards the first-degree tetrad in root position (see Table 3).

Duration

When a scale is built upon the cyclic principle, it responds to a specific combination of harmonics. Since the harmonic is a periodic frequency, it is a rhythm, and when it oscillates in simultaneity with others, the coincidence of the sinusoidal starting points is noticed at regular intervals, establishing a metre.[7] It is therefore important to reconsider the durational aspect of music when venturing into microtonal territory, so I shall now focus on the temporal relationships between notes, analysing metre and rhythm in semitonal music to study the possibility of analogies in quarter-tonal music.

The Metre

A metre is a regular cycle of beats, the first of which is strong and those which follow weak. Diatonic modal music employs duple and triple metres, referred to as tempus imperfectum (Latin: 'imperfect time') and tempus perfectum (Latin: 'perfect time') respectively. This may be justified by the following: the 3:2 frequency ratio corresponds to the interval used cyclically to generate the semitone scale, from which the diatonic scale is derived. As the second harmonic oscillates twice as fast as the fundamental and the third three times as fast, converting these harmonic oscillations into metres results in the second harmonic becoming a duple metre and the third a triple one. The proportion which enables the metric modulation from the one into the other without altering the total duration of the bar is referred to as proportio sesquialtera (Latin: 'sesquialtera proportion').

[7] Henry Cowell, *New Musical Resources* (New York: Alfred A. Knopf, 1930; New York: Cambridge University Press, 1996), pp. 45–81. This reference is to the Cambridge edition.

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Table 2:
Diatonic modal harmonic cadences

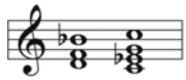
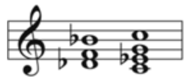
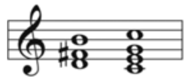
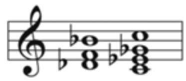
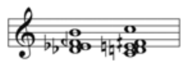
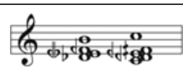
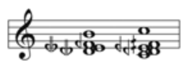
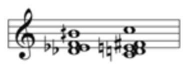
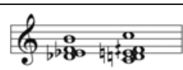
Diatonic Mode	Harmonic Cadence
C	
D/A	
E	
F	
G	
B	

Table 3:
Diahemitonic modal harmonic cadences

Diahemitonic Mode	Harmonic Cadence
C	
C-sharp/G-quarter-tone-sharp	
D/A-quarter-tone-flat	
D-sharp/A-quarter-tone-sharp	
E/B-quarter-tone-flat	
F/B-quarter-tone-sharp	
F-quarter-tone-sharp	
G-quarter-tone-flat	

Here I shall apply the reasoning followed above to quarter-tonally analogous numbers. The 11:8 frequency ratio corresponds to the interval used cyclically to generate the quarter-tone scale, from which the diahemitonic scale is derived. As the eighth harmonic

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oscillates eight times as fast as the fundamental and the eleventh 11 times as fast, converting these harmonic oscillations into metres results in the eighth harmonic becoming an octuple metre and the eleventh an undecuple one. Accordingly, diahemitonic modal music must employ octuple and undecuple metres as respective analogues of the imperfect and perfect times. I neologise the proportion which enables the metric modulation from the one into the other without altering the total duration of the bar as the octansdesesquialtera[8] proportion.

The Subdivision of Beats

Beat subdivision is the operation by which the beat is split into equal parts and is qualified according to its arity. Diatonic modal music employs binary and ternary beat subdivisions, referred to as prolatio minor (Latin: 'minor prolation') and prolatio major (Latin: 'major prolation') respectively. This may be justified by the following, which will echo the foregoing section: the 3:2 frequency ratio corresponds to the interval used cyclically to generate the semitone scale, from which the diatonic scale is derived. As the second harmonic oscillates twice as fast as the fundamental and the third three times as fast, converting these harmonic oscillations into beat subdivisions results in the second harmonic becoming a binary beat subdivision and the third a ternary one. What consists in the permutation of a two-beat ternary rhythm into a three-beat binary one is referred to as hemiolia (ancient Greek: 'hemiola').

As in the earlier section, here I shall apply the reasoning previously followed to quarter-tonally analogous numbers. The 11:8 frequency ratio corresponds to the interval used cyclically to generate the quarter-tone scale, from which the diahemitonic scale is derived. As the eighth harmonic oscillates eight times as fast as the fundamental and the eleventh 11 times as fast, converting these harmonic oscillations into beat subdivisions results in the eighth harmonic becoming an octonary beat subdivision and the eleventh an undenary one. Accordingly, diahemitonic modal music must employ octonary and undenary beat subdivisions as respective analogues of the minor and major prolations. I term the permutation of an eight-beat undenary rhythm into an 11-beat octonary one triogdoola.[9]

The Rhythmic Mode and the Ordo

A rhythmic mode is a rhythmic pattern whose ordo is formed by its expression in a phrase. This ordo indicates the number of its consecutive statements before a rest. The rhythmic modes employed in diatonic modal music are in major prolation and are usually believed to have originated in the metres used in ancient Graeco-Latin poetry. However, I suggest an alternative derivation. The second and third harmonics are those at the basis of the diatonic system. If the harmonic series is studied as a rhythmic source, then their selection

[8] Coined from the Latin octansdesesquialter, meaning '(in the ratio) of (one) and three-eighths (elements) to the other' (from octans, 'eighth', + de, 'off', + semis, 'half', + -que, 'and', + alter, 'other'), upon the model of 'sesquialtera'.

[9] Coined from the ancient Greek triogdoolios, meaning '(in the ratio) of the whole and three-eighths (to the whole)' (from treis, 'three', + ogdoos, 'eighth', + holos, 'whole'), upon the model of 'hemiola'.

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within it will be equivalent to that of a duplet and a triplet respectively (see Figure 31).

Their superposition results in two symmetric ternary beat subdivisions (see Figure 32).

These ternary rhythms are the first two rhythmic modes employed in diatonic modal music (see Figure 33).

When expressed in phrases, they form ordines which indicate the number of their consecutive statements before a rest. Each ordo begins and ends with a note of the same value. A perfect ordo starts on the beat (see Table 4).

An imperfect ordo starts on a weak part of the beat (see Table 5).

The other ternary rhythmic modes consist of the alternation of a one-beat note value and one of the first two ternary rhythmic modes, the succession of one-beat note values and the fusion of the first two ternary rhythmic modes, resulting in a triplet. Those belonging to the ante- and penultimate categories of rhythmic modes are ultra mensuram (Latin: 'beyond measure') in that their duration exceeds one beat, causing a modification of the rule of the ordo: given the composite nature of the rhythmic modes belonging to the former, their perfect ordines begin and end on the strong beat with their first rhythmic cell and their imperfect ones on a weak beat with their second. Similarly, given the exclusive composition of one-beat note values of the rhythmic mode belonging to the latter, its perfect ordo begins and ends on the strong beat and its imperfect one on a weak beat.

Undenary rhythmic modes can be devised in the same way. The eighth and eleventh harmonics are those at the basis of the diahemitonic system. If the harmonic series is studied as a rhythmic source, then their selection within it will be equivalent to that of an octuplet and an undecuplet respectively (see Figure 34).

Their superposition results in two symmetric groups, each of four undenary beat subdivisions (see Figure 35).

Following the earlier examples, these undenary rhythms must be the first eight rhythmic modes employable in diahemitonic modal music (see Figure 36).

Table 6 shows the perfect ordines they form when expressed in phrases which start on the beat.

Table 7 shows the imperfect ordines they form when expressed in phrases which start on a weak part of the beat.

Analogously, the other undenary rhythmic modes consist of the alternation of a one-beat note value and one of the first eight undenary rhythmic modes, the succession of one-beat note values and the fusion of the first eight undenary rhythmic modes, resulting in an undecuplet. Those belonging to the ante- and penultimate categories of rhythmic modes are beyond measure in that their duration exceeds one beat, causing the same modification of the rule of the ordo which we saw earlier in the beyond-measure ternary rhythmic modes.

The Rhythm in Minor Prolation

While the rhythm in major prolation is either configured as a rhythmic mode or is exempt from modal configuration, that in minor prolation is never configured modally. The binary rhythm is usually understood as the product of splitting the beat into two halves. However, I suggest an alternative origin based upon the hemiola, the permutation of a two-beat ternary rhythm into a three-beat binary one. A two-beat ternary rhythm is in major prolation and, by

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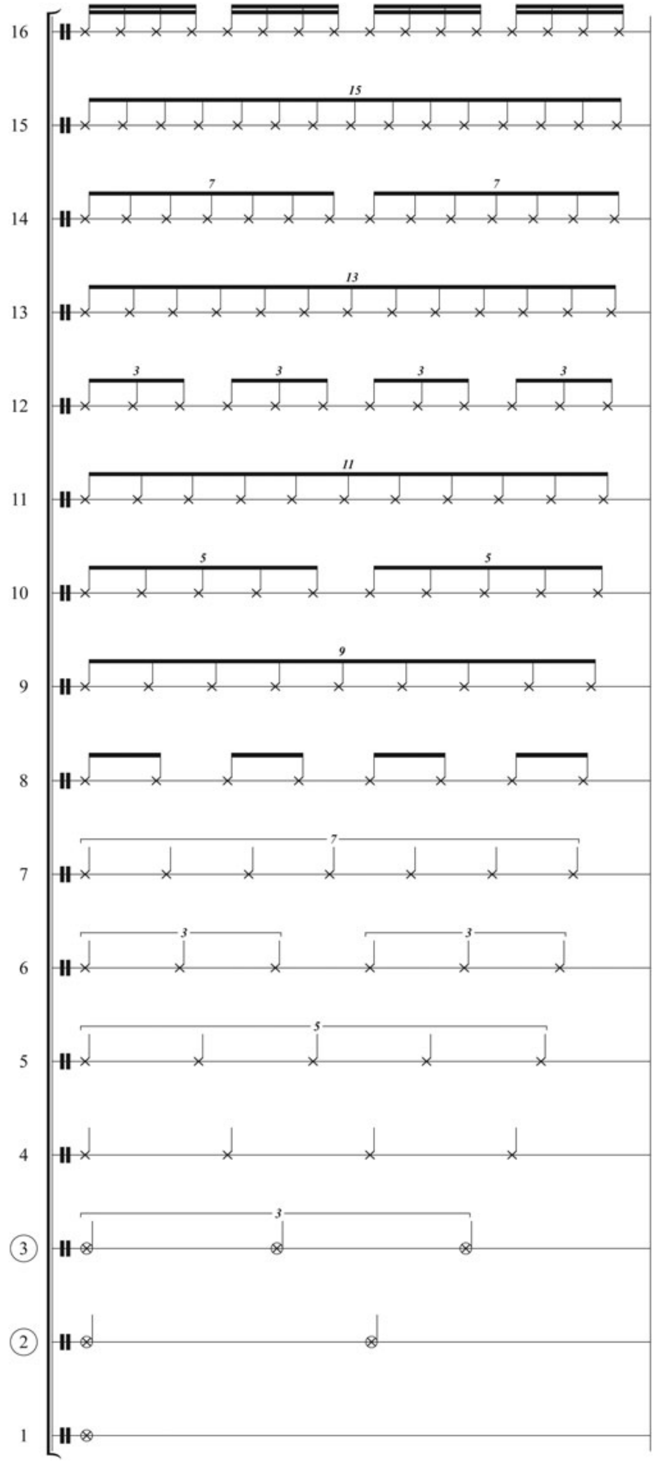

Figure 31:
Selection of the second and third harmonics within the rhythm-converted harmonic series.

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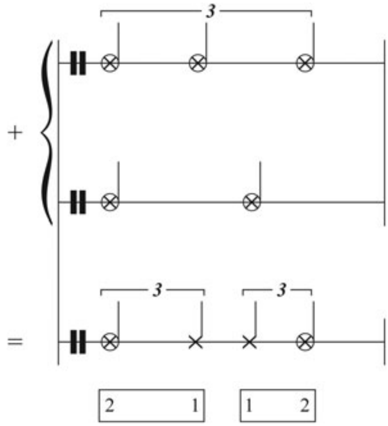


Figure 32:
Triplet against a duplet.

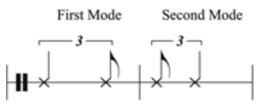


Figure 33:
First two ternary rhythmic modes.

Table 4:
Perfect ordines of ternary rhythmic modes

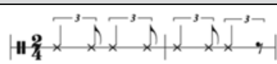
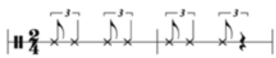
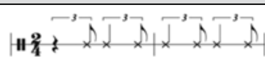
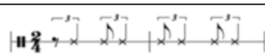
Ternary Rhythmic Mode	Perfect Ordo	
First	Third	
Second	Third	

Table 5:
Imperfect ordines of ternary rhythmic modes

Ternary Rhythmic Mode	Imperfect Ordo	
First	Third	
Second	Third	

permuting it into a three-beat binary rhythm, the hemiola makes it a rhythm in minor prolation. Since the juxtaposition of the first two ternary rhythmic modes results in a two-beat ternary rhythm, this rhythm can be read hemiolically (see Figure 37).

By reading it in this way, one obtains the binary rhythm (see Figure 38).

An octonary rhythm can be found in the same way by using the triogdoola, the permutation of an eight-beat undenary rhythm into an 11-beat octonary one. An eight-beat undenary rhythm is in major prolation and,

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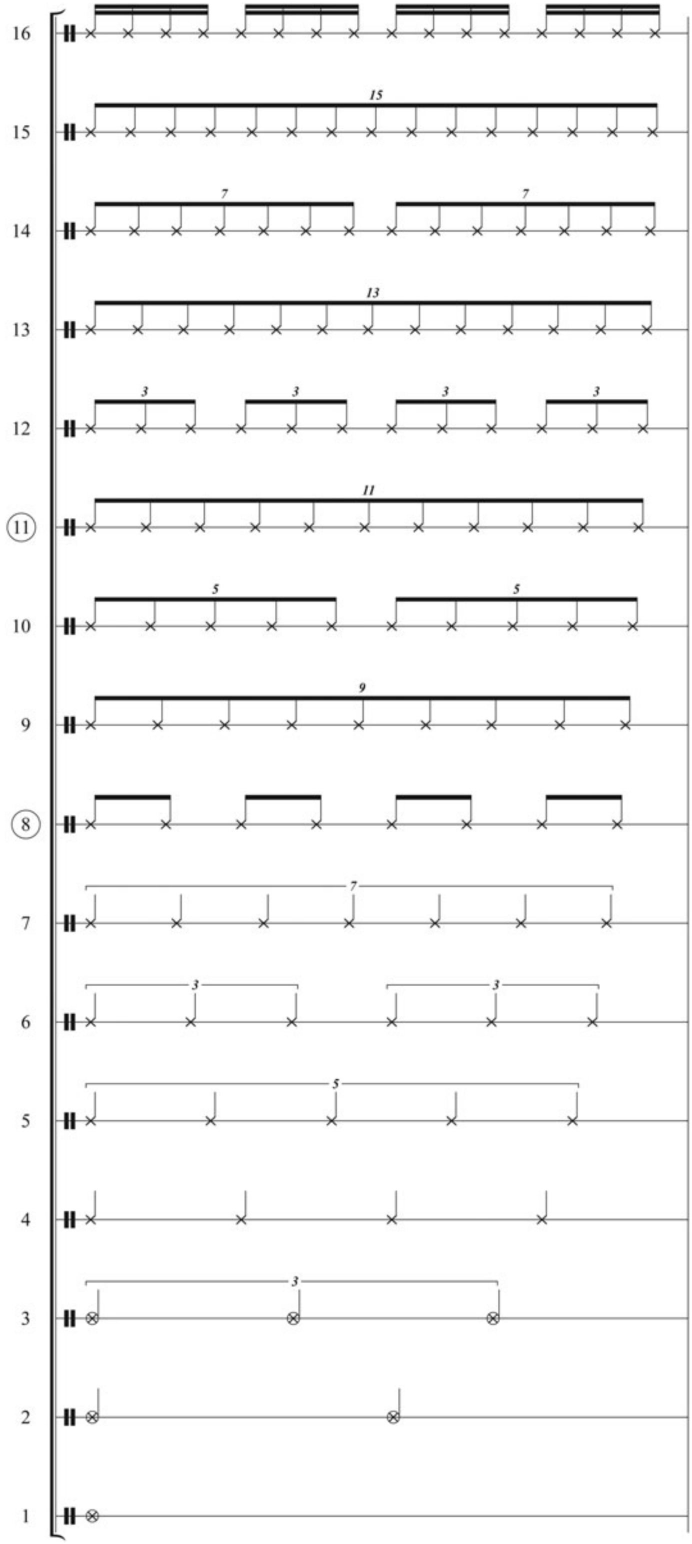

Figure 34:
Selection of the eighth and eleventh harmonics within the rhythm-converted harmonic series.

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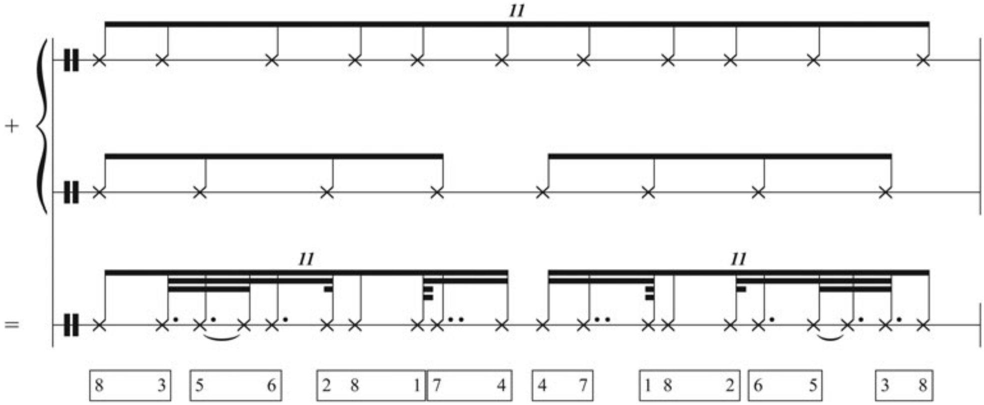

Figure 35:
Undecuplet against an octuplet.

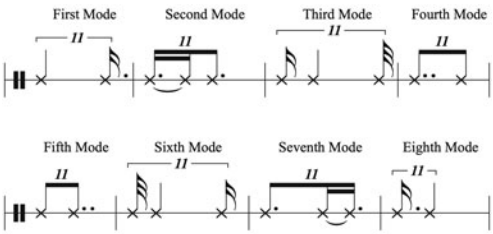

Figure 36:
First eight undenary rhythmic modes.

by permuting it into an 11-beat octonary rhythm, the triogdoola makes it a rhythm in minor prolation. Since the juxtaposition of the first eight undenary rhythmic modes results in an eight-beat undenary rhythm, this rhythm can be read triogdoolically (see Figure 39).

By reading it in this way, one obtains the octonary rhythm in its first seven kinds (see Figure 40).

The fusion of those whose smallest common value is equal to a quarter of a beat results in a quadruplet and that of those whose smallest common value is equal to an eighth of a beat in an octuplet. These two tuplets are added as the eighth and ninth kinds of the octonary rhythm.

The Appoggiatura

The appoggiatura is an ornamental note which delays the sounding of the main note and resolves onto it by conjunct motion. Although most embellishments are rhythmically unquantified, the appoggiatura has a fixed length. The diatonic appoggiatura conventionally has a length of one beat in duple metre, being half of the bar, leaving one beat to the main note (see Figure 41).[10]

This proportion corresponds to the octave ratio delimiting the frame of the semitone scale and mirrors that of the median pair of

[10] Carl Phillip Emmanuel Bach, *Essay on the True Art of Playing Keyboard Instruments*, tr. William J. Mitchell (New York: W. W. Norton, 1948), pp. 87–99; Johann Joachim Quantz, *On Playing the Flute*, tr. Edward R. Reilly, 2nd edition (London: Faber & Faber, 2001), pp. 91–100.

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Table 6:
Perfect ordines of undenary rhythmic modes

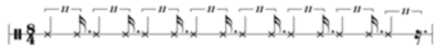
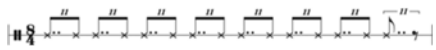
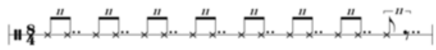
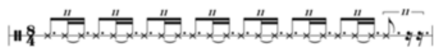
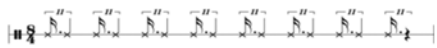
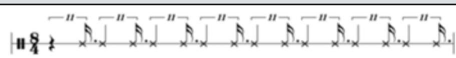
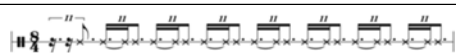
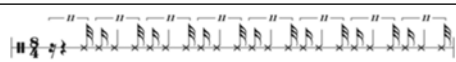
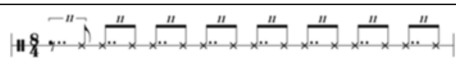
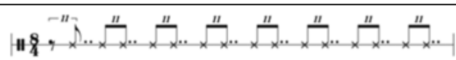
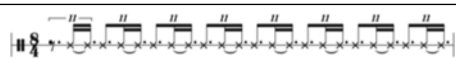
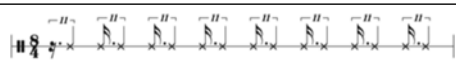
Undenary Rhythmic Mode	Perfect Ordo	
First	Seventh	
Second	Seventh	
Third	Seventh	
Fourth	Seventh	
Fifth	Seventh	
Sixth	Seventh	
Seventh	Seventh	
Eighth	Seventh	

Table 7:
Imperfect ordines of undenary rhythmic modes

Undenary Rhythmic Mode	Imperfect Ordo	
First	Seventh	
Second	Seventh	
Third	Seventh	
Fourth	Seventh	
Fifth	Seventh	
Sixth	Seventh	
Seventh	Seventh	
Eighth	Seventh	

note values of the hemiola (see Figure 37). The diatonic appoggiatura conventionally has a length of two beats in triple metre, being two thirds of the bar, leaving one beat to the main note (see Figure 42).[11]

[11] Bach, *Essay on the True Art*, pp. 87–99; Quantz, *On Playing the Flute*, pp. 91–100.

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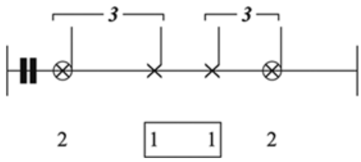

Figure 37:
Hemiola.

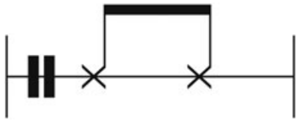

Figure 38:
Binary rhythm.

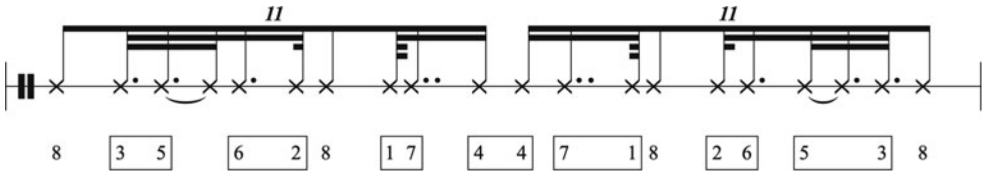

Figure 39:
Triogdoola.

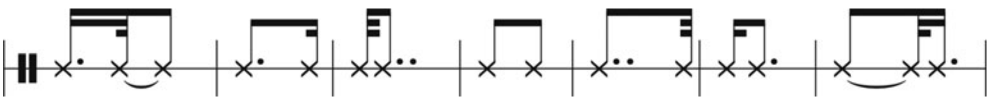

Figure 40:
First seven kinds of the octonary rhythm.

Figure 41:
Diatonic appoggiatura in duple metre.

This proportion corresponds to the perfect-fifth ratio at the origin of the semitone scale and mirrors that of the pair of note values of the first ternary rhythmic mode (see Figure 43).

Quarter-tonally analogous proportions can be derived from the median pair of note values of the triogdoola and the pair of note values of the first undenary rhythmic mode, thus establishing the length of the diahemitonic appoggiatura. The median pair of note values of the triogdoola is represented numerically as $4+4$, whose proportion corresponds to the octave ratio delimiting the frame of the quarter-tone scale (see Figure 44).

Thus, the diahemitonic appoggiatura must have a length of four beats in octuple metre, being half of the bar, leaving four beats to the main note (see Figure 45).

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Figure 42:
Diatonic appoggiatura in triple metre.

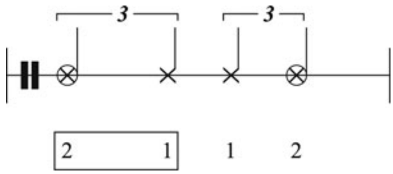

Figure 43:
Pair of note values of the first ternary rhythmic mode.

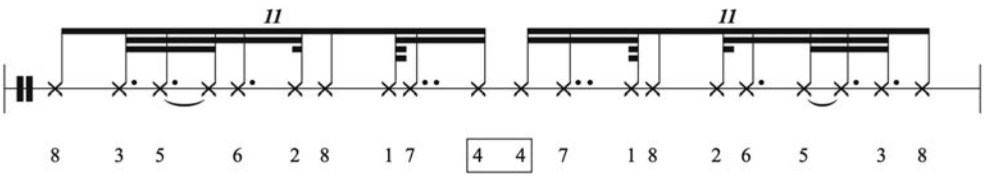

Figure 44:
Median pair of note values of the triogdoola.

Figure 45:
Diahemitonic appoggiatura in octuple metre.

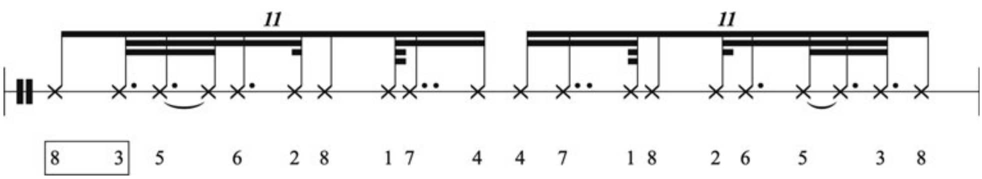

Figure 46:
Pair of note values of the first undenary rhythmic mode.

The pair of note values of the first undenary rhythmic mode is represented numerically as $8 + 3$, whose proportion corresponds to the major-fourth ratio at the origin of the quarter-tone scale (see Figure 46).

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Figure 47:
Diahemitonic appoggiatura in undecuple metre.

Thus, the diahemitonic appoggiatura must have a length of eight beats in undecuple metre, being eight elevenths of the bar, leaving three beats to the main note (see Figure 47).

Conclusion

In this article, I have analysed the attributes of diatonic modal music before mapping them onto quarter-tonality, thereby giving rise to diahemitonic modality, a quarter-tonal composition system which takes account of scale, mode, chord, cadence, metre and rhythm. I have demonstrated that each quality of diatonic modality finds its diahemitonic analogue, suggesting that diahemitonicism is neither more nor less than the natural evolution of diatonicism. This not only signifies an advance in quarter-tonal composition but also allows for a deeper comprehension of semitonal music and illustrates a method of microtonal composition system design which may, I hope, inspire future research. I believe I have created a new and coherent acoustic universe whose laws will guide anyone who chooses to explore it.

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