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On the magnetic flux conservation in the partially ionized plasma

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Abstract. Ohm, Hall, and ambipolar diffusions in the partially ionized plasma are considered. The statement of Pandey & Wardle (2011,2012) that only Ohmic diffusion is capable of destroying the magnetic flux is not sufficiently correct due to the formal dependence of the magnetic diffusion on a frame of reference.

Keywords. Plasmas, diffusion, magnetic fields

1. Introduction

The magnetic field freezing to the plasma is one of the most fundamental property of the cosmic plasma. The frozen–in conditions suggest that the magnetic flux as well as the topology of the magnetic field lines connected with plasma are conserved. However the notion of the magnetic freezing for the partially ionized plasma as distinguished from the fully ionized one becomes more complex. As a result inferences of some authors turn out to be not well–grounded. For example, Pandey & Wardle (2011,2012) concluded that only Ohmic diffusion is capable to destroy the magnetic flux, whereas Pedersen and Hall ones redistribute the flux in the medium. Moreover, "total flux is conserved even in the presence of Ohmic diffusion only if parallel current is absent in the medium" (Pandey & Wardle 2012).

The goal of this paper is to reconsider these results in the case of the collisional partially ionized within the framework the three fluid approximation.

2. On the magnetic flux conservation in different frames of reference

Using standard notation and neglecting by the viscosity, gas pressure, and gravitation, the momentum equations for the electrons (e), ions (i), and neutrals (n) can be written as

$$\frac{d\mathbf{V}_e}{dt} = -\frac{e}{m}\mathbf{E} - \frac{e}{mc}\mathbf{V}_e \times \mathbf{B} + \nu_{ei}(\mathbf{V}_i - \mathbf{V}_e) + \nu_{en}(\mathbf{V}_n - \mathbf{V}_e); \tag{1}$$

$$\frac{d\mathbf{V}_i}{dt} = \frac{e}{M}\mathbf{E} + \frac{e}{Mc}\mathbf{V}_i \times \mathbf{B} + \nu_{in}(\mathbf{V}_n - \mathbf{V}_i) + \nu_{ie}(\mathbf{V}_e - \mathbf{V}_i); \tag{2}$$

$$\frac{d\mathbf{V}_n}{dt} = \nu_{ni}(\mathbf{V}_i - \mathbf{V}_n) + \nu_{ne}(\mathbf{V}_e - \mathbf{V}_n). \tag{3}$$

It is convenient for partially ionized collisional plasma to introduce the velocity of fluid as whole $\mathbf{v} = (n_i\mathbf{V}_i + n_n\mathbf{V}_n)/(n_i + n_n)$ and the degree of the plasma ionization $F = n_n/(n + n_n)$. Then, taking into account the conservation of momentum: $n\mathbf{v}_i + n_n\mathbf{v}_n = 0$, where $\mathbf{v}_i = \mathbf{V}_i - \mathbf{v}$ $\mathbf{v_n} = \mathbf{V_n} - \mathbf{v}$ are the velocities of ions and neutrals relative to the center of mass, assuming $\mathbf{j} = \mathbf{j}_{\parallel} + \mathbf{j}_{\perp} + \mathbf{j}_{\perp} \times \mathbf{B}/B$, from (1)–(3) we find

$$\mathbf{E}_e^* + \frac{\mathbf{V}_O \times \mathbf{B}}{c} = \mathbf{E}_i^* + \frac{(\mathbf{V}_O + \mathbf{V}_H) \times \mathbf{B}}{c} = \mathbf{E}^* + \frac{(\mathbf{V}_O + \mathbf{V}_H + \mathbf{V}_A) \times \mathbf{B}}{c} = \frac{\mathbf{j}_\parallel + \mathbf{j}_\perp}{\sigma}. \quad (4)$$

Here

$$\mathbf{E}_e^* = \mathbf{E} + \frac{\mathbf{V}_e \times \mathbf{B}}{c}; \quad \mathbf{E}_i^* = \mathbf{E} + \frac{\mathbf{V}_i \times \mathbf{B}}{c}; \quad \mathbf{E}^* = \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c};$$

$$\sigma = \frac{ne^2}{m(\nu_{ei} + \nu_{en})}; \quad \mathbf{V}_O = -\frac{c\mathbf{j}_\perp}{\sigma B}; \quad \mathbf{V}_H = -\frac{\mathbf{j}}{en}; \quad \mathbf{V}_A = -\frac{F^2(\mathbf{j} \times \mathbf{B})}{cnM\nu_{in}}.$$

As it follows from equations (4) only Ohmic diffusion can change the field topology caused, for example, by the magnetic reconnection since at $(\mathbf{j}_\parallel + \mathbf{j}_\perp)/\sigma = 0$ the generalized Ohm's law can be written as (see, for example, Nickeler & Karlicky 2006)

$$\mathbf{E} + \frac{1}{c}\mathbf{V} \times \mathbf{B} = 0. \quad (5)$$

where $\mathbf{V}$ is the arbitrary velocity.

Moreover, at first glance equation (5) suggests that Hall and ambipolar diffusions are not capable to destroy the magnetic flux (Pandey & Wardle 2011). However this is not quite correct because the flux transport velocities $\mathbf{V}_e + \mathbf{V}_O$, $\mathbf{V}_i + \mathbf{V}_O + \mathbf{V}_H$, $\mathbf{v} + \mathbf{V}_O + \mathbf{V}_H + \mathbf{V}_A$ and velocities of particle species $\mathbf{V}_e, \mathbf{V}_i, \mathbf{v}$ are different. Therefore the magnetic flux is conserved in frames of reference, which are not connected with the plasma. Really, taking into account the total time derivative of the magnetic flux in the general case, according to Faraday's law

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},$$

and equations (4) and (5), for the magnetic flux in frames of reference connected with ions and fluid as a whole we respectively have

$$\frac{d\Phi_i}{dt} = \frac{d}{dt}\int\int \mathbf{B}d\mathbf{S}_i = -\frac{1}{en}\oint \mathbf{j} \times \mathbf{B}d\mathbf{l}, \quad (5)$$

$$\frac{d\Phi}{dt} = \frac{d}{dt}\int\int \mathbf{B}d\mathbf{S} = -\frac{1}{en}\oint \mathbf{j} \times \mathbf{B}d\mathbf{l} - \frac{F^2}{nMc\nu_{in}}\oint ((\mathbf{j} \times \mathbf{B}) \times \mathbf{B})d\mathbf{l}. \quad (6)$$

Since the real motion of the partially ionized plasma is determined by ions and neutral atoms, we can conclude from (5) and (6) that the magnetic flux is not conserved due to Hall and ambipolar diffusions.

3. Conclusions

(*a*) Formal peculiarities of the magnetic diffusion in the partially ionized plasma strongly depend on a frame of reference.

(*b*) The magnetic reconnection in the partially ionized plasma is possible only due to Ohmic dissipation of parallel and perpendicular electric currents.

(*c*) The magnetic flux is not conserved in the partially ionized plasma because of Hall and ambipolar diffusions.

References

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