THE EFFECTS OF ROTATION DURING STAR FORMATION

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ABSTRACT

Observations of molecular clouds show evidence of rotation and of fragmentation of subregions of the clouds into multiple stellar or protostellar systems. This review concentrates on the effects that rotation and pressure gradients have in a self-gravitating cloud to cause it to undergo the crucial process of fragmentation. Recent two-dimensional and three-dimensional numerical hydrodynamic calculations have made progress in determining these effects. In most cases the calculations are performed with modest spatial resolution and are limited to isothermal clouds with neglect of viscous and magnetic effects. The combined results of several calculations strongly suggest that rotating clouds that are unstable to collapse are also unstable to fragmentation.

1. INTRODUCTION

Among the fundamental problems that must be addressed by theoreticians who study star formation, the following are now being actively pursued:

1. What are the dominant physical processes that must be considered at each stage of star formation?

2. How does one predict the rate of star formation both in our galaxy and in external galaxies, and how does this rate vary with position in the galaxy and with time since the formation of the galaxy? Closely connected with this question is that of the efficiency of star formation: of the total mass of interstellar material available in a gravitationally bound cloud in the galaxy, what fraction actually is formed into stars during the lifetime of the cloud?

3. What is the mass spectrum of the stars that are formed and what are the maximum and minimum masses of stars? How do these quantities vary according to position in the galaxy and with time since the formation of the galaxy?

4. How are binary and multiple systems of stars formed? The

D. Sugimoto, D. Q. Lamb, and D. N. Schramm (eds.), Fundamental Problems in the Theory of Stellar Evolution, 5–26.

https://doi.org/10.1017/S0074180900073599 Published online by Cambridge University Press

important processes that must be considered include a) encounters and captures in a cluster or association of stars and protostars, b) fragmentation of a rotating, collapsing cloud into two or more protostellar objects during the earliest stages of stellar evolution, and c) fission of a rotating object in hydrostatic equilibrium during its pre-main-sequence contraction phase. What is the relative importance of each process and how are they interrelated?

5. What processes determine whether the end product of star formation is a single star, a planetary system, a binary system, or a multiple system? What is the probability that a planetary system will be formed?

Clearly a complicated network of physical processes must be considered in an attempt to answer questions 2 through 5. Although it is clear that certain subproblems related to star formation can be approached with analytical methods, the overall problem must be attacked with large-scale numerical computations involving a 3-D spatial grid. The physical processes that must be considered include at least the following: self-gravity, gas pressure, magnetic fields, rotation, radiative transfer, turbulence and convection, formation and dissociation of grains and molecules, and molecular and grain chemistry. If star formation is taken to start in a gravitationally bound cloud, the solution will depend on various parameters needed to define this initial state, which depends on the processes by which the dense molecular cloud complexes were formed in the interstellar gas. Finally, the solution to the star formation problem must involve observational predictions. For example, the emergent infrared spectrum of a collapsing protostar can be calculated, or the profiles of the lines emitted by the CO molecule can be calculated from the models and compared to the radio observations. A considerable amount of such work has been done in the spherically symmetric case, but studies of rotating objects are just beginning (see the review by Bertout and Yorke 1978).

The effects of rotation clearly can not be studied separately from the other physical processes that are important in star formation. However, the overall problem is so complicated that no attempt has been made to solve it in full generality; rather the interactions of a few physical effects (e.g., gravity, rotation, and pressure, or rotation and magnetic fields) have been studied under restrictive assumptions and idealized initial conditions. Those recent studies, primarily numerical in nature, that emphasize the effects of rotation will be discussed in this paper. Clearly, angular momentum must be one of the dominant effects in the solution of questions 4 and 5 above, and it undoubtedly has significant indirect, and largely unexplored, effects on the solution of questions 2 and 3.

The evolution of a star, up to the time where nuclear reactions become significant, can be divided into three periods. The first, star formation, refers to the approximate density range 10^{-23} - 10^{-19} g cm^{-3} and concerns the processes by which a massive interstellar cloud collapses and at the same time fragments into gravitationally bound pieces of order 1 solar mass. The second period, protostellar evolution, is assumed to begin at a density corresponding to the Jeans limit for a

https://doi.org/10.1017/S0074180900073599 Published online by Cambridge University Press

fragment of a given mass at a temperature of 10K. Densities increase from about 10^{-19} g cm^{-3} to 10^{-2} g cm^{-3} or more for fragments of 1 solar mass, and final temperatures are ~10^6K in the interior. This extreme compression, resulting from gravitational collapse, is due to two main mechanisms. The first applies to the star formation period and to the earlier part of the protostellar period, up to densities of about 10^{-13} g cm^{-3}. The protostellar material is optically thin to infrared radiation, and as collapse proceeds the heat generated by compression is immediately radiated by ions, atoms, molecules, or grains. Thus thermal pressure is unable to halt gravitational collapse unless rotation or magnetic fields begin to play an important role. The second mechanism occurs during the later part of protostellar collapse, starting at temperatures above 1800K and densities above 10^{-8} g cm^{-3} where dissociation of molecular hydrogen results in $\Gamma_1 < 4/3$ and consequent gravitational instability (in the absence of rotation). When collapse stops in the entire mass of the protostar, the third period of evolution begins, the pre-main-sequence contraction through a sequence of quasi-equilibrium states, continuing until nuclear-burning temperatures (~10^7K) are reached in the center. The evolution through periods two and three has been calculated in a continuous fashion only for the spherically symmetric case. However, when rotation is included, the protostar must evolve through analogous periods. Many of the recent calculations involving rotation have applied to the star formation period and the early part of protostar collapse, where an isothermal collapse at approximately 10K is a reasonable assumption. This review concentrates on these results. Note, however, that there have also been significant advances regarding the effects of rotation during the pre-main-sequence contraction phase by Lucy (1977), Gingold and Monaghan (1978, 1979), and Durisen and Tohline (1980). It has also been suggested (Larson 1980) that rotation plays a significant role in the explanation of the FU Orionis phenomenon.

2. OBSERVATIONAL DATA

The time scales for the three periods of evolution just referred to are approximately 10^7 years for star formation, 10^6 years for protostellar collapse, and 4 x 10^7 years for the quasi-static contraction of 1 solar mass. Consequently, there is abundant observational material in the visual and near infrared regions of the spectrum appropriate for study of the third period, and also a considerable amount of radio data appropriate for study of the star formation period. In the intermediate period, opportunities for comparison of theory and observation are very limited, due to the short time scale, expected heavy obscuration of protostars, and difficulties of observation in the relevant (far IR) spectral region.

From the observational material available regarding rotation it is clear that (1) angular momentum must play a significant role in star formation, and (2) there is a reduction of many orders of magnitude in the specific angular momentum of spin of particular mass elements between the molecular cloud phase (period 1) and the T Tauri

https://doi.org/10.1017/S0074180900073599 Published online by Cambridge University Press

phase (period 3). Typical values of specific angular momenta J/M for T Tauri stars are $10^{17} cm^2 s^{-1}$ to at most $10^{18} cm^2 s^{-1}$ (Herbig 1957, Kuhi 1978) if the stars are assumed to be uniformly rotating. On the other hand, a typical diffuse interstellar cloud would be expected to have J/M in the range 10^{23} to 10^{24} if it is corotating with its orbital motion about the center of the galaxy. Although there is little direct observational evidence of rotation in H I clouds (but note Gordon 1970) it is generally argued that the galactic magnetic field will be effective in maintaining corotation.

Much more observational evidence on rotation is available for massive dark clouds and globules which have densities of 10^{-20} - 10^{-22} g cm^{-3} and presumably correspond to the star formation period. Indirect evidence suggests that the so-called Hopper-Disney clouds, which are elongated dark clouds aligned with the galactic plane, are rotating as proposed by Heiles (1976) and further analyzed by Field (1978). More direct observational evidence for rotation in a few analogous objects is provided by velocity gradients in the microwave molecular emission lines of ^{13}CO. Milman's (1977) observation of the globule B361 gives an approximate angular velocity $\Omega = 10^{-13} s^{-1}$. Martin and Barrett (1978) have observed the two Bok globules B163 and B163 SW and find spins of $\Omega = 6 \times 10^{-14} s^{-1}$ and $1.0 \times 10^{-13} s^{-1}$, respectively. The two objects are suspected to be in orbital motion with the angular momenta of spin aligned with that of the orbit. Other globules observed by Martin and Barrett, for example B335, have no detectable rotation. Large dark clouds, such as Mon R2 with an estimated mass of 10^4 solar masses, have also been observed to rotate; ^{13}CO observations give $\Omega = 1.4 \times 10^{-14} s^{-1}$ (Loren 1977) while CS observations in the same cloud give $\Omega \geq 7.4 \times 10^{-14} s^{-1}$ (Kutner and Tucker 1975). Further examples, with similar values of Ω, are given by Field (1978) and Snell (1979). The specific angular momenta of these objects are in the range 10^{22} to $10^{23} cm^2 s^{-1}$ although there are also objects with lower values. Thus, apparently for many clouds where star formation is taking place, reduction in J/M by up to 6 orders of magnitude must take place by the time the T Tauri phase is reached. One of the chief aims of the numerical calculations to be discussed below is the resolution of this problem. In this connection there is additional observational evidence of interest concerning the question of fragmentation. Apart from the obvious fact that many young stars are found in clusters and associations, other observations suggest that fragmentation is a dominant process in star formation: (1) A considerable number of present main sequence stars are in multiple systems with two or more periods represented. Abt and Levy (1976) show that the typical main sequence star of spectral type F3-G2 has <u>both</u> a close and a distant companion. An example of such a system is κ Peg (ADS 15821), a visual binary with a period of 11.52 years each of whose components is a spectroscopic binary having periods of 4.77 days and 5.97 days, respectively (Beardsley and King 1976). A number of other systems with multiple periods in the same range have been observed, and the phenomenon is suggestive of a multiple fragmentation process during star formation. (2) Multiple infrared sources have recently been discovered in the cores of dense molecular clouds, where star formation is suspected to be taking place. For example, Beichman, Becklin and Wynn-Williams (1979) cite numerous observations that support the suggestion that infrared sources in such clouds commonly form in groups of two or more with a characteristic separation of 0.1 parsec. (3) The double Bok globule B163, B163 SW (Martin and Barrett 1978) mentioned above is suggested to be an example of a rotating cloud that has fragmented into orbiting subcondensations. Other molecular line observations also support the picture of fragmentation associated with rotation (Ho and Barrett 1980, Crutcher et al. 1978).

3. TWO-DIMENSIONAL NUMERICAL CALCULATIONS OF COLLAPSING CLOUDS

Three major suggestions regarding the solution of the angular momentum problem during star formation have come forward. (1) Stars form only from interstellar material that has much less angular momentum than the average. Although this effect may be significant, it has not been shown observationally that there is sufficient such material to account for the observed rate of star formation. Even material that is not rotating to present observational limits could have very significant J/M. (2) Magnetic fields result in braking of rotation through transfer of angular momentum from the cloud to the surrounding medium as a consequence of the propagation of Alfvén waves along the twisted magnetic field lines (Mestel and Spitzer 1956, Lüst and Schlüter 1955). Although it is clear, for example from the work of Mouschovias (1980), that this effect can reduce J/M by at least 2 or 3 orders of magnitude, eventually the braking becomes ineffective since the density in the cloud increases to the point where the degree of ionization is negligible and the field decouples from the matter. In fact, one would not expect magnetic braking to account for 100% of the angular momentum reduction, since then we could not account for the angular momentum of short and moderate-period binary systems that are unlikely to have formed by capture. (3) Angular momentum is converted from the spin of a collapsing cloud into orbital motion of binary or multiple systems that form as a result of fragmentation. Since the orbital J/M in binaries ranges from 4×10^{18} cm^2 s^{-1} (3-day period) to 10^{21} cm^2 s^{-1} (10^4 year period), the formation of binaries, perhaps through multiple fragmentation stages, naturally fills in the angular momentum gap between the rotating dark clouds and the T Tauri stars. The observations of rotation and fragmentation mentioned above support this suggestion, and many of the recent 2- and 3-D calculations of rotating clouds have been directed toward study of the process.

We first summarize recent calculations of collapsing rotating clouds in two space dimensions. The standard assumptions employed are : (1) axial symmetry; (2) global and local conservation of angular momentum, that is, no physical effects such as viscosity or magnetic fields transport angular momentum; (3) an ideal gas composed primarily of molecular hydrogen; (4) isothermal collapse, with certain exceptions noted below; (5) no mass flow in either direction through the outer boundary which is generally fixed in space. As long as the collapse remains isothermal the results may be scaled to any desired value of mass M, temperature T and molecular weight μ; thus the only parameters of the calculation are the two basic dimensionless quantities α, the initial ratio of thermal energy to gravitational energy, and β, the initial ratio of rotational energy to gravitational energy. Collapse occurs if $\alpha \leq 1. - 1.43\ \beta$ (Black and Bodenheimer 1976); this condition is essentially the Jeans criterion for a rotating cloud. The initial distributions of density and specific angular momentum are also parameters, although most calculations assume uniform density and angular velocity as initial conditions. The equations solved are the standard hydrodynamic equations of continuity, motion, and (where necessary) energy, along with the Poisson equation for the gravitational potential and an equation of state. Thus the physical effects included are self-gravity, gas pressure, and rotation. Radiation transport is included in some of the non-isothermal calculations.

The first question we may ask is whether there are any comparisons between 2-D calculations and observations. The principal study now available is a comparison with a set of six Bok globules, including the rotating ones B163, B163 SW, and B361 (Villere and Black 1980). The collapse models were generated from the code of Black and Bodenheimer (1975) while the observed parameters that were fit included the ^{13}CO column density at the center of the cloud, the ^{13}CO core radius, the axis ratio in the core, the ratio of the ^{13}CO core radius to the optical radius, the width of the ^{13}CO line profile, and the rotational velocity. In five of the six cases all of these parameters were consistent with collapsing cloud models. The derived masses are ~100 solar masses and the derived ages since the beginning of collapse are 50 to 90% of the initial free fall time. By this time the collapse has resulted in a centrally condensed density distribution but not yet in extreme rotational distortion or in ring structures. Predicted central densities of 10^4 to 2×10^5 H_2 molecules/cm^3 are also consistent with observations. Another important result is that the inferred initial density at the onset of collapse is much larger than that required for gravitational collapse ($\alpha \approx 0.1$). One globule (B361) is not fit by the models chiefly because of a large observed line width. The ratios of the abundances of ^{13}CO to H_2 in each globule are also determined by the model fit, and there is a clear trend of decreasing $^{13}CO/H_2$ with increasing density in the center of the cloud, in agreement with entirely independent measurements of this quantity in molecular clouds (Wootten et al. 1978). It was later found that all six globules could be fit if the ratio of $^{13}CO/H_2$ was allowed to vary spatially within the models as determined from the observations. Further calculations have also provided fits to six Lynds clouds, three of which are rotating.

A second important question concerns the degree to which the various 2-D calculations agree with each other. A detailed comparison has been carried out for the same initial and boundary conditions by Bodenheimer and Tscharnuter (1979). The former used an explicit "fluid-in cell" method involving differencing in two space dimensions and solution on a moving Eulerian grid; the latter used an implicit code involving differencing in the (spherical) radial direction only and a Legendre-polynomial expansion in the second (Θ) dimension. The case chosen had $\alpha = .46$, $\beta = .32$. Initially, Tscharnuter's code collapsed to a considerably higher central density than did Bodenheimer's. However, this discrepancy did not affect the later evolution since only a very small amount of mass was involved. In both cases a bounce occurred at the center, and after about 5 initial free-fall times, both calculations settled down to a near-equilibrium configuration that included a mild ring feature whose maximum density was about 4 times the central density. The isothermal equilibrium configuration that was obtained was quite flattened, with a polar-to-equatorial axis ratio of 1:7 and with a final β-value of 0.25. It had been previously suggested (Biermann and Michel 1978) that such nebulae, having about 2 solar masses and radii of about 10^4 AU, would provide a suitable location for the formation of cometary nuclei, provided that the nebula were stable for a long enough time to allow the dust grains to settle into a thin layer at the equatorial plane.

A second comparison calculation was performed by Boss (1980a) who used an explicit "fluid-in-cell" code which differed in several respects from the code of Black and Bodenheimer (1975) and also employed fewer grid points (220 versus 1600). A repeat of the Bodenheimer-Tscharnuter comparison resulted in a maximum density intermediate between those obtained by Bodenheimer and Tscharnuter. The long-term equilibrium structure was also quite similar and included a ring structure with moderate density contrast. However, Boss' model showed oscillations with shorter period and higher amplitude than those of Bodenheimer and Tscharnuter. Boss ran a second comparison case with $\alpha = .55$, $\beta = .02$, the same conditions used in one of the runs performed by Black and Bodenheimer (1976). Here the cloud is definitely gravitationally unstable, no equilibrium is possible, and after collapse and flattening a pronounced, self-gravitating ring structure develops in the central region of the cloud in both calculations. At this time the distributions of angular velocity, infall velocity, and density with distance from the center calculated by Boss and by Black and Bodenheimer are in excellent agreement. In summary, the available comparisons of 2-D hydrodynamic codes show overall fair agreement.

A third question that must be addressed is the physical mechanism for the origin of ring structures and the reality of their existence. This matter is one of considerable importance since such structures turn out to be sensitive checks on the accuracy of computer codes; furthermore, they have been shown to be unstable to fragmentation when three space dimensions are considered. Of the 2-D isothermal collapse calculations, those of Larson (1972), Black and Bodenheimer (1976), Nakazawa, Hayashi, and Takahara (1976), Regev (1979), Bodenheimer and Tscharnuter (1979), and Boss (1980a) have produced rings. A wide variety of numerical techniques is represented in these calculations. On the other hand, earlier calculations by Tscharnuter (1975) as well as those of Kamiya (1977) and Norman, Wilson and Barton (1980) do not show ring formation but rather a flattened disk-like structure in the interior of the cloud. When rings are found, they approach and later pass through a stage of hydrostatic equilibrium, at which time their properties agree well with those of equilibrium isothermal rings calculated analytically by Ostriker (1964). The controversial question is, however, the mechanism for excitation of the ring mode.

It would, of course, be desirable to find an analytic argument, entirely independent of the numerical codes, that would demonstrate the reality or non-reality of the ring structure. This approach has been explored by Tohline (1980a) and extended by Boss (1980a). An analytic approach becomes possible if the collapse is assumed to be pressure-free and the gravitational potential is fixed in time. In fact, the pressure does not play a dominant role in isothermal collapse, and the interplay between gravity and centrifugal effects is primarily responsible for the flow of material perpendicular to the rotation axis. Once rotational effects have produced a highly flattened structure, pressure effects do halt the collapse flow along the rotation axis. Tohline's approach was to integrate analytically the orbits of non-interacting particles. If the initial density distribution is uniform, and if the background static gravitational potential corresponds to this distribution, the particles all orbit in the potential well with the same period, maintaining a uniform but changing density. However, a typical collapsing cloud, due to the propagation of a rarefaction wave from the surface, always develops a non-uniform density distribution. Tohline also obtained an analytic expression for the particle orbits in the equatorial plane of a cloud collapsing in the potential of a $\rho \propto (1-r^2)$ distribution. In this case, the particles in the inner part of the cloud have shorter periods than those farther out; the inner ones reach their minimum radius sooner, then they begin to move with outward velocities, plowing into particles still falling in from larger radii. In this manner a density wave is excited that is ring-like in nature and that propagates outward. Although the ring self-gravity is not included in the analytic treatment, Tohline was able to show that enough mass is contained in the off-axis density enhancement to result in a self-gravitating ring; that is, the potential minimum would move into the ring.

Tohline supplemented his analytic calculations by solving the pressure-free collapse problem with a 2-D particle code. The gravitational potential was taken to be time-varying in a manner that closely approximated one of the collapses calculated by Black and Bodenheimer (1976). Again, the results show that ring formation occurred. The analytic approach was then extended by Boss (1980a) who obtained the particle orbits, not limited to the equatorial plane, of a rotating cloud with a static potential corresponding to $\rho \propto 1/r$. The development of phase differences and the resulting ring-like density wave that propagates outward is confirmed.

With a rather strong physical argument as well as diverse analytical and numerical calculations now supporting the existence of a ring in 2-D collapse calculations, what can be said about the calculations that do not produce rings? The absence of rings in Tscharnuter's (1975) calculation is now thought to be due to artificial numerical transport of angular momentum outwards from the center of the cloud. In this regard, small details in the difference equations representing the equation of motion in the azimuthal direction can strongly influence the results. Kamiya (1977) performed his calculation on a Lagrangian grid. Although local conservation of angular momentum must be exact in such a calculation, there are other difficulties. The zones become highly distorted (Kamiya did not rezone) so that difference representations become inaccurate, and the solution for the gravitational potential is likely to be in error. The pressure-free but non-uniform model of McNally (1976) was probably not evolved long enough in time for the ring to develop.

In the calculation of Norman _et al_. (1980), however, considerable attention was paid to the minimization of numerical inward diffusion of angular momentum in a Eulerian scheme, an effect they suggest is responsible for ring formation in other calculations. The collapse that they calculate ($\alpha = .52$, $\beta = .08$) is followed through a continuous increase of central density by more than ten orders of magnitude. Successively smaller fractions of the cloud's mass undergo a cycle of collapse, flattening, then a halt to the collapse due to pressure effects. The end result after 1.22 initial free-fall times is a flattened disk rather than the ring structure which Black and Bodenheimer (1976) obtained after a much smaller increase in central density for similar values of α and β. The reason for the discrepancy has not been clarified. Norman _et al_. do obtain ring formation for an initially differentially rotating but uniform-density cloud, and they suggest that the initial distribution of angular momentum versus mass is the critical parameter. The uniformly rotating uniform-density sphere may be a singular case that produces a disk, while only slight deviations from such a distribution (including those induced by numerical effects) could result in a ring solution (Norman 1980).

The later phases of protostellar collapse, during which an adiabatic approximation can be used in place of the isothermal approximation, have not been as thoroughly explored. The adiabatic phase is of particular importance, however, since the sizes and rotation periods of clouds starting collapse at densities above 10^{-12} g cm^{-3} are likely to be comparable to the separations and orbital periods of observed binaries. Furthermore, the primitive solar nebula undoubtedly formed in this density range.

Black and Bodenheimer (1976) calculated one collapse with an angular momentum appropriate for the solar nebula. The collapse started in the isothermal phase, but in the later stages the central regions became optically thick and began to heat adiabatically. A ring formed, involving only a tiny fraction of a solar mass at the center. Cameron (1978) speculates that the ring will fragment into a binary system but that as a consequence of accretion of mass from the outer collapsing region the fragments will merge and form a protosun. Tscharnuter (1978) also calculated a collapse of a cloud of 3 solar masses whose central regions entered the adiabatic regime. A nearly hydrostatic opaque core formed with temperature and density approximately 780K and 1.3 x 10^{-9} g cm^{-3}, respectively. The ring mode does appear in this core, but its properties and mechanism of formation differ from those of the rings that appear in the dynamically unstable regions of the isothermal portion of the collapse. Tscharnuter (1980) is now continuing the evolution of this "solar nebula" with the inclusion of viscous transfer of angular momentum. The possible mechanisms for angular momentum transfer at this point in the evolution are discussed by Safronov and Ruzmaikina (1978) and by Cameron (1978).

Other calculations have simply assumed a fully adiabatic rotating collapse, starting from a sphere with given density and angular velocity distribution. Takahara et al. (1977) take an ideal gas with adiabatic exponent $\gamma = 5/3$. Such a configuration is, in fact, not intrinsically unstable to gravitational collapse; thus the model by assumption starts out of equilibrium, but it tends to collapse toward the available equilibrium state. For slowly rotating clouds the core oscillates about an equilibrium oblate spheroid. A shock wave on the core boundary marks the region where the outer lower-density material is still being accreted. For rapidly rotating initial clouds, the core develops an off-axis density maximum, a ring-like structure that stays in equilibrium. Analogous calculations have been performed by Boss (1980c) with adiabatic exponents 5/3 and 7/5. If the final value of β for the equilibrium core is less than 0.43, the core forms a spheroid while if $\beta > 0.43$, the final model is ringlike. The ring structure in this case is probably analogous to the classical ring-mode instability for Maclaurin spheroids that starts at comparable values of β. The rings formed during isothermal collapse are formed through an entirely different process. Incidentally, the core equilibrium structures provide an excellent check on the accuracy of the numerical calculations, particularly angular momentum conservation, since they can be compared with independently calculated differentially rotating polytropes (Bodenheimer and Ostriker 1973). The agreement in Boss' work is good.

How does star formation proceed from such an equilibrium? The cores are probably unstable to fragmentation only if $\beta > 0.26$ (see below); also the fragments still need $\gamma < 4/3$ if they are to collapse. The equation of state provides this condition at a temperature of about 2000K when H_2 dissociates. If the equilibrium core reaches this temperature collapse will occur. Otherwise either external compression or internal angular momentum transport is required to allow the evolution to proceed. Bodenheimer (1978) has calculated one collapse in the adiabatic regime including H_2 dissociation. A ring structure forms at the center by a process similar to that which forms rings in the isothermal collapse.

https://doi.org/10.1017/S0074180900073599 Published online by Cambridge University Press

4. THREE-DIMENSIONAL NUMERICAL CALCULATIONS OF COLLAPSING CLOUDS

The critical process of fragmentation in rotating clouds, which is related to the origin of binary, multiple, and planetary systems, can be studied only with calculations that take into account non-axisymmetric effects. A number of important questions can then be addressed:

1. Under what conditions will a rotating collapsing cloud fragment?
2. Is ring formation simply a consequence of the assumption of axial symmetry in 2-D calculations or does it play a role in fragmentation as well?
3. What is the role of the Jeans length in the fragmentation process?
4. What are the properties of the fragments? Will they undergo further collapse and fragmentation?
5. How reliable are the numerical calculations?

This last question is particularly important since the calculations are restricted to rather coarse spatial grids.

The standard assumptions used in the 3-D calculations include a) symmetry about the equatorial plane, b) isothermal or adiabatic collapse, c) ideal gas equation of state, and d) no viscous or magnetic effects. Thus, local transport of angular momentum can occur only through gravitational torques. Most of the calculations involve direct solution of the equations of hydrodynamics on numerical grids containing up to 10^4 cells. Since such calculations are expensive, other workers have adopted a technique where the flow is represented by a set of "fluid elements" whose motions are followed by means of an N-body calculation. Pressure effects are represented either by smoothing over the density distribution or by introduction of a repulsive force term. The main part of the following discussion is based on the results from solutions of the hydrodynamic equations.

One of the first calculations of this type was that of Norman and Wilson (1978), performed on a 40 x 40 x 26 grid. The initial configuration was a near-equilibrium isothermal ring that resulted from one of the axisymmetric calculations of Black and Bodenheimer (1976). The density distribution in the ring was perturbed non-axisymmetrically and the evolution followed to test for stability. If perturbations of 10% amplitude were introduced, corresponding to pure modes of m = 2 through 6, the ring fragmented into m blobs, symmetrically located according to the density maxima of the initial perturbations. The fragmentation occurs within half a rotation period and the growth rate is most rapid for m = 2. Other calculations were performed starting from a super-position of such modes, with randomly chosen phases and amplitudes. The lower-order modes dominated the fragmentation, and in four of the five cases calculated the end result was expected to be a binary system, while in the final case three blobs formed, equally spaced around the ring circumference. In the binary systems, the residual spin angular momentum of a fragment was approximately 20% of its orbital angular momentum. The instability

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of rings to fragmentation was confirmed by Tohline (1980b) for the m = 2 case. Cook and Harlow (1978) performed similar experiments (using a 10 x 12 x 5 grid) on equilibrium polytropic rings. The calculation was performed adiabatically. If the initial perturbation (in velocity) was of mode m = 2 and had an amplitude of 1%, the end result was fragmentation into a binary. Other modes were also investigated.

The important question still remains, however, of whether rings form at all during the collapse of a cloud. Tohline (1980b), using a 34 x 34 x 16 grid, started from a uniform-density, uniformly rotating isothermal cloud into which he introduced a m = 2 density perturbation of 50% amplitude. The parameters of such a calculation include α, β, and the type, mode, and amplitude of the initial perturbation. For $\alpha = .05$, $\beta = .28$ the cloud fragmented directly into a binary. For $\alpha = 0.5$ with two different values of β the result was that pressure effects damped the initial perturbation, and a nearly axisymmetric structure with a ring developed. Although the perturbation amplitude in the ring began to grow, no significant fragmentation occurred before the ring passed through its equilibrium configuration and began to collapse axisymmetrically. Tohline (1980b) has shown that the perturbations left by the time the ring stage is reached are different in nature from those introduced directly into the ring by Norman and Wilson (1978) and that fragmentation is less likely to occur.

Cook and Harlow (1978) made analogous calculations (but with a smaller initial perturbation) of the collapse of an adiabatic cloud with $\gamma = 5/3$. Varying the initial value of α, they found that the higher-pressure case ($\alpha = .15$) resulted in damping of the perturbation while a lower value of $\alpha(=.1)$ led to growth and fragmentation. Recently Boss (1980d) has made more extensive adiabatic calculations on a finer grid, using $\gamma = 7/5$ and starting again from uniform density and uniform rotation. The perturbation imposed was of mode 2 with an amplitude of 50%. For low $\beta(0.05-0.1)$ fragmentation occurs only for $\alpha < 0.075$, while for higher $\beta(.2-.3)$ fragmentation occurs for $\alpha<0.15$. The higher-α runs settle down into near-equilibrium spheroids with final $\beta<.27$, the critical value above which they would be dynamically unstable to non-axisymmetric modes.

Returning to the isothermal case, Narita and Nakazawa (1978) calculated collapses with $\alpha = .3$ and two values of β starting from a highly non-axisymmetric cloud resembling an ellipsoid of non-uniform density and uniform angular velocity. A ring formed that eventually broke up into a binary system. A comparison calculation between two independent 3-D computer codes starting from the same initial conditions ($\alpha = .25$, $\beta = .2$, 50% perturbation of m = 2) was performed by Boss and Bodenheimer (1979). The codes both used the "fluid-in-cell" techniques but in general they were based on different coordinate systems, number of grid points, and difference methods. In both cases a binary system formed directly; the masses of the components were each 15% of that of the original cloud, their ratio of spin to orbital angular momentum was about 0.2, and their value of α was about 0.05 so that they contained many Jeans masses. Boss (1980b) has performed a number of other isothermal calculations, whose results contain two notable differences from other calculations. In particular, a calculation with $\alpha = .25$, $\beta = .2$ and no initial perturbation except for small numerical effects collapses, forms an axisymmetric ring, and then fragments into a binary. Initially axisymmetric calculations performed with the Tohline code remain axisymmetric. The second difference lies in the occurrence of single blobs in some runs. For example, from the initial condition of $\alpha = .24$, $\beta = .18$ with a centrally condensed density distribution and with no initial perturbation the collapse developed into a ring which then broke up into a single off-axis condensation. A similar result occurred for $\alpha = .63$, $\beta = .2$ with a 50% $m = 2$ perturbation, but the blob was not well defined. In other, more standard cases, Boss obtains the normal binary.

Boss (1981) has also performed calculations with a tidal perturbation induced by the presence of a nearby protostar, located 2 cloud radii away. For $\alpha = .25$, $\beta = .20$ and for various values of the mass of the distorting object, the collapsing cloud is distorted into a bar-like shape and it then fragments into a binary. The fragments all have spin J/M about a factor 20 lower than that of the original cloud. For $\alpha = .63$, however, the tidal forces again result in a bar-like configuration but the thermal energy is sufficient to prevent fragmentation; rather, a single dense fragment results. Another variation on the initial conditions was studied by Różyczka et al. (1980a) who imposed random subsonic variations on the velocity field of the uniform-density isothermal cloud. Fragmentation results when $\alpha = .02$ but not for $\alpha = 0.1$ and 0.5.

A comprehensive summary of the 3-D collapse of an isothermal cloud is provided by the work of Bodenheimer, Tohline, and Black (1980). A wide range of α and β is studied starting from two different types of perturbations with mode 2 and amplitude 10% and 50%. The general results show that the cloud first collapses toward a centrally condensed thin disk (Figure 1). When pressure effects become important in slowing the collapse parallel to the rotation axis, a shock forms on the edge of the disk. Only after about one initial free-fall time, when rotation has begun to stabilize the disk in the direction perpendicular to the rotation axis does fragmentation begin. Two different types were noticed. For $\alpha<0.3$ the non-axisymmetric perturbations grow during collapse, although slowly at first, and the cloud fragments directly, usually into a binary, but occasionally into four fragments. This type of fragmentation is illustrated in Figure 2; the bar-like nature of the overall structure is evident. On the other hand, for $\alpha>0.3$ the general result is damping of the initial perturbation and formation of a nearly axisymmetric ring structure, which then fragments (Figure 3). Clouds with α up to 0.6 were found to fragment, with the exception of an intermediate range around $\alpha = 0.5$, where the ring began to collapse on itself on a time scale shorter than the fragmentation time scale. In these cases the numerical code was unable to follow the fragmentation, although its occurrence is likely.

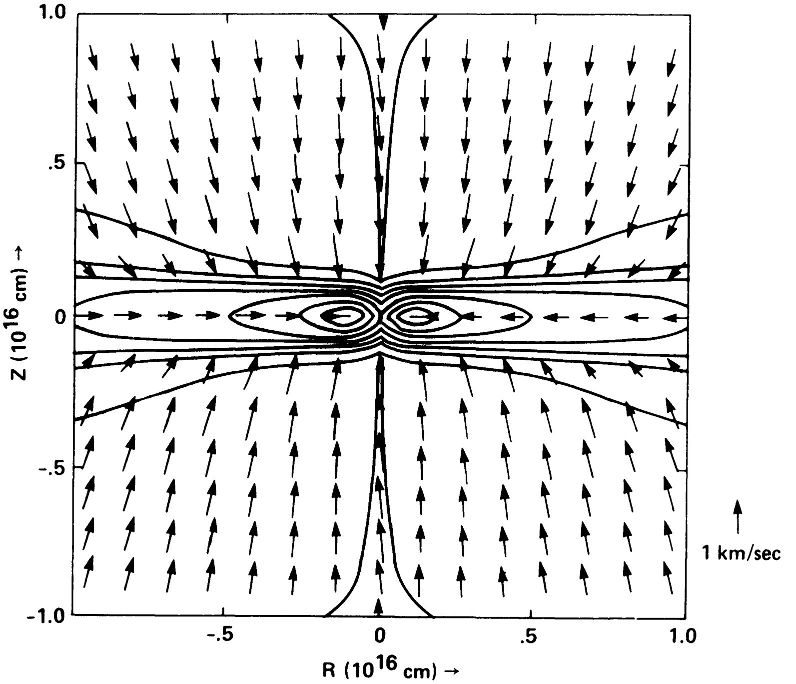

Figure 1. Cross section of the inner portion of a collapsing cloud through the rotation (z) axis after 1.21 initial free-fall times. R is the distance to the rotation axis. Equidensity contours (solid lines) with an interval of a factor of 3.2 are shown along with velocity vectors (arrows) with length proportional to speed. The calculations were done in 3 space dimensions (Bodenheimer, Tohline, and Black 1980).

A somewhat different approach is taken by Larson (1978) who uses a finite particle scheme with repulsive forces between neighboring particles to represent the thermal pressure. Dissipation due to shocks is represented by an additional viscous term which results in more transport of angular momentum than in the fluid-dynamic schemes. The initial conditions are similar to those in the other calculations with no explicit perturbation other than that given by a random initial distribution of the particles in a sphere. With $\alpha = 0.25$ and $\beta = 0.3$ a binary forms, for $\alpha = 0.35$, $\beta = 0.19$ a single dense condensation without a ring forms in the center, and with $\alpha = 0.075$, $\beta = 0.3$, the result is multiple fragmentation into 5-10 sub-condensations. The general outcome is that the number of fragments obtained is approximately equal to the number of Jeans masses contained in the original cloud.

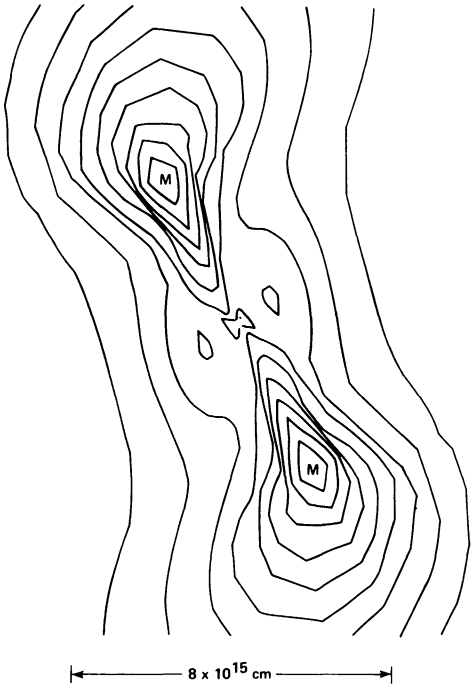

Figure 2. Equidensity contours in the equatorial plane (interval: factor 1.7) for a cloud that has undergone direct fragmentation. The maximum density is denoted by M; the center is a local minimum. (Bodenheimer, Tohline, and Black 1980).

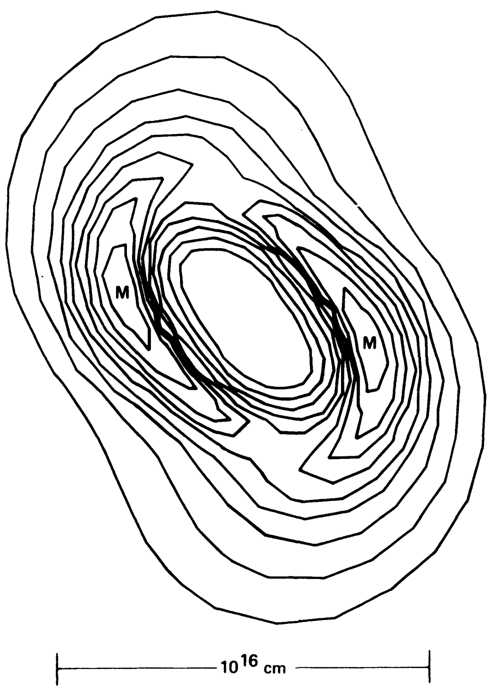

Figure 3. Equidensity contours in the equatorial plane (interval: factor 1.7) for a cloud that has undergone a "ring-mode" fragmentation. The maximum density is denoted by M; the center is a density minimum. (Bodenheimer, Tohline, and Black 1980).

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Multiple fragmentation of this type has not been observed in other calculations, partly because short-wave-length perturbations tend to be numerically and physically damped in the fluid dynamic calculations, and partly because Larson's fragments become well-developed only after several free-fall times, beyond the point where the other calculations are forced to stop for numerical reasons. Larson has deduced that a power-law mass spectrum will be the outcome. However, the finer details of these calculations must be taken with a good deal of caution because of the approximations and the small number of particles involved.

An improved numerical particle scheme that adopts some of the features of Lucy's (1977) method has been developed by Wood (1980). The smoothing length scale is defined locally which is an advantage since the development of fragments can be followed with reasonable resolution wherever they occur. The calculation starts with random 5% density fluctuations on an otherwise uniform-density uniformly rotating sphere. For $\alpha = 0.3$, $\beta = 0.23$ the cloud collapses, bounces in the z-direction, becomes unstable to a bar mode, and then fragments into a binary. For other cases, for example $\alpha = .2$, $\beta = .3$, the result was three symmetrically located condensations.

Most of the above-described calculations assumed an initial configuration that was gravitationally unstable. However, it is also of interest to mention briefly some fragmentation calculations that start from equilibria. Relevant structures include the equilibrium rings that were subjected to the analysis of Norman and Wilson (1978), the isothermal equilibrium disks (Bodenheimer and Tscharnuter 1979), and the quasi-spheroidal cores that result from adiabatic collapse (Takahara et al. 1977, Boss 1980c). If star formation is to proceed beyond such equilibria, some physical process must occur, and fragmentation is a strong possibility. Różyczka et al. (1980b) performed 3-D calculations on the isothermal equilibrium disk of Bodenheimer and Tscharnuter (1979) which has a final $\beta = 0.25$. Density perturbations of mode 2 and 4 with amplitudes of 10%, 20%, and 40% were applied. The $m = 4$ perturbations result in oscillations but no clear fragmentation, while in most cases the $m = 2$ perturbations result in fragmentation into a binary. Similar calculations were performed by the author, but the initial 50% perturbation was applied to the cloud before the onset of collapse rather than at the equilibrium phase. When the equilibrium was approached, the perturbation had damped considerably, but after about 3 initial free-fall times the central part of the cloud had clearly fragmented into a binary.

A 3-D numerical analysis (Durisen and Tohline 1980) of the dynamical stability of rapidly rotating polytropes applies to the adiabatic equilibria (Takahara et al. 1977 and Boss 1980c) which closely approximate polytropes of index 1.5 when $\gamma = 5/3$. Their analysis, of course, also is relevant to the possible fission of pre-main-sequence stars at a much later stage of their evolution. Starting with a density perturbation of 10% or 33% amplitude and mode 2, they follow the evolution of polytropes of n = 0.5 and n = 1.5 with the angular momentum distribution of a uniformly rotating uniform sphere, using the 3-D fluid-dynamic code of Tohline (1980b). For n = 0.5 the perturbation damped for $\beta = 0.25$ and grew for $\beta = 0.33$. For n = 1.5 it damped for $\beta = 0.08$, 0.18, and 0.27, grew for 0.33, and remained roughly constant for 0.30. Within the limits of the numerical results, the point of dynamic instability of the polytropes agrees roughly with the earlier tensor virial equation analyses which showed that the point occurs at $\beta \cong 0.26$ (e.g., Ostriker and Bodenheimer 1973). It is not yet conclusive, however, that the instability leads to fragmentation. Extensive evolution of the high-β configurations shows that an incipient binary forms at the center, but that the density contrast there does not increase noticeably during the ensuing phases, which involve the development of a spiral arm pattern in the outer regions and its evolution into an exponential disk and then into an expanding ring.

5. CONCLUSIONS

The overall results of the 2-D and 3-D hydrodynamic calculations with rotation applied to the star formation and protostar collapse periods of stellar evolution allow us to conclude the following:

1. Rotating collapsing interstellar clouds are unstable to fragmentation over a wide range of initial conditions in the isothermal phase, given even a small initial non-axisymmetric perturbation. If the cloud is unstable to collapse almost any combination of the α- and β-parameters will result in fragmentation. Flattened isothermal equilibria with β about 0.25 are also unstable.

2. The process of fragmentation differs from that suggested by Hoyle (1953) and Hunter (1962) in which fragments spontaneously appear when their mass exceeds the local Jeans mass. Perturbations can in fact damp initially due to pressure effects. After about one initial free-fall time, when the cloud collapse is slowed by pressure effects parallel to the rotation axis and primarily rotational effects perpendicular to the axis, the fragmentation begins.

3. Fragmentation can occur either directly as a consequence of the initial perturbation imposed on the cloud, or through an intermediate ring stage. The dividing line is roughly at $\alpha = 0.3$, but it also depends on the amplitude of the perturbation.

4. The dominant mode of fragmentation is the binary (m = 2) mode, although results with one, three, and four fragments have also been reported.

5. Among the results discussed above, there are some disagreements between authors on cases that were run with essentially the same initial conditions. Numerical difference techniques and numerical accuracy undoubtedly are of importance in this regard. For example, on relatively coarse numerical grids some numerical damping of perturbations inevitably occurs. Thus, some of the details of the results of the various calculations may change in the future as a consequence of improved numerical methods.

6. The properties of the fragments in the isothermal case are

such that they are unstable to collapse. The mass of a fragment in many cases is 10 to 15% of the cloud mass, the value of α is a few percent, and the ratio of spin to orbital angular momentum is very roughly 20%. The fragments form in the innermost part of the cloud which has lower J/M than the average for the cloud. This effect, combined with the conversion of spin to orbital motion, results in a reduction of spin J/M by a factor 10 to 20 from that of the initial cloud. Thus, after a series of several collapses and fragmentations the J/M as well as the fragment masses can be reduced by considerable factors. Bodenheimer (1978) showed that such a process could result in direct evolution from a massive interstellar cloud to main-sequence binary and multiple systems within the observed range of masses and orbital angular momenta.

7. During the adiabatic phase the tendency to fragment is not as universal as it is in the isothermal phase. The collapse approaches an equilibrium, and fragmentation is likely if β in the equilibrium configuration is above 0.27. Initial conditions set up at the end of the isothermal phase, which involve fragments with $\alpha<0.1$, suggest that the conditions for further fragmentation during the adiabatic phase are in general satisfied. If they are not, two possibilities are open. First, the adiabatic evolution could result in temperatures above the dissociation temperature for H_2. Fragmentation would be as likely to occur during the ensuing collapse as it is during the isothermal phase. Second, if the temperature remains too low, the equilibrium configuration can evolve only as a consequence of angular momentum transport. This final set of circumstances may provide the conditions necessary to form a single star and a surrounding rotating nebula.

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https://doi.org/10.1017/S0074180900073599 Published online by Cambridge University Press

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DISCUSSION

Nakamura: How have you checked the accuracy of your 2-D and 3-D numerical calculations.

Bodenheimer: A large number of tests of the accuracy of the solutions have been performed, and new tests are continually being devised. The one currently applied are
1. Comparison with analytic solutions in limiting cases, where available.
2. Intercomparison between different numerical techniques on the same physical problem. Example: Boss and Bodenheimer, Astrophys. J. 234, 289.
3. Examination of the effect of increasing the number of grid points.
4. Tests of the conservation of the distribution of angular momentum with mass, in the 2-D case (see Norman, Wilson, and Barton, Astrophys. J. August 1980).
5. Comparison with the detailed structure of equilibrium differentially rotating polytropes (Bodenheimer and Ostriker, Astrophys. J. 180, 159) in the case of adiabatic 2-D collapses, which should reach such equilibria.

Nariai: For the case of binary-type fragmentation, the loss of mass from the region outside of the outer Lagrangian points greatly reduces the J/M ratio (Nariai and Sugimoto 1976, Publ. Astron. Soc. Japan 28, 593). Is this effect included in your calculations?

Bodenheimer: This effect is included, but it has not been analyzed separately from the other effects. It would seem that this mechanism provides additional help in solving the argular momentum problem.

Mouschovias: You mentioned semi-analytical results which explain the formation of rings as due to a "density wave". Was there an implication that rotation is not important in ring formation?

Bodenheimer: Rotation is critically important in the generation of the wave. Because of rotation and a non-uniform density distribution, the particles in the inner regions overshoot their equilibrium positions, rebound, interact with particles from the outer regions that are still falling in, and so generate the wave.

Tscharnuter: According to your fragmentation scheme, the multiple stellar system should be strictly co-planar. Which processes would you consider to be responsible for the observed finite inclinations?

Bodenheimer: It is true that in some observed multiple stellar systems

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the short-period system and the long-period system are not co-planar. The direction of the angular momentum vectors could be influenced by (1) close encounters and interactions with other protostellar systems during the formation or early evolution of a dense cluster, or (2) magnetic fields which could affect the angular momentum of parts of a diffuse cloud before fragmentation actualy takes place.

Sugimoto: Is the number of final fragments related to the number of modes, for example, in the initial perturbation? If not, what do you expect for the number spectrum of the fragments?

Bodenheimer: In our calculations, we impose an m=2 perturbation and in most cases we get a binary as a result. However, we have also obtained 4 fragments. On general grounds I would expect that the growth rate of the lower-order modes would be faster than that of higher modes. Calculations by Norman and Wilson (Astrophys. J. 224, 497, 1978), starting from equilibrium rings, show that even if a mixture of modes (from m=2 to 6) is imposed in the initial perturbation, the final outcome is usually a binary. We still need to do more numerical calculations with better spatial resolution (to adequately represent the higher modes) to answer this question, but I expect that 2 fragments will be the usual result.

Tayler: In each stage of fragmentation only a small fraction of the mass is included in the fragments. Does this mean that you believe that star formation is very inefficient if only rotation, pressure and self-gravitation are important?

Bodenheimer: That is an important point. If only 20% of the mass of the cloud fragments at each stage (10% per fragment in a binary), there is considerable mass left over that has too much angular momentum to join the fragments. After several stages only a very small fraction of the original material would end up in stars, so the process would in fact be quite inefficient (but efficient in solving the angular momentum problem).

Unno: How many steps of fragmentation are needed for star formation in a realistic situation? Are there numerical simulations for that?

Bodenheimer: Approximately four stages of fragmentation are required for evolution from the interstellar cloud state to a main-sequence multiple system. Some approximate simulations based on the 2- and 3-D numerical solutions can be found in a paper by Bodenheimer (Astrophys. J. 224, 488).

Schatzman: What is the present situation regarding the mass spectrum problem?

Bodenheimer: From the theoretical point of view, I think that the question is completely open. The 3-D calculations that I have been discussing show that fragmentation occurs, but they make no predictions regarding the mass spectrum. Larson suggests (Mon. Not. R.A.S. 184, 69, 1978) that his fragmentation calculations result in a power-law mass spectrum. However a number of approximations are involved. Silk and Scalo, and co-workers, have investigated models involving coalescence of numerous small fragments plus accretion of the surrounding gas. However, there

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are problems with this model as well. For example, the origin of the original fragments is obscure: It is based on the Hoyle-Hunter fragmentation theory, which is no longer believed, and furthermore rotation is not included at all in the analysis.