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STABILITY OF LINE GRAPHS

To George Szekeres on the occasion of his 65th birthday

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Abstract

We present in this paper a discussion on some stability properties of line graphs. After relating the semi-stability properties of the line graph of a graph to a concept of Sheehan, we proceed to deduce that, with fully characterised lists of exceptions, the line graphs of trees and unicyclic graphs are semi-stable. We then discuss the problem of deciding which line graphs are stable. Via a discovery of the finite number of graphs G such that both G and its complement have stable line graphs, we show that P_4 is the only self-complementary graph whose line graph is stable.

1. Introduction

Throughout this paper, all graphs that we consider will be finite, will have no loops or multiple edges, and will be undirected. Some basic definitions and terminology to be found in Behzad and Chartrand (1972) will not be given here. On the other hand, as concepts relating to the semi-stability of graphs are not yet well known, we shall explain them fully below.

If G is a graph, we denote by $V(G)$ its vertex set and $E(G)$ its edge set. $L(G)$ denotes the line graph of G; here we identify $V(L(G))$ with $E(G)$. By $\Gamma(G)$ we denote the automorphism group of G, and by $\Gamma_1(G)$ the automorphism group of $L(G)$. $\Gamma^*(G)$ denotes the subgroup of $\Gamma_1(G)$ induced by $\Gamma(G)$.

A *block* of G is a maximal non-separable subgraph of G. If B is a block of G which contains at most one cutvertex of G, then we say that B is an *end-block* of G. If B_1 and B_2 are end-blocks of G which have a common vertex, we say that they are *intersecting end-blocks*.

If u and $v \in V(G)$ are adjacent, we write $u \sim v$; similarly if e and $f \in E(G)$ are adjacent, we write $e \sim f$. If $v \in V(G)$, $N_G(v)$ is the set $\{u \in V(G): u \sim v \text{ in } G\}$ and $\overline{N_G}(v)$ is $N_G(v) \cup \{v\}$; if $e \in E(G)$, $N_G[e]$ is the set $\{f \in E(G): e \sim f \text{ in } G\}$ and $\overline{N_G}[e]$ is $N_G[e] \cup \{e\}$. If $W \subseteq V(G)$, then by G_W we denote the subgraph $\langle V(G) - W \rangle$ of G induced by $V(G) - W$; for simplicity, if $W = \{v\}$, we write G_v rather than $G_{\{v\}}$. If $F \subseteq E(G)$, then by $G - F$ we denote the spanning subgraph of G with edge set $E(G) - F$; if $F = \{e\}$, we write $G - e$ rather than $G - \{e\}$. If $f \in E(\bar{G})$, then $G + f$ is the spanned supergraph of G with edge set $E(G) \cup \{f\}$.

If $W \subseteq V(G)$, then by $\Gamma(G)_W$ we denote the maximal subgroup of $\Gamma(G)$ each element of which fixes each vertex in W; here we consider $\Gamma(G)_W$ to act only on $V(G) - W$. If $W = \{v\}$, we write $\Gamma(G)_v$ rather than $\Gamma(G)_{\{v\}}$. Similar notions hold regarding $\Gamma_1(G)_F$, for $F \subseteq E(G)$. We note that if $\alpha \in \Gamma_1(G)_e$, then $\alpha(e)$ denotes the unique element of $\Gamma_1(G)$ which fixes e and which restricts to α when acting on $E(G) - \{e\}$.

We say that G is *semi-stable* at $v \in V(G)$ if $\Gamma(G_v) = \Gamma(G)_v$; if G is semi-stable at some vertex, we say that G is a *semi-stable graph.* (We define the semi-stability of rooted graphs similarly.) For $k \geqq 1$, a sequence $S = \{v_1, \cdots, v_k\}$ of distinct vertices of G such that $\Gamma(G_{S_j}) = \Gamma(G)_{S_j}$ for $j = 1, \cdots, k$, where $S_j = \{v_1, \cdots, v_j\}$, is called a *partial stabilising sequence* for G. The empty sequence is also considered to be a partial stabilising sequence for G. The *stability index, s.i.* (G), of G is the maximum cardinality of a partial stabilising sequence for G. If *s.i.* $(G) = |V(G)|$, then we say that G is *stable.*

In Sheehan (1973) the author introduced what he called "stable graphs". As we use that terminology, as explained above, to describe a concept rather different from that of Sheehan, we use the phrase "*S-stable*" to describe Sheehan's concept. Thus if $e \in E(G)$, G is *S-stable* at e if $\Gamma(G - e) \leqq \Gamma(G)$; if G is S-stable at some edge, we say that G is an *S-stable graph.*

Having introduced the above ideas, we can now state just what this paper accomplishes. First of all, we note that $L(G)$ is semi-stable at e if and only if $\Gamma_1(G - e) = \Gamma_1(G)_e$. It seems reasonable to hope that this condition may often be equivalent to $\Gamma(G - e) \leqq \Gamma(G)$. This is in fact the case and in Section 3 we discover when $L(G)$ being semi-stable at e implies that G is S-stable at e, and vice-versa. We then use these results in Section 4 to discover those trees and unicyclic graphs whose line graphs are not semi-stable.

In Section 5, we investigate stable line graphs. We find all those G which have the property that $L(G)$ and $L(\bar{G})$ are both stable. This enables us to show that P_4 is the only self-complementary graph whose line graph is stable.

2. Preliminary results

In this section we state, without proof, several results which are needed in the remainder of the paper. First of all we describe some consequences of the definitions of $\Gamma(G)$, $\Gamma_1(G)$ and $\Gamma^*(G)$.

LEMMA 2.1. (a) (See Behzad and Chartrand (1972), Theorem 13.5). *Let G be a non-empty graph. Then $\Gamma_1(G) = \Gamma^*(G)$ if and only if*:

(i) *not both G_1 and G_2 (of Figure 2.1) are components of G, and*

(ii) *none of the graphs G_3, G_4, G_5 (of Figure 2.1) is a component of G.*

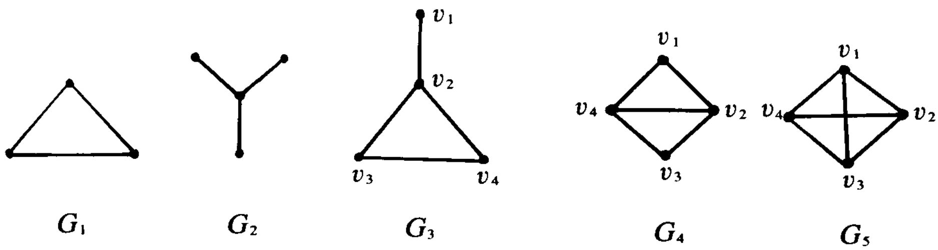

Figure 2.1

(b) (Harary and Palmer; see Behzad and Chartrand (1972), Corollary 13.3a.) *For a non-trivial graph G, $\Gamma(G) \cong \Gamma^*(G)$ if and only if G contains neither K_2 as a component nor two or more isolated vertices.*

We now restate, as a lemma, a remark made at the conclusion of Section 1.

LEMMA 2.2. *If $e \in E(G)$, then $L(G)$ is semi-stable at e if and only if $\Gamma_1(G-e) = \Gamma_1(G)_e$.*

Taking for granted the standard results about the automorphism groups of unions of graphs (see Harary (1969), for example), the following results are fairly obvious; we omit their proofs (see Grant (1974) for the proof of a similar theorem).

LEMMA 2.3. *Let $e \in E(G)$, and let H be the component of G which includes e.*

(i) *$L(G)$ is semi-stable at e if and only if (a) $L(H)$ is semi-stable at e, (b) no component of $H-e$ which has at least 2 vertices is isomorphic to a component of G and (c) there do not exist both a component of G isomorphic to K_3 and a component of $H-e$ isomorphic to $K_{1,3}$, whose vertex of degree 3 is incident with e.*

(ii) *G is S-stable at e if and only H is S-stable at e and no component of $H-e$ is isomorphic to a component of G.*

3. S-stability and the semi-stability of line graphs

If G is S-stable at e, then it seems reasonable to hope that $L(G)$ might be semi-stable at e. This is usually the case, but, as we shall see, Lemma 2.1 implies that there are exceptions. The following theorems fully explain the situation.

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THEOREM 3.1. *Suppose $L(G)$ is semi-stable at e. Let H be the component of G which includes e. Then G is not S-stable at e if and only if at least one of the following four statements is true*:

(i) *$H-e$ has a component isomorphic to K_2*;

(ii) *both G and $H-e$ have isolated vertex*;

(iii) *H is the graph G_3 of Figure* 2.1 *and e is one of the edges (v_2, v_3) or (v_2, v_4)*;

(iv) *H is the graph G_4 of Figure* 2.1 *and e is the edge $(v_2\, v_4)$*.

PROOF. If any of (i), (ii), (iii) or (iv) holds, then clearly G is not S-stable at e.

Suppose that G is not S-stable at e, and that neither (i) nor (ii) holds. It follows that $\Gamma(G-e) \not\leq \Gamma(G)$. Since (ii) does not hold, and $L(G)$ is semi-stable at e, it follows by the first part of Lemma 2.3 that no component of $H-e$ is isomorphic to a component of G. From this and the fact that $\Gamma(G-e) \not\leq \Gamma(G)$, it follows by the second part of Lemma 2.3 that $\Gamma(H-e) \not\leq \Gamma(H)$. Let $\theta \in \Gamma(H-e)-\Gamma(H)$. As (i) does not hold, θ induces a non-identity permutation $\alpha \in \Gamma_1(H-e)$. By hypothesis, $L(G)$ is semi-stable at e, so that by Lemma 2.3, $L(H)$ is semi-stable at e; therefore by Lemma 2.2, $\alpha \in \Gamma_1(H)_e$. It follows that $\alpha(e) \in \Gamma_1(H)$. If there exists ψ in $\Gamma(H)$ such that ψ induces $\alpha(e)$ in $\Gamma_1(H)$, then $\psi \in \Gamma(H-e)$ and ψ induces α in $\Gamma_1(H-e)$. Thus θ and ψ are elements of $\Gamma(H-e)$ which induce the same element α of $\Gamma_1(H-e)$. As $\Gamma(H-e) \not\leq \Gamma(H)$, it follows that H is not K_2. As (i) does not hold, we deduce by Lemma 2.1(b) that $\theta = \psi$, so that $\theta \in \Gamma(H)$, a contradiction. It follows that there is no ψ in $\Gamma(H)$ which induces $\alpha(e)$ in $\Gamma_1(H)$. By Lemma 2.1 (a) we deduce that H is one of the graphs G_3, G_4, G_5 shown in Figure 2.1. If H is G_3, then e must be (v_2, v_3) or (v_2, v_4), since $L(H)$ is semi-stable at e. If H is G_4, then e must be (v_2, v_4), since $L(H)$ is semi-stable at e. Moreover, H is not G_5, since we have seen that $\Gamma(H-e) \not\leq \Gamma(H)$. Thus if neither (i) nor (ii) holds, and G is not S-stable at e, then either (iii) or (iv) holds. This completes the proof.

THEOREM 3.2. *Suppose G is S-stable at e. Let H be the component of G which includes e. Then $L(G)$ is not semi-stable at e if and only if one of the following two statements is true*:

(i) *$H-e$ is disconnected and has a component isomorphic to the graph G_3 of Figure* 2.1, *whose vertex of degree* 1 *or* 3 *is incident with e*;

(ii) *$H-e$ is disconnected and has a component isomorphic to $K_{1,3}$ whose vertex of degree* 3 *is incident with e, and G has a component isomorphic to K_3*.

PROOF. If either (i) or (ii) is true, then by inspection, $\Gamma_1(G)_e \neq \Gamma_1(G-e)$, so that by Lemma 2.2 $L(G)$ is not semi-stable at e. Suppose that G is S-stable at e and $L(G)$ is not semi-stable at e. By Lemma 2.3, H is S-stable at e and either (a)

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$L(H)$ is not semi-stable at e or (b) $H-e$ has a component isomorphic to $K_{1,3}$ whose vertex of degree 3 is incident with e and G has a component isomorphic to K_3. If (b) is the case, then (ii) is true. Suppose, then that (a) is the case. If β is any element of $\Gamma(H-e)$, then $\beta \in \Gamma(H)$ since H is S-stable at e. It follows that β must map the set of vertices incident with e onto itself, and so must induce an element of $\Gamma_1(H)$ which fixes e. This element then restricts to an element of $\Gamma_1(H)_e$. Since $L(H)$ is not semi-stable at e, it follows that $\Gamma_1(H)_e \neq \Gamma_1(H-e)$. We conclude that not all elements of $\Gamma_1(H-e)$ are induced by elements of $\Gamma(H-e)$, that is $\Gamma_1(H-e) \neq \Gamma^*(H-e)$. We deduce from Lemma 2.1(a) that either (I) $H-e$ has both of the graphs G_1 and G_2 of Figure 2.1 as components or (II) $H-e$ has one of the graphs G_3, G_4, G_5 of Figure 2.1 as a component. In fact (I) cannot hold; if it did hold, then $H-e$ would be isomorphic to $K_3 \cup K_{1,3}$, and no feasible graph $H=(K_3 \cup K_{1,3})+e$ is S-stable at e. Thus (II) holds. Moreover, $H-e \not\cong G_3$ since H is S-stable at e, $H-e \not\cong G_4$ since (a) holds and obviously $H-e \not\cong G_5$. Hence $H-e$ is disconnected. Moreover, again because H is S-stable at e, and because G_4 and G_5 have no fixed vertices we conclude that $H-e$ has a component isomorphic to G_3 whose vertex of degree 1 or 3 is incident with e. In other words, (i) must hold.

4. The semi-stability of line graphs of trees and unicyclic graphs

In Sheehan (1973), the author found those trees and unicyclic graphs which are not S-stable at any edge. He proved

LEMMA 4.1. *If T is a tree which is not S-stable, then either T is P_n for some $n \geqq 4$, or is one of the trees E_7 or J shown in Figure* 4.1.

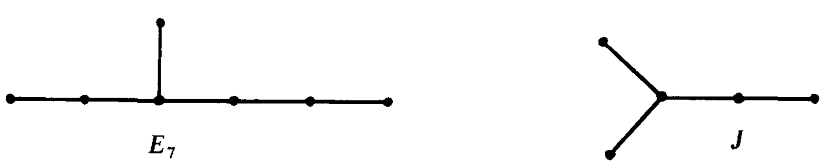

Figure 4.1

LEMMA 4.2. *The only unicyclic graphs which are not S-stable are those shown in Figure* 4.2.

We shall now proceed to find those trees and unicyclic graphs whose line graphs are not semi-stable.

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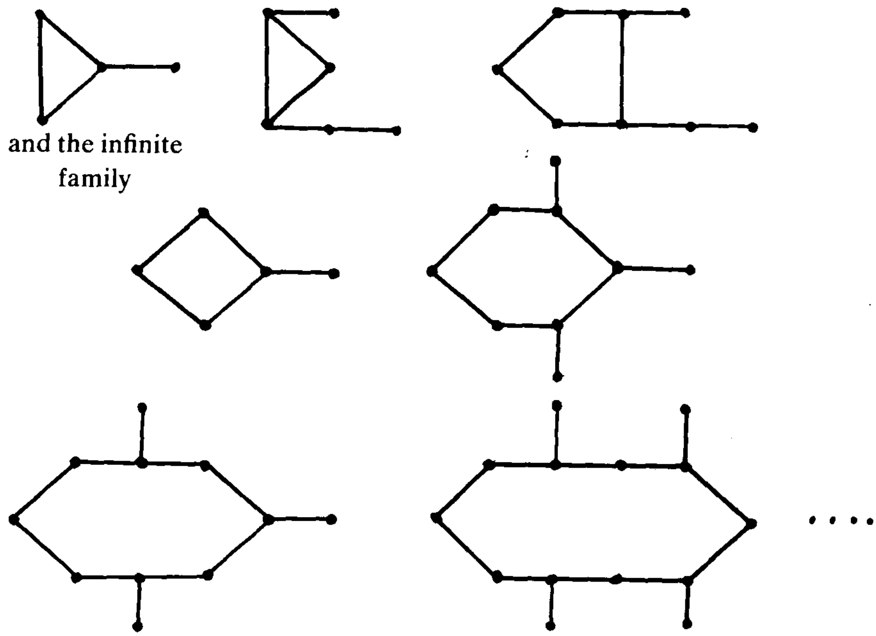

Figure 4.2

It is convenient to state here a result of Heffernan (1972) regarding the semi-stability of rooted trees.

LEMMA 4.3. *If T is rooted tree, with $|V(T)| \geqq 2$, then T is semi-stable at an end-vertex other than its root.*

As a final preliminary, we note the following lemma which helps us to decide whether or not a given connected graph is S-stable.

LEMMA 4.4. *Let G be a connected graph which is semi-stable at an end-vertex w. Let x be the vertex of G adjacent to w. Then G is S-stable at (w, x).*

PROOF. The result clearly holds if $G = K_2$. Thus assume $G \neq K_2$. $G-(w,x) = G_w \cup \langle\{w\}\rangle$, where G_w is connected, and $|V(G_w)| > 1$. Thus $\Gamma(G-(w,x))_w = \Gamma(G_w) = \Gamma(G)_w$ as G is semi-stable at w. As w is fixed by $\Gamma(G-(w,x))$, it follows that $\Gamma(G-(w,x)) \leqq \Gamma(G)$, so that G is S-stable at (w,x).

Our results on semi-stability of line graphs are:

THEOREM 4.5. *The only trees whose line graphs are not semi-stable are P_n for $n \geqq 5$ and E_7.*

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PROOF. (i) Let T be a tree whose line graph is not semi-stable. Suppose that T is S-stable at e. As $L(T)$ is not semi-stable, it is not semi-stable at e. By Theorem 3.2 it follows that either $T-e$ must have a component isomorphic to G_3 or T must have a component isomorphic to K_3, a contradiction as T is acyclic. Thus T is not S-stable. By Lemma 4.1, T is P_n for some $n \geqq 4$ or is E_7. However $L(P_4)$ is semi-stable, so that in fact T is P_n for some $n \geqq 5$ or is E_7.

(ii) By inspection, if $n \geqq 5$, $L(P_n)$ is not semi-stable, and $L(E_7)$ is not semi-stable.

THEOREM 4.6. *The only unicyclic graphs whose line graphs are not semi-stable are the graphs shown in Figure* 4.2, *with the exception of the first of these graphs.*

PROOF. (i) Let U be a unicyclic graph whose line graph is not semi-stable. Suppose U is S-stable at e. Since $L(U)$ is not semi-stable, it is not semi-stable at e. By Theorem 3.2, bearing in mind that U is unicyclic, it follows that U is of the form $(G_3 \cup M)+e$, where M is a tree and e is incident with either v_1 or $v_2 \in V(G_3)$ (see Figure 2.1) and a vertex v of M. We may consider U to consist of the cycle C with $V(C)=\{v_2, v_3, v_4\}$, at the vertex v_2 of which is adjoined a rooted tree which has at least 3 vertices. By Lemma 4.3, this rooted tree is semi-stable at an end-vertex $w(\neq v_2)$, which is adjacent to a vertex x of U, say. It follows that U is also semi-stable at w. By Lemma 4.4, U is S-stable at (w,x). As $L(U)$ is not semi-stable, we deduce from Theorem 3.2 that the component of $U-(w,x)$ other than $\langle\{w\}\rangle$ must be G_3, with either $v_1 = x$ or $v_2 = x$ Thus U is one of the graphs U_1, U_2 shown in Figure 4.3. However in these cases, $L(U)$ is semi-stable (at the vertex representing edge g in both cases). This is a contradiction. It follows that U cannot be S-stable. Therefore, by Lemma 4.2, U is one of the graphs shown in Figure 4.2. Moreover, U cannot be the first of these graphs because $L(U)$ is by hypothesis not semi-stable.

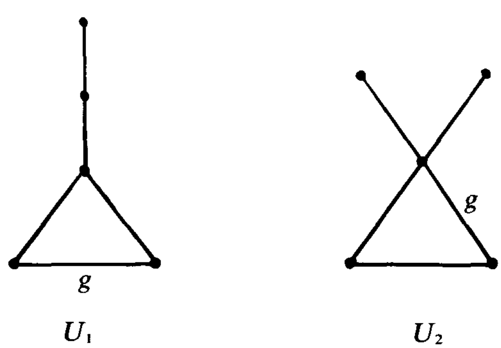

Figure 4.3

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(ii) If U is one of the graphs shown in Figure 4.2, with the exception of the first of these graphs, then $L(U)$ is not semi-stable, this fact falling out of Sheehan's proof of Lemma 4.2, (Sheehan (1972)), together with Lemma 2.2.

5. Stable line graphs

In this section, we investigate the problem of finding which line graphs are stable. Our results all have proofs which rely heavily on the structural implications of a graph being stable.

First of all, it is easy to see that the following is true.

LEMMA 5.1. *If $L(G)$ is stable, then either $|E(G)| = 1$ or $\Gamma_1(G)$ contains a transposition.*

PROOF. Suppose $L(G)$ is stable, and suppose $|V(L(G))| = |E(G)| > 1$. Then $L(G)$ has a partial stabilising sequence S of cardinality $|E(G)| - 2$. Suppose $E(G) - S = \{e_1, e_2\}$. Then $L(G)_S$ is isomorphic either to K_2 or $2K_1$, and $\Gamma(L(G)_S)$ contains the transposition (e_1e_2). As S is a partial stabilising sequence for $L(G)$, it follows that $(e_1e_2) \in \Gamma(L(G))_S$, so that $\Gamma(L(G)) = \Gamma_1(G)$ contains a transposition.

To help us apply Lemma 5.1, we note the following structural interpretation of the possession of a transposition by $\Gamma_1(G)$.

LEMMA 5.2. *Let e and f be edges of a graph G. Then $(ef) \in \Gamma_1(G)$ if and only if*

(i) *in the case that $e \sim f$ in G, $\overline{N_G}[e] = \overline{N_G}[f]$*

(ii) *in the case that $e \not\sim f$ in G, $N_G[e] = N_G[f]$.*

We can now deduce the following result, which is more explicit than Lemma 5.1.

THEOREM 5.3. *If $L(G)$ is stable, then either*

(i) $|E(G)| = 1$, *or*

(ii) *some component of G is P_4, C_4, K_4 or one of the graphs G_3, G_4 of Figure 2.1, or*

(iii) *two components of G are K_2, or*

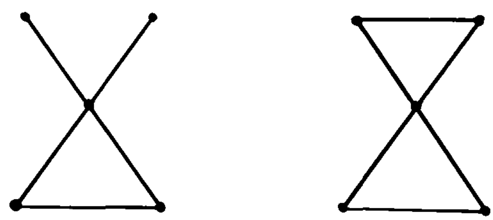
(iv) *G has either an end-block isomorphic to K_3 or two intersecting end-blocks both isomorphic to K_2.*

PROOF. By Lemma 5.1, either $|E(G)| = 1$ or $\Gamma_1(G)$ contains a transposition. In the former case (i) holds. Thus assume that $\Gamma_1(G)$ contains a transposition, say (ef). Suppose first of all that $e \not\sim f$ in G. By Lemma 5.2, $N_G[e] = N_G[f]$. If $N_G[e] = N_G[f] = \varnothing$, then (iii) holds. If $N_G[e] = N_G[f] \neq \varnothing$, then clearly the component of G containing e and f is P_4, C_4, K_4, G_3 or G_4 and so (ii) holds. Now suppose that $e \sim f$ in G. Then $e = (u, v)$, $f = (u, w)$, where u, v, w are distinct vertices. By Lemma 5.2, $\overline{N_G}[e] = \overline{N_{\bar{G}}}[f]$, and so clearly this set must either consist entirely of edges incident with u or be of the form $F \cup \{(v, w)\}$ where F is a set of edges incident with u. In the former case, G has two intersecting end-blocks isomorphic to K_2 and, in the latter case, G has an end-block isomorphic to K_3. Therefore (iv) holds.

COROLLARY. *Let G be a connected graph such that $L(G)$ is stable. Then either* (i) *G is K_2, P_4, C_4, K_4, G_3 or G_4, or* (ii) *G is separable and has either an end-block isomorphic to K_3 or two intersecting end-blocks both isomorphic to K_2.*

We can now deduce the following interesting theorem which was mentioned in the introduction.

THEOREM 5.4. *$L(G)$ and $L(\bar{G})$ are both stable if and only if G is $K_2, P_3, P_4, K_3, K_4, C_4, K_{1,3}, K_{1,4}, G_3, G_4$, one of the graphs G_6, G_7, shown in Figure* 5.1, *or the component of one of these graphs.*



G_6 G_7

Figure 5.1

PROOF. (i) By inspection, the graphs listed above and their complements all have stable line graphs.

(ii) Now suppose that both $L(G)$ and $(L(\bar{G})$ are stable. As at least one of $G, \bar{G}$ is connected, we may suppose, without loss of generality, that G is connected. By (i), and Theorem 5.3, Corollary, we need only consider the cases where G has (a) two intersecting end-blocks, both isomorphic to K_2 or (b) an end-block isomorphic to K_3.

(a). Assume first of all that $u, v, w \in V(G)$ are such that $\langle\{u, v\}\rangle$ and $\langle\{u, w\}\rangle$ are two end-blocks of G both isomorphic to K_2. If $|V(G)| \leqq 4$, then G is P_3 or $K_{1,3}$. Suppose, then, that $|V(G)| \geqq 5$. For each $x \in V(G) - \{u, v, w\}$, $\bar{G}$ contains a circuit with vertices x, v, w. It follows that all vertices in $V(G) - \{u\}$ belong to the same block (H, say), of $\bar{G}$. Since $L(\bar{G})$ is stable, Theorem 5.3 implies that $\bar{G}$ must have a component or block with 2 or 3 vertices or a

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component with 4 vertices. Hence H must be a component of $\bar{G}$ such that $|V(H)| = 4 < |V(G)|$. Therefore $V(H) = V(G) - \{u\}$ and u is adjacent in G to all four vertices in $V(H)$. Since $\langle\{u, v\}\rangle$ and $\langle\{u, w\}\rangle$ are end-blocks of G, it follows that G is $K_{1,4}$ or G_6.

(b) Now assume that there exist $u, v, w \in V(G)$ such that $\langle\{u, v, w\}\rangle$ is an end-block of G isomorphic to K_3, and that if $|V(G)| > 3$, u is a cutvertex of G. If $|V(G)| \leqq 4$, then G is K_3 or G_3. Suppose, then, that $|V(G)| \geqq 5$. For each pair of vertices $x, y \in V(G) - \{u, v, w\}$, $\bar{G}$ contains a circuit with vertices x, y, v, w. If follows that all vertices in $V(G) - \{u\}$ belong to the same block (H, say), of $\bar{G}$. As in (a), H is then a component of $\bar{G}$ with 4 vertices, and u is adjacent in G to all vertices in $V(H)$. As $\langle\{u, v, w\}\rangle$ is an end-block of G, it follows that G is G_6 or G_7. This completes the proof of the theorem.

COROLLARY. *If G and $\bar{G}$ are both connected, and $L(G)$ and $L(\bar{G})$ are both stable, then G is P_4. In particular, the only self-complementary graph whose line graph is stable is P_4.*

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