ON THE DATE OF CAUCHY'S CONTRIBUTIONS
TO THE FOUNDING OF THE THEORY OF GROUPS

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Dedicated with love and respect to my father
for his eightieth birthday, 15 October 1989

Evidence from published sources is used to show that Cauchy's group-theoretical work
was all produced in a few months of intense activity starting in September 1845.

1. INTRODUCTION

Had he lived so long A.-L. Cauchy would have been 200 years old in August this
year. He and B.H. Neumann could have given us all very great pleasure by attending
each other's birthday celebrations. In this note I propose to prove three assertions
about a paper [4] that Cauchy wrote when he was 56. The first is that he *was* 56 years
old when he wrote it; the second, that, contrary to their publication dates, Bertrand's
work [3] preceded Cauchy's; and the third, that it was Bertrand's manuscript which
triggered Cauchy's interest.

It is quite common to identify scientific articles by year of publication. This some-
times transposes priorities or mis-represents the facts by a year or two, but it rarely
matters. And in exceptional cases the rule is broken—the famous *Premier Mémoire*,
for example, which was submitted by Galois to the Paris Academy in January 1831
but first published by Liouville towards the end of 1846, is always thought of as being
a product of that earlier time. The case that I propose to examine in some detail is
another unusual one in that traditional dating places it too early. Coincidentally, it is
related circumstantially, though not in any way directly, to the *Premier Mémoire*: for
Galois' work and the 1845 papers by Cauchy are the two sources that introduced group
theory to mathematics.

The paper in question 'Mémoire sur les arrangements ...', is one of Cauchy's major
contributions. According to long established bibliographical convention the year of
publication was 1844, but I believe that it was written in September and October 1845
at the same time as the long series of *Comptes rendus* articles [5–22]. One might feel

that this small discrepancy should be worth no more than a footnote in my long-planned, half-written larger essay on the development of group theory in the nineteenth century. The justification for a free-standing paper is that this apparently minor bibliographical point has disproportionate consequences for our understanding of Cauchy's impetus and of the relationship between Cauchy's work and that of Bertrand. Besides, there is a substantial credibility gap to be bridged because the 1844 dating has been accepted by many authoritative scholars. Thus G.A. Miller [27], Hans Wussing [30, pp. 61–68, 86, English translation, pp. 86–95, 118], Luboš Nový [28, p. 210], B.L. van der Waerden [29, p. 85], for example, refer to his work on group theory as stretching over the years 1844–46. That would suggest a much longer period for his preoccupation with the subject than the few months that I will show to have been the case. Of course it is a small mistake and one which need not worry us greatly were it not for two points. The first is this. Every time we read an item that takes the 1844 date seriously, as for example, those quoted above, or Josephine Burns [25], or Amy Dahan [26], we may be a little doubtful whether the author has properly read Cauchy's text because the evidence against 1844 is clear in the published record. The second and more serious point is that from time to time incorrect inferences have been drawn from the incorrect premiss. Luboš Nový, for example writes [*loc. cit.* fn]

> First Cauchy published an extensive treatise [4] in the 3rd volume of the
> *Exercices d'analyse et de physique mathématique* in 1844, and in the autumn
> of 1845 he decided to publish the main results again in the *Comptes Rendus*;
> however, he soon became dissatisfied with repeating his earlier results and he
> developed new methods and results.

Since the chronology is wrong Nový's suggestions as to Cauchy's state of mind are unlikely to be right. A more significant example is to be found in [26, p. 310] where Amy Dahan writes

> Ces travaux de CAUCHY sur les substitutions, et en particulier les Notes aux
> C. R. de l'Académie, furent pratiquement ignorés des autres mathématiciens
> jusque vers 1860. Plusieurs contributions de J. BERTRAND [3], de J.A. SERRET
> [23] n'y font aucune référence et n'intègrent aucune des notions développées
> par CAUCHY.

> [These articles by CAUCHY on substitutions, and in particular the notes in the
> C. R. of the Academy, were practically ignored by other mathematicians until
> about 1860. Several contributions by J. BERTRAND [3] and by J.A. SERRET
> [23] make no reference to them and incorporate none of the ideas developed
> by CAUCHY.]

This comment makes good sense in relation to Serret but it is quite inappropriate (though true) in relation to Bertrand because, as I have already indicated, Bertrand's work was written *before* Cauchy's.

2. THE NUMBER OF VALUES OF A FUNCTION

Although the meaning of the titles of various of the papers that I shall discuss is not important for my argument, it is perhaps worth a short digression to ensure that there is no misunderstanding about the context of Cauchy's work. When Bertrand, Cauchy, Serret and others spoke of 'le nombre de valeurs d'une fonction' (see, for example, the titles of [1, 2, 3, 5, 7, 8, 24]) it had a very different meaning from what it does now. It referred to the number of functions that can be obtained from $f(x_1, \ldots, x_n)$ by permuting the variables $x_1, \ldots, x_n$ amongst themselves. Thus the number of values is 1 if and only if f is a symmetric function, it is 2 if and only if f is an alternating function; and the 'Theorem of Lagrange' was that the number of values always divides $n!$. In his early work [1] (published 1815 but written three years earlier), which contains no group theory, Cauchy sets up a good notation for substitutions and their algebra, and he proves that the number of values of a function which is neither symmetric nor alternating will always be at least p, where p is the largest prime number $\leq n$; he conjectures furthermore that the number of values of a function of n variables that is neither alternating nor symmetric will in fact always be at least n if $n \geq 5$, and he proves this conjecture for the case where $n = 6$. In his later work Cauchy realises that what matters is that the collection of permutations of $x_1, \ldots, x_n$ which leave f unchanged is closed under composition, that is it is what we would now call a (permutation) group; Cauchy's terminology for such a collection is 'système de substitutions conjuguées'. When he refers to 'le nombre de valeurs égales' (see, for example the title of [5]) he means the number of permutations that leave the function invariant, that is to say the order of its group; when he refers to 'le nombre de valeurs inégales' he means the number of values in the sense explained above, that is to say, the index of its group in the symmetric group of degree n.

3. CHRONOLOGY

The dating of the relevant works is, I believe, as follows. In March 1845 Bertrand submitted a manuscript, 'Mémoire sur le nombre de valeurs que peut prendre une fonction quand on y permute les lettres qu'elle renferme' to the Paris Academy. It took up the theme of the 1815 paper by Cauchy [1] and, in particular, it contained a proof (subject however to assuming true the postulate that for all $n \geq 7$ there is a prime number between $n/2$ and $n - 2$, a postulate which Bertrand had verified for all n up to 6 000 000 but which was not proved until seven years later by Chebyshef) of the

conjecture mentioned in Section 2 above. Record of receipt and of the appointment of Poinsot, Cauchy and Lamé as 'Commissaires' (perhaps best translated into modern terms as 'referees') is published in the *Comptes rendus* for 17 March 1845 in the form of an abstract [2]. There is a report in the *Comptes rendus* for 10 November 1845 ostensibly by the committee but evidently written by Cauchy as 'rapporteur'. The paper was finally published in 1848 (see [3]).

In September 1845 Cauchy started writing on the theory of substitutions. He produced a long series of *Comptes rendus* papers [5–22], and, as I contend, the great paper [4] in *Exercices*. The material fills nearly 400 quarto pages of Cauchy's collected works. There are 25 *Comptes rendus* contributions all told. They appeared regularly in each weekly issue, sometimes two or more in one issue, from 15 September 1845 until 19 January 1846, after which there was a three-week gap until 9 February and finally a two-month gap before the last paper appeared on 11 April 1846. It should be noted that the number of the *Comptes rendus* containing the record for a given Monday's meeting was usually published in time for the following Monday's meeting, as is recorded in the 'Bulletin bibliographique' which appears at the end of each issue. Thus if we think of Cauchy writing his mathematics during one week, giving his manuscript to the secretaries of the Academy at the Monday meeting the next week, and seeing his *Comptes rendus* notes published the next Monday after that, we will have a pretty clear and accurate picture of the pace of his work. It is more like journalism, reporting Cauchy's weekly prolific invention, than scholarly writing as most of us know it.

The conventional dating of [4] as 1844 comes, presumably, from the fact that Volume III of *Exercices* in which it appears bears the date 1844 on its title page. The fact is, however, that the *Exercices* appeared in parts ('livraisons' or 'fascicules'). That was as common for academic periodicals in the nineteenth century as it is now. The only difference was that the parts were on the whole smaller, and they were not thought of as self-contained individual 'issues' as is common nowadays; a part was distributed to subscribers when it had reached the relevant size (16 or 32 or ... pages) and the publisher would have no great sensitivity about ensuring that it ended at the end of an article — or even a paragraph or sentence. Nowadays many publishers distribute the title page as a separate item included with the last issue, so that librarians can have it inserted at the appropriate place in the bound volume. But this was certainly not common practice last century and the first issue of Volume III of *Exercices* included, not unnaturally, the title page. Since it appeared in 1844 that is the date that it carries. Nevertheless, that certainly does not imply that everything in the volume dates from 1844.

There were twelve livraisons to each volume of *Exercices*. The first, which included the title page and some subscription information, and the twelfth, which included an

index, were made up of five 8-page sections; all other livraisons were made up of four 8-page sections as is witnessed by the printer's line that the reader will find at the foot of each page whose number is congruent to 1 modulo 8 in the original edition. The 'Mémoire sur les arrangements ...' [4] starts on the 15th page of the 29th livraison and continues through to the 20th page of the 32nd livraison. The public record shows that the 29th livraison was given by Cauchy to the Academy on 8 December 1845 (see *C. R.* 21 (1845), p. 1255 and 'Bulletin Bibliographique' on p. 1284). The 30th, 31st and 32nd livraisons are recorded as having been received by the Academy on 23 March, 27 April and 18 May 1846 respectively (see *C. R.* 22 (1846), pp. 557, 698, 862)[1]. Of course it is just possible that these parts were issued earlier and reached the Academy only after some delay, but that seems very unlikely. It is also possible that the 'Mémoire sur les arrangements' was written earlier, but the fact that it did not begin to appear until December 1845 is quite sufficient to excuse Bertrand from any duty (or possibility) of referring to it in his work.

4. EVIDENCE

My argument in the preceding section shows only that the appearance of 1844 on the title page of the third volume of *Exercices d'Analyse* is inconclusive. Of course it does not prove that Cauchy's work was in fact written in the five months from September 1845 to January 1846, apart from the two last items which date from February and April 1846. For that we shall consider internal evidence in Cauchy's writings. The very first [5] of the *Comptes rendus* contributions opens with the following paragraph

> Je m'étais déjà occupé, il y a plus de trente années, de la théorie des per-
> mutations, particulièrement du nombre des valeurs que les fonctions peu-
> vent acquérir; et dernièrement, comme je l'expliquerai plus en détail dans
> une prochaine séance, M. Bertrand a joint quelques nouveaux théorèmes à
> ceux qu'on avait précédemment établis, à ceux que j'avais moi-même obtenus.
> ...   Je me propose de publier, dans les *Exercices d'Analyse et de Physique*
> *mathématique*, les résultats de mon travail avec tous les développements qui
> me paraîtront utiles; je demanderai seulement à l'Académie la permission d'en

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[1] I am most grateful to Professor René Taton who, in a letter from Paris dated 30 May 1989, confirms these datings (as well as the general conclusions of this note) and draws attention to the chronology published on pages 604, 635, 636 of Volume XV of the second series of Cauchy's *Oeuvres* (Gauthier-Villars, Paris 1974). I am equally grateful to Dr Frank Smithies who, in a letter from Cambridge dated 3 June 1989, writes "Not everybody gets the dating wrong. In Belhoste's biography of Cauchy (Belin, Paris, 1985) he devotes a section (pp. 65–74) to 'Le calcul des substitutions'. In this (p. 73) he says: 'Ses notes aux *Comptes rendus* furent rédigées en fait à l'occasion de l'examen d'un mémoire de J. Bertrand ...'. ... On looking at Belhoste again I see that (earlier on p. 73) he says: *Le Mémoire sur les arrangements*, publié dans les livraisons de Décembre 1845 et de Mars et Avril 1846 ...'. So he certainly has it right."

insérer des extraits dans le *Compte rendu*, en indiquant quelques-unes des propositions les plus remarquables auxquelles je suis parvenu.

[I have already worked, more than thirty years ago, on the theory of permutations, and in particular, on the number of values that functions can take; and lately, as I shall explain in more detail in a later session, Mr Bertrand has added some new theorems to those that had previously been established, to those which I myself had obtained. ... I propose to publish, in the *Exercices d'Analyse et de Physique mathématique*, the results of my work with all the developments which appear to me to be useful; I shall simply ask the Academy for permission to insert extracts of it in the *Compte rendu*, while indicating some of the most noteworthy propositions at which I have arrived.]

Note first the reference to more than thirty years earlier, which is (as the editor of this part of the *Oeuvres* indicates) a reference to the 1815 paper [1] . If Cauchy had published [4] in 1844 one would have expected him to refer back to his work of this more recent time and to this paper. Note also the reference to the *Exercices*. If [4] had already been published then his use of the future tense and his announcement that he proposes to publish in the *Exercices* must refer to a project which was never carried out for there is only the one article on the subject in that journal. And turning to Bertrand, why is it that Cauchy refers to Bertrand rather than Bertrand to Cauchy? Or rather, to be more precise, why is it that Bertrand refers to Cauchy's 1815 paper [1], and rehearses quite a bit of the notation and general theory that it introduces, whereas the later paper [4] would have been just as relevant, if not more so? All these difficulties disappear when we realise that [4] is the paper referred to by Cauchy and that he began writing it in September 1845 at the same time as he was beginning his series of *Comptes rendus* notes.

Another such passage is to be found in the second sentence of the second *Comptes rendus* item: in [5, p.668] Cauchy writes in the future tense 'Les propositions que j'énoncerai ici se trouveront d'ailleurs démontrées et développées dans les *Exercices d'Analyse et de Physique mathématique*' [the propositions which I shall state here will, moreover, be proved and developed in the *Exercices d'Analyse et de Physique mathématique*].

The second piece of evidence that we should consider is the overlap between [4] and the *Comptes rendus* papers. The material in the first and fourth parts of [5], the first and second parts of [6], in [9] and in [10] is almost all duplicated in the *Exercices* paper. Sometimes there are substantial passages where the text is duplicated word for word. For example,

$$[4, \text{p.}152, \text{l.}1 - \text{p.}155, \text{l.}18] = [5, \text{p.}595, \text{l.}28 - \text{p.}599, \text{l.}18]$$

except that at one point 'Pour abréger, nous représenterons souvent' has become 'Rien n'empêche de représenter' and at another point three words have changed from roman to italic type; or again,

$$[4, \text{p. } 170, \text{l. } 8 - \text{p. } 173, \text{l. } 6] = [9, \text{p. } 1125, \text{l. } 32 - \text{p. } 1129, \text{l. } 8]$$

except that a few lines on $[4, \text{p. } 172] = [9, \text{p. } 1128]$ have been changed. (in the *Oeuvres* these equations become

and

$$(2)\text{XIII, p. } 172, \text{l. } 5 - \text{p. } 176, \text{l. } 5 = (1)\text{IX, p. } 280, \text{l. } 14 - \text{p. } 284, \text{l. } 15$$

$$(2)\text{XIII, p. } 192, \text{l. } 6 - \text{p. } 195, \text{l. } 23 = (1)\text{IX, p. } 420, \text{l. } 12 - \text{p. } 424, \text{l. } 6.$$

Inferences drawn from such duplication must be treated with caution since, as is well-known, there is a very great deal of it scattered throughout Cauchy's works. And of course Nový's suggestion [28, p. 210] that Cauchy published [4] in 1844, decided in the autumn of 1845 to publish the main results again, and became dissatisfied with repeating his earlier results, is perfectly consistent with this evidence. But it seems unlikely that the secretaries and other members of the Academy would have countenanced the filling of their pages with material that had already been published. My hypothesis, that the duplications result from the fact that the *Exercices* paper was written at the same time as the *Comptes rendus* work, seems to meet this point.

The third piece of evidence is this. The 29th livraison of volume III of *Exercices d'Analyse et de Physique mathématique*, which, as I have written above, contains the first 18 pages of [4] and which was received by the Academy on 8 December 1845, begins with two short papers. The first of these appears, again verbatim except for the addition of a couple of short paragraphs at the end of the first section and the addition of an extra three pages at the end, in the *Comptes rendus* for 4 August 1845. The second, entitled 'Note sur quelques propositions relatives à la théorie des nombres', contains the proof of a proposition related to the euclidean algorithm, with application to a version of the Chinese Remainder Theorem. This proposition is stated without proof as a footnote in [5, p. 606]; furthermore, at the end of the paper Cauchy writes that the theorems that it contains are particularly useful in the theory of permutations, 'ainsi qu'on le verra dans les Mémoires qui suivront la présente Note'. Thus this note is contemporary with Cauchy's work on permutations, and probably written at the same time as the first of the *Comptes rendus* papers, that is to say, in the second week of September 1845. Its relevance to my argument is that the last paragraph of its penultimate page (p. 149 in the original edition) contains a reference to a theorem 'énoncé par M. Poinsot dans le *Journal des Mathématiques* de M. Liouville [février 1845]' (compare Section 15 on p. 43 of Poinsot's paper in vol. 10 of Liouville's Journal). Therefore the 29th livraison must have been written some time after February 1845, and, in light of its relationship with the *Comptes rendus* papers, September to October or November 1845 seems the most likely time.

5. Conclusion

As promised in Section 1, I have attempted to prove three propositions. The primary one is that Cauchy's works on substitutions and group theory were all written during a period of intense activity that began in or around the second week of September 1845 and died away some four or five months later. The second is that Bertrand's work [2, 3] preceded Cauchy's; the third that it was the 'refereeing' of Bertrand's paper which triggered Cauchy's interest. The first two assertions are, I believe, demonstrated by the evidence. The last is a little more speculative (but see the quotation from Belhoste in the footnote in Section 3). Nevertheless, it seems very plausible in virtue of the chronology. It is, moreover, confirmed by the opening sentence of a paper by Camille Jordan. On p. 40 of Vol. I of *Bulletin de la Société mathématique de France*, in the record of the meeting on 8 January 1873 (= *Oeuvres de Camille Jordan*, I, p. 365) we read 'Vers l'année 1845, époque où les travaux de M. Bertrand ramenèrent l'attention de Cauchy sur la théorie des substitutions, ... ' [About the year 1845, at a time when the work of Mr Bertrand revived Cauchy's interest in the theory of substitutions, ...]. Jordan, though not exactly contemporary, was writing less than thirty years later and knew Bertrand well. His evidence added to that of the opening sentence of [5] quoted in Section 4 above is probably as much as we can expect to find on this point.

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   Fourth paper: 779–797 (6 Oct.) = *Oeuvres* (1), IX, 323–341.

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   Third paper: 931–933 (27 Oct.) $\doteq$ *Oeuvres* (1), IX, 368–371;
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