ACCUMULATION OF CHONDRULES ON ASTEROIDS

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It is suggested that aerodynamic forces played a significant role in the selective accumulation of chondrules on asteroids moving with respect to the gas in a primeval solar nebula. Particles smaller than millimeter chondrules would sweep by an asteroid moving in a critical velocity range, whereas larger particles could be accumulated by impact. Theory and calculation cover the case of subsonic velocity and asteroidal diameter up to 50 km for a nebula density up to 10^{-6} *g/cm*^{3}*, or higher for smaller asteroids.*

Chondrules, roughly millimeter spherules found abundantly in many meteorites, have long been aptly described in Eucken's (1944) terms as products of a "fiery rain" in a primeval solar system nebula. Chondrules are clearly mineral droplets that have cooled rapidly, some showing evidence of supercooling. On the basis of the quantitative loss of volatile elements, Larimer and Anders (1967) deduced that chondrules were formed in an ambient temperature of some 550 K. Because melting temperatures are roughly 1300 K greater, some violent heating mechanism must have been involved. Noteworthy is a suggestion by Wood (1963) that the quick heating was produced by shock waves in a primitive solar nebula. Volcanic and impact processes have been suggested, as has the pinch effect in lightning (Whipple, 1966).

Whatever the source of droplet formation, a major evolutionary problem concerns the high abundance of chondrules among several classes of meteorites; in some the percentage of chondrules exceeds 70 percent by mass. Accepting the concept that meteorites are broken fragments of asteroids that were originally accumulated from solids in a gaseous solar nebula, one's credulity is taxed by the added assumption that a substantial fraction of the solid material should have been in the form of spherules. Thus, the purpose of this paper is to explore the possibility that chondrules may have been selectively accumulated on some asteroidal bodies, thereby eliminating the undesirable supposition that chondrules constituted a major fraction of the dispersed solids in any part of the nebula.

Almost axiomatic is the assumption that the accumulation process for smaller asteroids essentially ceased when the solar nebula was removed,

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presumably by the effect of the solar wind from the newly formed Sun in its brilliant Hayashi phase (Hayashi et al., 1960). Possibly the largest asteroids can still continue to grow in vacuum conditions, but the relative velocities of particle impact on asteroids less than perhaps a hundred kilometers in dimension would be generally dissipative rather than accumulative because of the low velocities of escape against gravity.

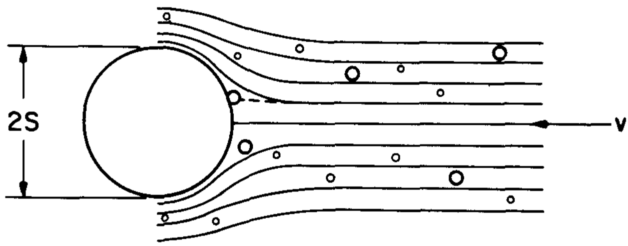

Figure 1.–Flow pattern in which larger particles may strike the moving body. S = radius of sphere.

While the solar nebula was present, however, small bodies moving through the gas would have exhibited aerodynamic characteristics. At a given body velocity and gas density, solid particles having a mass-to-area ratio below a certain value would be carried around the body by the inertia and viscosity of the gas currents so as not to impinge or accumulate on the moving body (fig. 1). The physical conditions for certain such accumulation processes will be established in the following sections of this paper.

IMPACT OF SMALL PARTICLES ON A SPHERE MOVING THROUGH A GAS

Taylor (1940) dealt with this basic problem for a cylinder. Langmuir and Blodgett (1945) derived numerical results by theory and calculation for cylinders, wedges, and spheres moving through air containing water droplets or icy spheres. Fuchs (1964) and Soo (1967) summarized the subject for subsonic flow and included both theoretical and experimental results by various investigators. Probstein and Fassio (1969) investigated "dusty hypersonic flows." The transonic case has apparently not been attacked seriously. The following discussion is based on the presentations by Langmuir and Blodgett augmented by the summaries by Fuchs and Soo.

A sphere of radius S is assumed to move at velocity v through a gas of density ρ and viscosity η containing in suspension small spheres of radius s and density ρ_s. The gas viscosity is given by the classical approximation

$$\eta = \frac{1}{2} vL\rho \tag{1}$$

where L is the mean free path of the atoms or molecules, assumed to be neutral.

Because the flow about the forward surface of the moving body is relatively streamlined at rather high values of the Reynolds number, the reference Reynolds number R_e is calculated for the small particles and given by the expression

$$R_e = \frac{2s\rho v}{\eta} \tag{2}$$

The applicable Reynolds number is reduced greatly from this value because the small particles will not be thrown violently into the full velocity of the gasflow v, except perhaps near the stagnation point. As will be seen, the relatively small value of the applicable Reynolds number permits the application of the simple Stokes' law of particle drag at values of v far above those for which the law might intuitively appear to be valid. Because the Stokes force F on a sphere of radius s moving at a velocity v_s through a gas

$$F = 6\pi\eta s v_s \tag{3}$$

is independent of the gas density when $L < s$, the impact equation for particles impinging on the larger sphere of radius S is widely applicable in a solar nebula where the density cannot be accurately specified. The Stokes approximation begins to fail significantly for $R_e > 10$, but the drag force is overestimated only by about a factor of 3 at $R_e = 10^2$. (See Probstein and Fassio, 1969.) An inertia parameter ψ is defined as

$$\psi = \frac{s^2 \rho_s v}{9\eta S} \tag{4}$$

which is the ratio of the inertia force to the viscous force for small particles in the stream. Theory and experiment show (fig. 2) that particles of radius $< s$ do not impact the sphere for $\sqrt{\psi} < 0.2$ for potential flow and 0.8 for viscous flow.

At higher values of R_e, when Stokes' law deteriorates, the behavior of the impact changes as a function of another parameter ϕ defined by

$$\phi = \frac{R_e{}^2}{2\psi} = \frac{18\rho^2 v S}{\rho_s \eta} \tag{5}$$

Figure 2 illustrates the changes in impact efficiency for values of ϕ up to $\phi = 10^4$. Note that the limiting value of $\sqrt{\psi}$, initiating the impacts, is nearly independent of ϕ, whereas the efficiency of impact is not greatly dependent on ϕ. Hence limiting conditions for impaction of particles of radius s, on a sphere of radius S, up to R_e somewhat less than 10^2, can be confidently given by equation (4) when $\psi_{\text{limit}} \sim 0.04$.

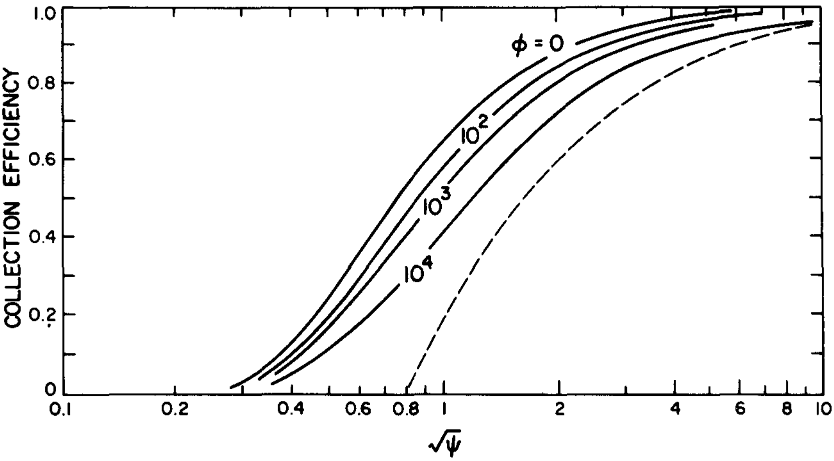

Figure 2.–Collection efficiency versus inertia parameter ψ as function of parameter ϕ. Dotted curve: viscous flow.

Let us then assume that chondrules have a radius of 0.05 cm and decide that particles of 1/3 this radius (diameter 1/30 cm) should not impact our asteroid of radius S moving at velocity v_l through the primeval nebula of viscosity η. Then $s = 1/60$ cm, $\sqrt{\psi} = 0.2$, and

$$v_l = \frac{0.36\eta}{s^2\rho_s} S = 432\eta S = 0.069S \tag{6}$$

if we take ρ_s as 3 g/cm^3; employ cgs units; and adopt a "solar mix" of primeval gas with mass distribution $X_{\mathrm{H}} = 74$ percent, $Y_{\mathrm{He}} = 24$ percent, and $Z_{\mathrm{other}} = 2$ percent, hydrogen being in the form of neutral molecules at a temperature of 550 K. The viscosity becomes approximately

$$\eta = 1.6 \times 10^4 \text{ dyne-s-cm}^{-2}$$

Numerical values for equation (6) are given in table I.

TABLE I.–*Numerical Values for Equation* (6)

Diameter asteroid, $2S$, km	Limiting velocity v_l, km/s	R_e
0.1	0.0034	0.04
1.0	.034	.4
10.0	.34	4.0
100.0	3.4?	40.0

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DISCUSSION OF RESULTS

Before drawing conclusions from the first two columns of table I, we must check to see that the Reynolds numbers involved are not too high for the Stokes extrapolation to be valid; i.e., $R_e < 10^2$. This check involves an assumption as to the density near the plane of the solar nebula in the asteroid belt, say at 2.5 AU. Few theorists place the gas pressure here much greater than 1 kN/m^2 (10^{-2} atm). (See, for example, Cameron, 1962.) For a central mass equal to the Sun and allowing for the gravitational attraction of the gas itself, we find the corresponding density, $\rho \sim 5 \times 10^{-7}$ g/cm^3, and the surface density integrated perpendicular to the plane throughout the nebula some 3×10^5 g/cm^2, or about 1/30 solar mass per square astronomical unit. The Reynolds number (eq. (2)) is then given at $s = 1/60$ cm by $R_e = 11 \times 10^{-5}\, v$ cm/s, values of which are tabulated in the third column of table I, safely within our limits for asteroids up to 100 km in diameter. The limiting velocity, however, becomes supersonic at $v_l \sim 1.8$ km/s. Hence the condition of subsonic velocity limits our present conclusions to asteroids less than about 50 km in diameter.

The condition of molecular mean free path not exceeding the limiting dimensions of the chondrules restricts the theory to $\rho > 10^{-8}$ g/cm^3, a somewhat higher density than is sometimes assumed for the solar nebula in the asteroid belt. It is evident that a more complete theory is needed to cover the case of low densities in the solar nebula and that the transonic case should be developed before the present suggestion for the selective accumulation of chondrules by asteroids can be wholeheartedly accepted. The latter situation probably requires numerical analysis.

The case of lower density can be roughly approximated by means of Epstein's law of drag (see, e.g., Kennard, 1938), which, for mean free paths that are large compared with the dimension of the body, gives a drag force roughly $\rho/3\rho_c$ that of Stokes' law, where ρ_c is the critical gas density at which the mean free path of the molecules equals the dimension of the body. Because ψ varies inversely as the drag force, the critical value of ψ for accumulation in equation (4) will also vary approximately as $\rho/3\rho_c$ for relatively low gas densities. Hence the limiting velocity in equation (6) and in table I can be corrected as to order of magnitude by a factor of $\rho/3\rho_c$, or about $\rho \times 10^7$ g-cm^{-3}, for $\rho < 10^{-8}$ g-cm^{-3}.

The simple solution involving Stokes' law covers a considerable range of possible physical conditions and fairly reasonable ranges for planetoid velocities. In equation (6) the square of the radius of the limiting particle size varies inversely as the velocity and directly as the radius of the asteroid or planetoid. This indicates that the process of selective accumulation is fairly sharply defined in particle size when measured by velocity or size of the planetoid. If the process of chondrule formation (e.g., by lightning) is inherently limited for large dimensions, but not at small dimensions, the aerodynamic selection factor could frequently produce a fairly narrow range in chondrule dimensions. Furthermore, extremely small chondrules could easily lose their identity in some meteorites by chemical differentiation during subsequent heating of the asteroid. Together, upper limits to dimensions in chondrule formation, a selective accumulation process, and perhaps some partial differentiation seem capable of relieving the theorist from the undesirable postulate that chondrules once constituted a sizable fraction of the mineral content in any part of the solar nebula.

Note that aerodynamic forces will prevent the accumulation of finely divided minerals on planetoids in motion with respect to the gaseous medium, thus greatly reducing the accumulation rates calculated on the basis of simple cross-sectional areas and velocities.

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