

Part 5

Micro Lensing



Joachim Wambsgans and Peter Schneider



Philippe Jetzer

Microensing Search for Dark Matter at all Mass Scales

Joachim Wambsganss

Astronomisches Rechen-Institut & Universität Heidelberg, Germany;
jkw@ari.uni-heidelberg.de

Abstract. Gravitational microlensing is a powerful tool to detect compact matter on mass scales ranging from roughly $10^{-6}M_{\odot}$ to 10^6M_{\odot} . Of particular importance is the fact that lensing is sensitive to both luminous *and* dark matter. There are two practically important regimes of microlensing: cosmological and Galactic. The former deals with the effect of stellar mass objects at cosmological distances on even more distant quasars, the latter treats stellar mass objects in the Milky Way or its Halo on stars in the Magellanic Clouds, the Galactic Bulge or M31. Microlensing has been detected in about ten quasars, roughly a dozen microlensing events towards stars in the Magellanic Clouds are on record, more than 1000 events have been measured towards stars in the Bulge. The large majority of microlensing events towards quasars and towards stars in the Magellanic Clouds or the Bulge can be explained by ordinary stellar mass objects. A small fraction of the Galactic microlensing events ($< 5-10\%$) could potentially be due to dark matter objects, including stellar mass Black Holes. Current and planned experiments will clarify the question whether any dark matter objects are necessary at all to explain microlensing events.

1. What is Microlensing?

1.1. The Basics

The light deflection effects of *compact* objects with *small* masses along the line of sight to distant sources is usually called microlensing. What does ‘compact’ mean in this context? It means much smaller than the Einstein radius of the object. What does ‘small’ refer to? The angular Einstein radius ought to be below the resolution limit of the telescope. The Einstein radius is defined as

$$r_E = \sqrt{\frac{4GM}{c^2} \frac{D_S D_{LS}}{D_L}},$$

where G and c are the gravitational constant and the velocity of light, respectively, M is the mass of the lens, and D_L, D_S, D_{LS} are the distances to the lens, to the source, and between lens and source. For a quasar at a redshift of about $z_S = 2$ and a lens at $z_L = 0.5$, this results in

$$R_{E,\text{cosmo}} \approx 4 \times 10^{16} \sqrt{M/M_{\odot}} \text{ cm},$$

or an Einstein angle of

$$\theta_{E,\text{cosmo}} = R_E/D_S \approx 10^{-6} \sqrt{M/M_{\odot}} \text{ arcsec}.$$

For sources at 8 kpc and lenses at $D_L = d * D_S$, the physical and angular Einstein radii are roughly:

$$R_{E,\text{galactic}} \approx 1 \times 10^{14} \sqrt{M/M_{\odot}} \sqrt{(1-d)} \text{ cm},$$

or

$$\theta_{\text{E,galactic}} \approx 10^{-3} \sqrt{M/M_{\odot}} \sqrt{1-d} \text{ arcsec.}$$

Applying the criteria above, this results in a very broadly defined mass range of $10^{-6} \leq m/M_{\odot} \leq 10^6$ for microlensing[†].

1.2. Two regimes of Microlensing

Looking at the spatial distribution of compact objects in the stellar mass range, one easily identifies two astrophysical regimes in which microlensing is at work (and which has already been used in the previous subsection):

(a) Compact objects in the Milky Way or its halo (or elsewhere in the Local Group) acting as lenses on stars in the Galactic Bulge, the LMC/SMC or M31. This variant is referred to under the names

- stellar microlensing,
- Galactic microlensing, or
- local group microlensing.

The lightcurves of this kind of microlensing are usually simple: a background star is lensed by a foreground star. To first order, the shape of the lightcurve is a one-parameter family depending on the impact parameter, once the parameters are expressed in units of Einstein radii (Paczynski 1986b). In roughly 10% of the cases, the shape is affected by the nature of the lens being a physical binary.

(b) Compact objects in a distant galaxy, or its halo acting on even more distant (multiple) quasars. This type is referred to by the names

- quasar microlensing,
- extragalactic microlensing, or
- cosmological microlensing.

In quasar microlensing, the lightcurves are much more complicated than in stellar microlensing. The lenses do not act individually any more. Due to the high density of objects in the lens galaxy, the coherent light deflection effect produces a wealth of different lightcurves (Paczynski 1986a).

1.3. How can we observe microlensing?

Due to the relative motion between source, lens and observer, the impact parameter $u(t)$ varies as a function of time, and so does the lensing magnification

$$\mu(t) = \frac{u(t)^2 + 2}{u(t)\sqrt{u(t)^2 + 4}}.$$

As a consequence, some observable quantities are changing as a function of time. There are several ways to detect microlensing:

- photometrically – due to the changing apparent magnitude as a function of time,
- spectroscopically – due to the difference in the shape of a broad emission line with time or between the various (time-delay corrected) images of the same quasar (cf. Lutz Wisotzki, these proceedings)
- astrometrically – due to the position (i.e., the center-of-light) of the quasar changing as a function of time.

Hence: microlensing is a *dynamical* phenomenon!

[†] Occasionally, parts of this mass range are also called “nanolensing”, “mesolensing”, or “millilensing”

The typical time scale involved in microlensing is the *Einstein time*, the duration for crossing of the Einstein radius. For the above chosen parameters,

$$t_{E,\text{cosmo}} = \frac{r_E}{v_{\perp}} \approx 15 \sqrt{M/M_{\odot}} v_{600}^{-1} \text{years},$$

is the cosmological Einstein time† and

$$t_{E,\text{galactic}} \approx 70 \sqrt{M/M_{\odot}} \sqrt{(1-d)} v_{200}^{-1} \text{days},$$

is the corresponding galactic Einstein time, where v_{600} and v_{200} indicate relative transverse velocities of 600 km/sec and 200 km/sec, respectively.

2. (Pre-)History of microlensing

Right after the discovery of the first multiply imaged quasar (Walsh *et al.* 1979), Chang & Refsdal (1979) suggested that the flux of the two quasar images can be affected by stars close to the line of sight. Gott (1981) proposed that a massive galaxy halo could be made of low mass stars and “should produce fluctuations of order unity in the intensities of the QSO images on time scales of 1-14 years.” Young *et al.* (1981) were the first to use numerical simulations in order to explore the effect of quasar microlensing.

The bottom line of all these early investigations was: for a multiply imaged quasar, the surface mass density (or “optical depth”) at the position of an image is of order unity. If this matter is made of (dark) compact objects in the range described above, microlensing is expected to be going on basically “all the time”, due to the relative motion of source, lens(es) and observer. In addition, this means that the lens action is due to a coherent effect of many microlenses, because the action of two or more point lenses whose projected positions is of order their Einstein radii combines in a very non-linear way (cf. Wambsganss 1998). An illustration of this coherent action can be found in Figures 1: Even if detected, the lensing objects could be either “normal” stars or low-mass objects like brown dwarfs or Jupiter-like planets, or some kind of compact dark matter, including stellar mass black holes. The lensing signal alone could not be used for or against dark matter as the microlensing agents.

In Figure 1, the magnification distribution produced by an ensemble of lenses is indicated in the quasar plane by different colors. The three dashed lines show the tracks of a quasar. In Figure 2 the corresponding lightcurves are displayed, for two different source sizes. If the size of the quasar is small compared to the inter-caustic spacing, each caustic crossing is resolved individually, which results in relatively high maxima in the lightcurves (solid line). For a larger source (dashed line, factor 10 larger than solid line), the peaks are smoothed out, the character of the lightcurve is different.

The lens action of more than two point lenses cannot be easily treated analytically any more. Hence numerical techniques were developed in order to simulate the gravitational lens effect of many compact objects. Paczyński (1986a) had used a method to look for the extrema in the time delay surface. Kayser, Refsdal & Stabell (1986), Schneider & Weiss (1987) and Wambsganss (1990) had developed and applied an inverse ray-shooting technique that produced a two-dimensional magnification distribution in the source plane.

In 1986, Paczyński (1986b) had suggested a “local” microlensing experiment: Assuming that the halo of the Milky Way is made of compact dark matter objects, he suggested that

† This results in depressingly large values of years to decades for typical quasars. However, what is really relevant is the *crossing time*, the time it takes a quasar to cross its own diameter: $t_{cross} = R_{source}/v_{\perp} \approx 4R_{15}v_{600}^{-1}$ months. Here the quasar size R_{15} is parametrized in units of 10^{15} cm. In fact, there can be multiple caustic crossings per Einstein radius.

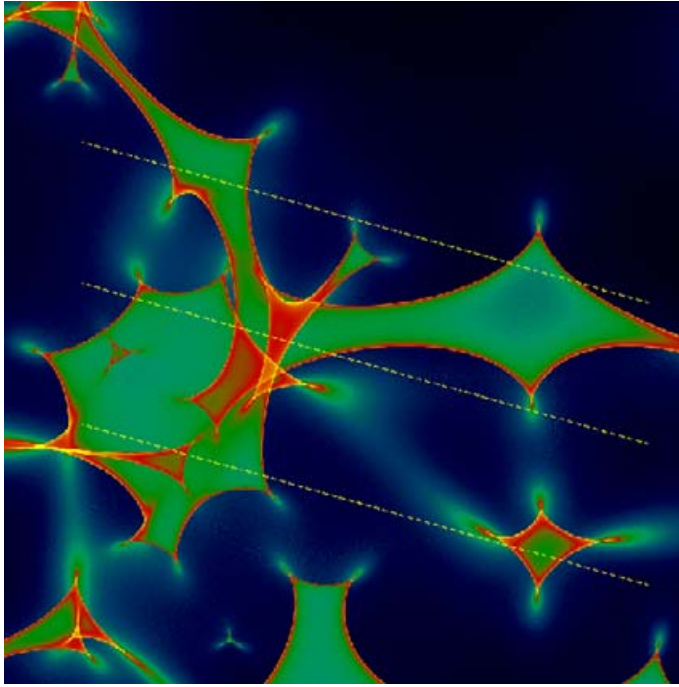


Figure 1. Microlensing magnification pattern produced by stars in a lensing galaxy. The colors/gray steps represent different magnifications, with the sharp caustic lines corresponding to the highest magnification. The dashed lines indicate three tracks along which a background quasar moves. The corresponding lightcurves are displayed in Figure 2

occasionally one of them should pass in front of a star in the Large or Small Magellanic Cloud (LMC/SMC) and hence introduce a characteristic brightening of the background star.

3. Microlensing Detections: are they caused by dark matter?

Over the last two decades, a large number of microlensing searches were and are being performed. In both regimes, cosmological and Galactic microlensing, some clear signatures were detected. Here these observations are presented and discussed with respect to the question whether they were or could be caused by dark matter objects.

3.1. Cosmological Microlensing I: The Einstein Cross, Quadruple Quasar Q2237+0305

In 1989 the first evidence for cosmological microlensing was found by Irwin *et al.* (1989) in the quadruple quasar Q2237+0305: one of the components showed fluctuations, whereas the others stayed constant. In the mean time, Q2237+0305 has been monitored by many groups (Corrigan *et al.* 1991; Østensen *et al.* 1996; Lewis *et al.* 1998). The most recent (and most exciting) results (Figure 3, and cf. Woźniak *et al.* 2000a,b) show that all four images vary dramatically, going up and down like a rollercoaster in the last three years: $\Delta m_A \approx 0.6$ mag, $\Delta m_B \approx 0.4$ mag, $\Delta m_C \approx 1.3$ mag, $\Delta m_D \approx 0.6$ mag. Comparison of these lightcurves with simulations (cf. Figures 1, 2) show that the continuum emitting region of the quasar is relatively small, of order 10^{14} cm (see, e.g., Wambsganss, Paczynski & Schneider 1990; Wyithe *et al.* 2000a,b; Yonehara 2001, Kochanek 2003).

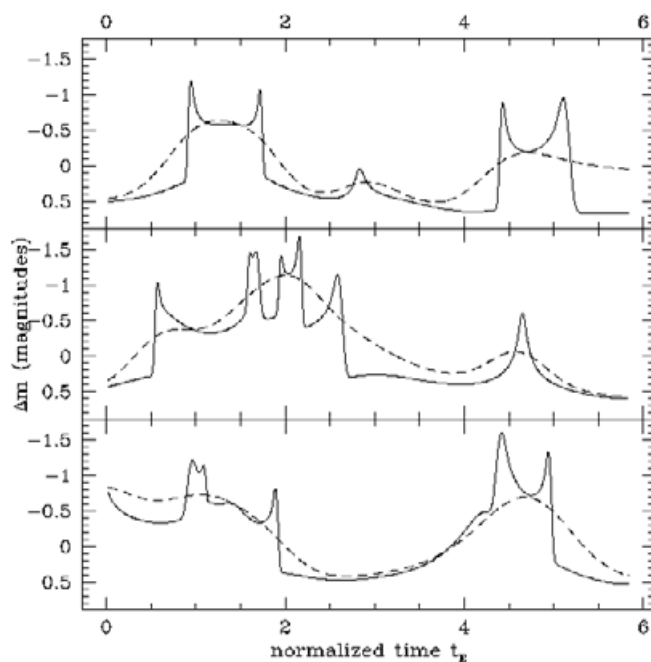


Figure 2. Microlensing lightcurves for the three tracks shown in Figure 1. The solid line correspond to a small source (Gaussian shape with width of about 3% of the Einstein radius), the dashed line represents a source that is a factor of 10 larger.

3.2. Cosmological Microlensing II: The Double Quasar Q0957+561

The microlensing results for the double quasar Q0957+561 are not quite as exciting. In the first few years after its discovery, there is an almost linear change in the (time-shifted) brightness ratio between the two images (Schild 1996): $\Delta m_{AB} \approx 0.25$ mag over 5 years. But since about 1991, this ratio stayed more or less “constant” within about 0.05 mag, so not much microlensing has been going on in this system recently (Schild 1996; Pelt *et al.* 1998; Schmidt & Wambsganss 1998). The possibility for some small amplitude very rapid microlensing was pointed out by (cf. Colley & Schild 2000). However, both a very well determined time delay and highly accurate photometry is required, in order to confirm it.

With numerical simulations and limits obtained from data of three years of Apache Point monitoring data of Q0957+561 (see Figure 7), Wambsganss *et al.* (2000) exclude a whole range of “Machos” masses as possible dark matter candidates in the halo of the lensing galaxy in 0957+561. They extracted simulated lightcurves according to the timing of the observed ones and evaluated 100000 cases for seven different values for the lens mass (from $m/M_{\odot} = 10^{-7}$ to 10^0) and four different quasar sizes (10^{14} cm to 3×10^{15} cm): The small “difference” between the time-shifted and magnitude-corrected lightcurves of images A and B ($|\Delta m_{A-B, Q0957}| \leq 0.05$ mag) excludes a halo of the lensing galaxy made of compact objects with masses of $10^{-7} M_{\odot} - 10^{-2} M_{\odot}$ (cf. Figures 8 and 9).

3.3. Cosmological Microlensing III: OGLE monitoring of HE 1104-1805

The OGLE team had collected data points for 102 separate nights in three years (Schechter *et al.* 2003). Both images show fluctuations, but with substantial differences when

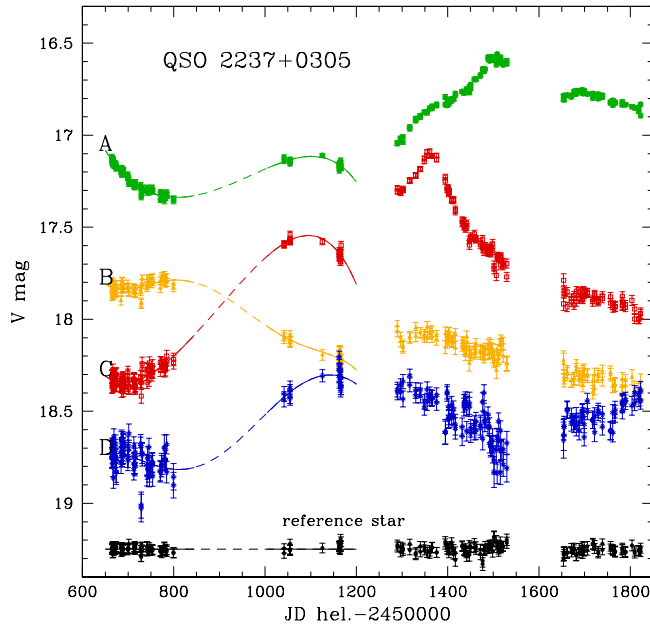


Figure 3. Microensing lightcurve of the quadruple quasar Q2237+0305, as measured by the OGLE team (Woźniak *et al.* 2000a,b; updated version see at <http://bulge.princeton.edu/~ogle/ogle2/huchra.html>).

correcting for the time delay. They interpret the short time scale of the variability (4 weeks) as requiring relativistic motion of one or more of the the source components, potentially relativistically moving knots. Is the microlensing caused by dark matter? Not necessarily, ordinary stars in the lensing galaxy could do the job.

3.4. *Cosmological Microlensing IV: Microlensing by cosmologically distributed lensing objects of stellar mass?*

Dalcanton *et al.* (1994) did a study to search for such a cosmological population of objects in the mass range of $0.001 \leq M/M_{\odot} \leq 120$. They investigated the equivalent width distribution of 200 quasars, based on the assumption that microlensing of such objects would magnify the continuum emission of the quasars, because it emerges from a much smaller spatial region than the broad and narrow line regions. If microlensed, hence, this would reduce the equivalent width of such affected quasars. In particular, one would expect this to occur for quasars with higher redshift, since the optical depth of such a distribution of compact objects would increase with increasing source redshift. No such effect was found (Dalcanton *et al.* 1994). Their conclusion was that $\Omega_{10^{-3}M_{\odot}-10^{1.3}M_{\odot}} < 0.1$.

3.5. *Cosmological Microlensing V: Few (if any) million solar mass black holes: kinky VLBI jets in Q0957+561, gamma-ray bursts, double radio sources*

There are various arguments that halos of galaxies could also be made by black holes with masses around one million solar masses. This is an interesting mass range, because the Einstein radius of such objects at cosmological distances is of order a milliarcsecond, hence accessible to observations with VLBI. And there are also objects out there to measure the effect: the radio jets of lengths 50 to 100 milliarcseconds are perfect targets

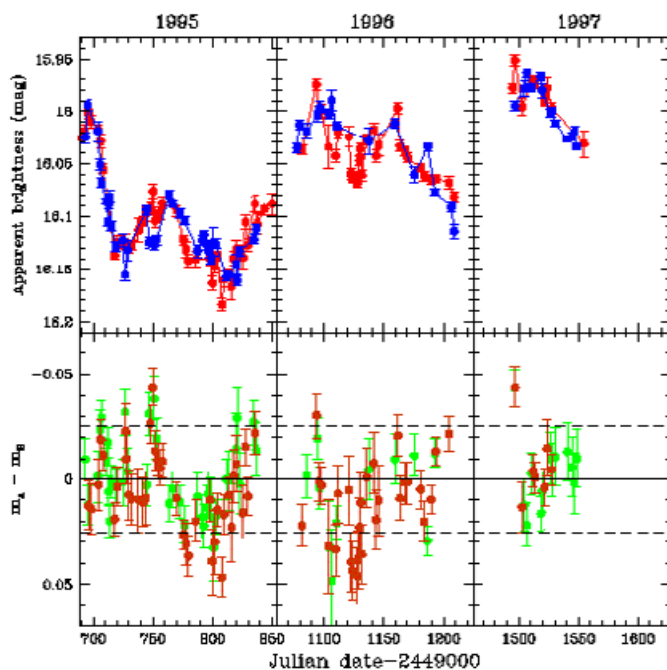


Figure 4. Observed lightcurves of the double quasar Q0957+561; top: superposition of lightcurves of image A and (time shifted and magnitude shifted) image B; bottom: difference lightcurves (Wambsganss *et al.* 2000).

for such a test. If there is a significant population of lenses in this mass range between such a radio jet and the observer, they would produce bends and kinks and holes in such a jets. The problem is that some/most jets have naturally bends and kinks, hence the lensing signature is not unique. However, nature provides us with a good test lab anyway: for multiply-imaged quasars, we have two or more images of such a radio jet. And since this lensing effect acts differently on either of these jets, we are able to see whether such millilensing objects exist from comparing the two radio jets (Figure 7, Wambsganss & Paczyński 1992). In the case of the double quasar Q0957+561, this test was done (Garrett *et al.* 1994). The close similarity of the two jets excludes scenarios in which more than 10% of the halo is made of objects with $M > 3 \times 10^6 M_\odot$.

A study by Wilkinson *et al.* (2001) investigated 300 compact radio sources with VLBI for possible double sources. They did not find any multiple images with angular separations between $1.5 \leq \Delta\theta/\text{milliarcsec} \leq 100$, corresponding to a mass range of $10^6 \leq M/M_\odot \leq 10^8$. They used this null result to put limits on the matter content in this form of supermassive objects: $\Omega_{10^6-10^8} < 0.01$ (2σ).

However, recently Metcalf (2002) showed that millilensing by substructure may explain differences in the structure of the radio jets in B1152+199 (cf. Metcalf, these proceedings).

3.6. Cosmological Microlensing VI: Quasar Microlensing at high magnification/supressed saddle points and the role of dark matter

A subset of all quadruply imaged quasars have an image geometry with one close image pair (e.g., PG 1115+080, SDSS0924+0219, MG0414+0534). Simple (macro-)models predict that the source sits inside but close to a (macro-)caustic, and that the two images should be highly magnified with very similar magnification. In most of the observed

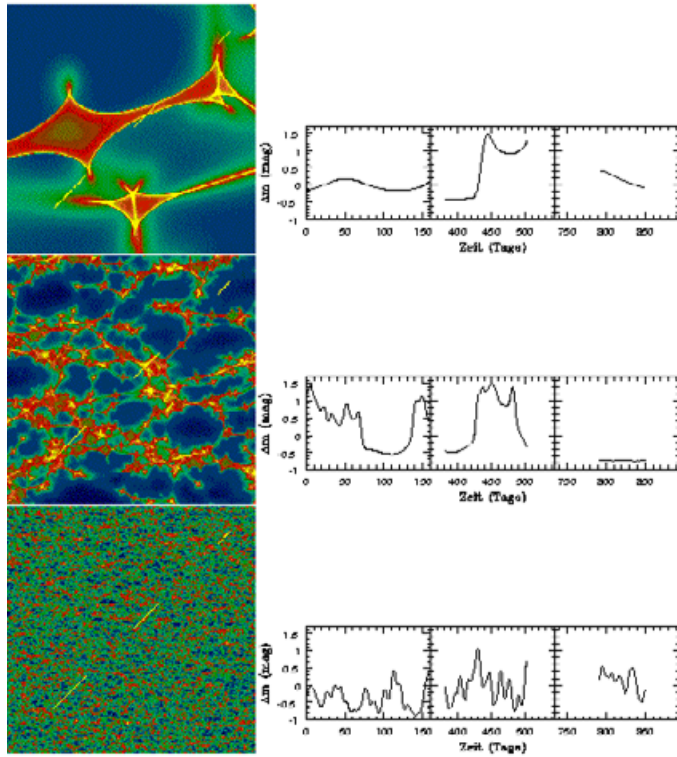


Figure 5. Simulated microlensing lightcurves for the double quasar Q0957+561; Left: Magnification patterns for compact objects in three different mass ranges; the three-part straight line indicates the track of the background quasar. Right: corresponding three-part microlensing lightcurves (Wambsganss *et al.* 2000).

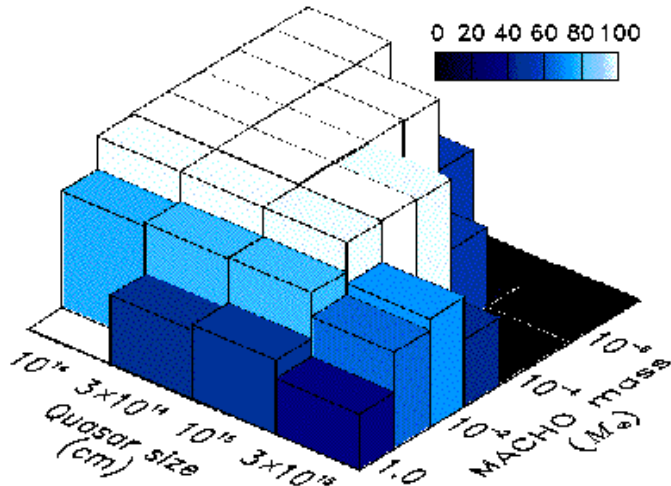


Figure 6. Exclusion diagram: the highest (white) columns indicate values of quasar source size and macho mass which are excluded by more than 99.9% probability; the other columns show exclusion probabilities of between 40% and 85% (Wambsganss *et al.* 2000).

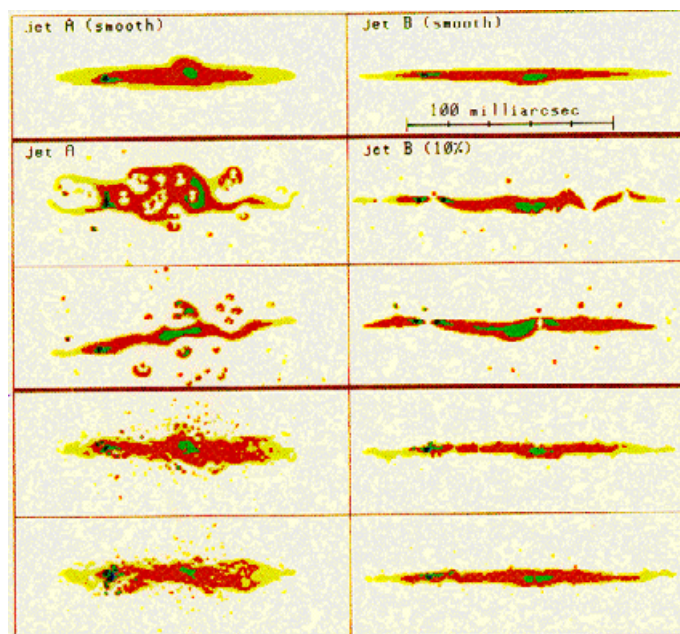


Figure 7. Millilensing by million solar mass black holes affects the VLBI jets of multiply imaged quasars: the top row shows model VLBI jets for images A and B of the double quasar Q0957+561, as produced by the smooth lensing potential. If the halo of the lensing galaxy is made of million solar mass black holes (here: $m/M_{\odot} = 3 \times 10^5$), then the two jets should be affected by them *differently*, as shown in the four rows below: kinks, holes, additional milli-images should appear in both images uncoherently (Wambsganss & Paczynski 1991).

cases, however, this is not the case: close image pairs tend to have quite different magnifications. In almost all cases, the fainter (or demagnified) image seems to be the saddle point image. Currently two competing explanations are being discussed: Substructure in the macro-lens (galaxy) could introduce this flux ratio anomaly (Dalal & Kochanek 2002; Metcalf & Madau 2001; Metcalf & Zhao 2002), or microlensing by compact stellar mass objects plus smoothly distributed (dark) matter (Schechter & Wambsganss 2002).

The two proposed mechanisms make different predictions for the behaviour of the two images: If substructure is the cause for the flux ratio anomaly, due to the large mass and corresponding Einstein scale, it should act the same way in basically all wave bands and the flux ratio should be constant in time. If microlensing plus smoothly distributed dark matter is the origin of the discrepancy, then we expect different behaviour in different wavebands, due to the fact, that source size changes as seen in different energy bands. In general, small source sizes (shorter wavelengths) should be affected more drastically than larger sources (which smooth out the microlensing caustics). A second consequence of this microlensing explanation is that the flux ratio should change with time (cf. section 1), because the relative positions of the microlenses change over the course of a few years and hence produce fluctuations in the magnification. More observations are necessary to definitely distinguish between the two explanations. It appears possible or even likely that both are part of the story.

3.7. Galactic Microlensing I: Macho Experiments towards LMC/SMC:

Following Paczyński's (1986b) original suggestion, two collaborations started to search for dark matter objects in the Galactic Halo. Alcock *et al.* (2000a,b) summarized the results

of the MACHO experiment which had run for 5.7 years and searched for microlensing events towards the Magellanic Clouds, potentially revealing the nature of compact dark matter objects in the Galactic Halo. The MACHO result is: depending on their two sets of criteria, they identified 13 or 17 events. This is consistent with a 20% macho contribution to dark matter halo, assuming they are caused by dark matter objects.

In a conference proceedings contribution, Sahu (2003) discussed the question whether the microlensing towards the Magellanic Clouds could be caused by dark matter objects.

Sahu looked at the information on the distance of the 17 MACHO events towards the Magellanic Clouds. He concludes:

- for one of them, a binary-lens event, the distance could be determined securely via its caustic crossing time scale: it is within the SMC.
- for three more, the lens location could be estimated. This estimate is less certain than for the SMC event, but the evidence suggests that it is *most likely* that the lenses are located within the Magellanic Clouds as well.

As an independent second step, assuming that most of the events are dark matter objects in the Galactic Halo: the time scales of the events towards the LMC imply that masses of order $0.5 M_{\odot}$ (Alcock *et al.* 2000a). However, the most likely masses for the events towards the SMC are in the range 2 - 3 M_{\odot} . No model of the Galaxy is consistent with such an inhomogeneous mass distribution. On the other hand, if one assumes that most of the events are caused by foreground objects in the LMC/SMC, then the expected masses would be of order $0.2 M_{\odot}$ for both LMC and SMC.

A third line of argument uses the frequency of binary lenses. Two of the 17 events are caused by binary lenses. Both are (most likely) caused by objects located within the LMC/SMC. Assuming that roughly 50% of the potential lenses in the LMC/SMC are in binary systems (similar to the stars in the solar neighbourhood), one would expect that 10% of all microlensing events would show binary characteristics (cf. Mao & Paczyński 1991). This implies that of order 20 events are expected to be caused by single stars within the LMC/SMC. So this would be perfectly consistent, if most of the observed microlensing events are caused by stars in LMC/SMC.

A fourth argument of Sahu's (2003): If the microlensing are caused by $0.5 M_{\odot}$ in the Galactic Halo (as claimed from the LMC observations), one would have expected to detect about 15 events in the direction towards the SMC, with time scales of about 40 days. Not a single event of this kind was detected: in fact, both SMC events are shown to be due to self-lensing.

Although each individual of these four conclusions are not very strong, the combination of them provide relatively strong argument against them being interpreted as mostly due to halo objects. Sahu (2003)'s conclusions are: "*Close scrutiny of the microlensing results towards the Magellanic Clouds reveals that stars are major contributions as lenses, and the contribution of MACHOs to dark matter is 0% to 5%.*". This view might not be shared by everyone working in the field. However, it is certainly a viable one. Sahu (2003) analysis of MACHO data: consistent with 0% to 5% macho contribution to dark matter halo. A recent analysis of the EROS data (Afonso *et al.* 2003 and this conference) the conclusion is: the dark matter (macho) contribution to the dark matter halo of the Milky Way is $\leq 3\%$.

3.8. Galactic Microlensing II: Microlensing towards the Galactic Bulge

The microlensing searches towards the Galactic Bulge provided microlensing events galore: more than 1500 events so far, this year alone more than 500 by OGLE and MOA. Most of them are single lens events, but quite a number of double lens events with or without caustic crossings were found as well. In general: there is no need for dark matter

to explain these events. By and large, known (sub)stellar contribution can account for them. Due to the degeneracy of the three parameters lens mass, lens distance and relative transverse velocity, the duration of the event does not directly relate to the lens mass: long-duration events can be caused by an moderate mass object which has only a small velocity component perpendicular to the line of sight. However, massive objects produce long events.

In an analysis of some very long events, Bennett *et al.* (2002) investigated the lightcurves of many MACHO events towards the Bulge and came up with events MACHO-96-BLG-5, MACHO-98-BLG-6, MACHO-99-BLG-22 as good candidates for the lenses being black holes. Agol *et al.* (2002) quantified the (a posteriori) black hole probabilities to 4%, 16% and 76%, respectively, for these three events.

3.9. Galactic Microlensing III: “Dark” Matter Detection:

Bond *et al.* (2004) detected the first extrasolar planet with microlensing. The event OGLE 2003-BLG-235 or MOA 2003-BLG-52 showed a clear deviation from the single-lens-single-source lightcurve. It has double peak which is very characteristic for a caustic crossing typical for a binary lens event. Modelling of the system resulted in a unique fit for a mass ratio $q = 0.0039^{+0.0011}_{-0.0007}$. If the primary is a low mass main sequence star in the galactic disk, the separation would correspond to very roughly 3 AU, and the mass of the companion would be about $M_2 \approx 1.5M_{Jup}$. This is the first clear detection of a planet with the microlensing technique, and hence can be counted as some kind of dark matter, though not the one cosmologists prefer.

4. Summary and Outlook

Over the last 15 years, gravitational microlensing has been detected in both the cosmological and the Galactic regimes: Regarding cosmological microlensing, of order ten multiple quasars show uncorrelated microlensing fluctuations in their lightcurves, a subset of them displays spectroscopic signatures as well. There is not much convincing evidence for microlensing of single quasars.

In the Galactic microlensing regime, more than a dozen events were detected towards the LMC/SMC, and more than 1000 events towards the Galactic bulge. Most of these microlensing events can be explained by ordinary stellar mass objects. However, there is still room for a small fraction (less than 5-10%) which could also be due to dark matter objects, including stellar mass black holes. Ongoing and future experiments like RETRO-CAM, COSMOGRAIL, SUPERMACHO, MAGNUM (all described at this conference) will contribute further to the question of whether there is a significant population of compact dark matter objects.

Acknowledgements

It is a pleasure to thank Georges Meylan and Yannick Mellier and their LOC members for having planned and organized a very pleasant meeting in Lausanne.

References

- Afonso, C., Albert, J. N., Alard, C., Andersen, J., Ansari, R., Aubourg, E., Bareyre, P., Bauer, F., Beaulieu, J. P., Blanc, G., *et al.* (The EROS Team): 2003, *Astron. Astrophys.* 404, 144
Agol, E., Kamionkowski, M., Koopmans, L.V.E., Blandford, R.D.: 2002, *Astrophys. J.* 576, L131
Alcock, C., Allsman, R.A., Alves, D.R., Axelrod, T.S., Becker, A.C. *et al.*: 2000a, *Astrophys. J.* 541, 734

- Alcock, C., Allsman, R.A., Alves, D.R., Axelrod, T.S., Becker, A.C. *et al.*: 2000b, *Astrophys. J.* 542, 281
- Alcock, C., Allsman, R.A., Alves, D.R., Axelrod, T.S., Becker, A.C., Bennett, D.P., Cook, K.H.: 2001, *Nature* 414, 617
- Bennett, D.P., Becker, A.C., Quinn, J.L., Tomaney, A.B., Alcock, C. *et al.* (The MACHO and MPS Collaborations): 2002, *Astrophys. J.* 579, 639
- Bond, I. A., Udalski, A., Jaroszynski, M., Rattenbury, N. J., Paczynski, B., *et al.* 2004, *Astroph. Journ.* 606, L155
- Chang, K., Refsdal, S.: 1979, *Nature* 282, 561
- Dalal, N., Kochanek, C.S.: 2002, *Astroph. Journ.* 572, 25
- Dalcanton, J.J., Canizares, C.R., Granados, A., Steidel, C.C., Stocke, J.T.: 1994, *Astrophys. J.* 424, 550
- Garrett, M.A., Clader, R.J., Porcas, R.W., King, L., Walsh, D., Wilkinson, P.N.: 1994, *Monthly Notices Roy. Astron. Soc.* 270, 457
- Gott, J.R. III: 1981, *Astroph. Journ.* 243, 140
- Irwin, M.J.: 1989, *Astron. J.* 98, 1989
- Kayser, R., Refsdal, S., Stabell, R.: 1986, *Astron. Astrophys.* 166, 36
- Kochanek, C.S.: 2004, *Astroph. J.* 605, 58
- Metcalfe, R.B., Madau, P.: 2001, *Astroph. Journ.* 563, 9
- Metcalfe, R.B., Zhao, H.: 2002, *Astroph. Journ.* 567, L5
- Metcalfe, R.B.: 2002, *Astroph. Journ.* 580, 696
- Østensen, R., Refsdal, S., Stabell, R., Teuber, J., Emanuelson, P.I. *et al.*: 1996, *Astron. & Astroph.* 309, 59
- Paczynski, B.: 1986a, *Astrophys. J.* 301, 503
- Paczynski, B.: 1986b, *Astrophys. J.* 304, 1
- Pelt, J., Schild, R., Refsdal, S., Stabell, R.: 1998, *Astron. & Astroph.* 336, 829
- Sahu, K. in: Proceedings of the STScI Symposium on "Dark Universe: Matter, Energy, and Gravity", also astro-ph/0302325
- Schechter, P.L., Wambsganss, J.: 2002, *Astrophys. J.* 580, 685
- Schechter, P.L., Udalski, A., Szymanski, M., Kubiak, M., Pietrzyński, G., *et al.* 2003, *Astrophys. J.* 584, 657
- Schild, R.: 1996, *Astroph. Journ.* 464, 125
- Schmidt R., Wambsganss, J.: 1998, *Astron. & Astroph.* 335, 379
- Schneider, P., Weiss, A.: 1987, *Astron. Astrophys.* 171, 49
- Walsh, D.: 1979, *Nature* 279, 381
- Wambsganss, J.: 1990, PhD thesis (Munich University), also MPA report 550
- Wambsganss, J., Paczyński, B., Schneider, P.: 1990, *Astroph. Journ.*, 358, L33
- Wambsganss, J.: 1999, *Journ. Comp. Appl. Math.* 109, 353
- Wambsganss, J., Schmidt, R.W., Colley, W.N., Kundic, T., Turner, E.L.: 2000, *Astron. Astrophys.* 362, L37
- Webster, R.L., Corrigan, R., Irwin, M., Hewett, P., Jedrzejewski, R.: 1989, *Gemini* 25, 1
- Woźniak, P.R., Alard, C., Udalski, A., Szymański, M., Kubiak, M., *et al.*: 2000, *Astrophys. J.* 529, 88
- Wyithe, J.S.B., Webster, R.L., Turner, E.L.: 2000a, *MNRAS*, 318, 762
- Wyithe, J.S.B., Webster, R.L., Turner, E.L.: 2000b, *MNRAS*, 318, 1120
- Woźniak, P.R., Udalski, A., Szymański, M., Kubiak, M., Pietrzyński, G., *et al.*: 2000, *Astrophys. J.* 540, L65
- Yonehara, A.: 2001, *Astrophys. J.* 548, L127
- Young, P., Gunn, J.E., Kristian, J., Oke, J.B., Westphal, J.A.: 1981, *Astrophys. J.* 244, 736