NON-AXISYMMETRIC MAGNETIC STRUCTURE GENERATION IN PLANETS SUN AND GALAXIES

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Abstract. The problem of preferential excitation of non-axially symmetric magnetic fields in most of the observed spiral galaxies and in planets as Uranus and Neptune is studied. It is suggested that a dynamo acting in a thin conductive shell is possible in these objects.
It is shown that in this thin shell axisymmetric and non-axisymmetric magnetic field modes can be excited with equal efficiency. The preferential excitation of non-axially symmetric magnetic fields is possible, when the gradient of the angular velocity of rotation is comparatively small and has an essential component perpendicular to the shell.

1. Introduction

The problem of the excitation of planetary and galactic non-axially symmetric magnetic structure is not yet resolved in the MHD dynamo theory. The well known theoretical models (Parker 1979; Krause and Rädler 1980; Ruzmaikin et al. 1988a) give a good explanation of a preferential generation of mean axisymmetric magnetic fields in the Earth, Jupiter, Saturn and some of the spiral galaxies. But they can not give satisfactory explanation of the generation of strong non-axisymmetric structures. Nevertheless, such structures were recently detected in some nearby spiral galaxies. On the Sun one can see strong non-axisymmetric structures in the form of the solar sector structure, corona holes and active longitude. Also, non-axisymmetric magnetic structures dominate over axisymmetric ones in the magnetic fields of Uranus and Neptune. A magnetic dipole is inclined to the rotation axis by 58^o for Uranus and 47^o for Neptune (Ness et al. 1986, 1989).

In this work conditions for the excitation of non-axiallly symmetric mean magnetic fields are studied. The special attention is paid to the conditions for a preferential exitation on non-axially symmetric magnetic fields, when non-axisymmetric magnetic structures can dominate over axisymmetric ones.

The problem is solved in approximation of a strong generation and for an $\alpha\omega$-dynamo, where the differential rotation Ω dominates over the mean helicity α.

2. Approximation of a strong generation

Let us measure the values of the main generation sources by their characteristic values: mean helicity α by α_* and differential rotation Ω by Ω_*. A turbulent magnetic diffusivity η is opposed to the generation. So, the less η/α_* and η/Ω_* are, the stronger the generation is. The one third power of the product of these two terms has length dimension. And for a strong generation

$$\varepsilon = \left(\eta^2/\alpha_*\Omega_*\right)^{1/3}/r_*$$

is a small parameter. Here r_* - is a characteristic scale of the problem. Also, one can introduce ε as the cubical root from the inverse dynamo-number. For the successful generation the intensity of it must be large enough. Otherwise, ε must be smaller

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than a certain small threshold value. The existence of the mean magnetic fields in planets and galaxies is the reason for using the approximation of a strong generation, when $\varepsilon \ll 1$. Really, ε is not so small, about $10^{-1} - 10^{-2}$ in order, but it is very convinient to use the approximation of a strong generation. In this approximation we can obtain general solutions of dynamo problems in a very simple form.

Three dimensional values α_*, Ω_* and η form the dimensionless parameter $q = (\Omega_* \eta/\alpha^2)^{1/3}$, which is not dependent from the typical size of the problem r_*. This parameter determines the generation regime. When q tends to zero, it is α^2-dynamo generation, and when q tends to infinity, it is pure $\alpha\omega$-dynamo. Here we examine only the case of $\alpha\omega$-dynamo, we do not take into consideration the generation of the toroidal mean magnetic field by the α-effect.

In the approximation of strong generation, the equation for the generation of a large-scale magnetic field $\mathbf{B}$ can be written in dimensionless form:

$$\frac{\partial \mathbf{B}}{\partial t} + q\frac{\partial \mathbf{B}}{\partial \phi} = qr\sin\theta(\nabla\Omega \cdot \mathbf{B})\hat{\phi} + \frac{\varepsilon}{q}\nabla \times (\alpha\mathbf{B}) + \varepsilon^2 \triangle \mathbf{B}. \tag{1}$$

Here (r, θ, ϕ) is a spherical coordinate system, and $\varepsilon^2 r_*^2/\eta$ is chosen as the unit time.

3. Local asymptotic solution

We consider the magnetic field generated by the given flow. The generation sources are assumed to be axisymmetrically distributed . In this case time and azimuthal angle ϕ do not enter explicitly the dynamo equation and the asymptotic solution can be represented as

$$\mathbf{B} = \mathbf{F}e^{im\phi + \gamma t + iS/\varepsilon} \tag{2}$$

The physical magnetic field is given by the real part of this complex quantity. The complex constant γ is an eigenvalue dependent both on ε and q. A slowly varing function $\mathbf{F}$ and the eigenvalue γ are explanded in series in ε:

$$\mathbf{F} = \mathbf{F}_0 + \varepsilon\mathbf{F}_1 + ..., \quad \gamma = \gamma_0 + \varepsilon\gamma_1 + \ldots . \tag{3}$$

The asymptotic forms, which we use below are similar to WKB asymptotics for differential equations with a small parameter at the highest-order derivative (Maslov and Fedorjuk, 1981). When the generation is strong, the generated magnetic field is strongly concentrated in space: the typical dimensionless scale is of order ε. Considering highly concentrated field distributions, we replace the boundary conditions by a requirement for the field to decay at infinity. And for this reason we restrict ourselves by using the Taylor expansions for the functions S and $\mathbf{F}$ near the generation maximum, which is near the extremum of the local dynamo number $D = r\sin\theta\alpha|\nabla\Omega|$.

Substituing Eq. (2-3) into Eq. (1), and equating the coefficients of equal powers of ε, we obtain equations for the first, second etc., approximations. Solving these equations consequently, we determine the functions $S, \mathbf{F}_0, F_1 ...$ and the numbers γ_0, γ_1 ... . For our purposes, it suffices to determine the magnetic field to the principal order in ε, and the eigenvalue γ to the second order. The appropriate equations were solved approximately for galaxies with the Brandt rotation curve (Starchenko and Shukurov, 1989); for the Sun (Ruzmaikin et al., 1988b) and for Uranus (Ruzmaikin and Starchenko, 1989) with taking into consideration only the radial gradient of angular rotation rate. Here we shall use these solution, but without any restriction on the form of the angular rotation rate.

4. The excitation conditions

This formula

$$Re\gamma = \frac{3}{8} - 16P^2\,(1 + (8B - \sqrt{3})^2)^{-1} -$$

$$\left[\frac{R^{-1}}{4} \cdot \sqrt{1 + \sqrt{1 + (8B - \sqrt{3})^2}} + \frac{3}{4\sqrt{2}}\right] z_1^{-1}\varepsilon \quad , \tag{4}$$

where $P \equiv mq\rho_1$, $B \equiv P\rho_1\Omega''$, and $R \equiv \rho_1/z_1$, represents the two lowest approximations to the real part of the eigenvalue for the magnetic mode that grows at the greatest rate for a given azimuthal number m. Here P, B and R are the main parameters controlling excitation of non-axially symmetric magnetic fields. These parameters are determined by quantities, which are calculated at the extremum of the local dynamo-number D: $\rho_1 \equiv \left(-D/\partial^2 D/\partial\rho^2\right)^{1/2}$ and $z_1 \equiv \left(-D/\partial^2 D/\partial z^2\right)^{1/2}$ are the characteristic half-width of the extremum of D along (ρ) and perpendicular (z) to the gradient of the angular rotation rate; $\Omega'' \equiv |\partial^2\Omega/\partial\rho^2|/|\partial\Omega/\partial\rho|$ is the typical change of the gradient of angular rotation rate. The distance from the axis of rotation to the extremum of D is taken as a characteristic scale r_*.

It is first of all for the excitation of non-axially symmetric magnetic fields necessary that the main order (ε^0) of the expression (4) must be greater then zero. This is always true when P is less than about 0.15. This is only possible for the thin shell along gradient of the angular rotation rate, because ρ_1 is about the typical half-thickness of the generation shell in this direction and q, m are greater or equal one in our approximation.

More stronger restriction is necessary for the preferential excitation of non-axially symmetric magnetic structure ($m > 0$) over axisymmetric ($m = 0$) ones.

On figure 1 one can see the region of the parameters in which preferential excitation is possible.

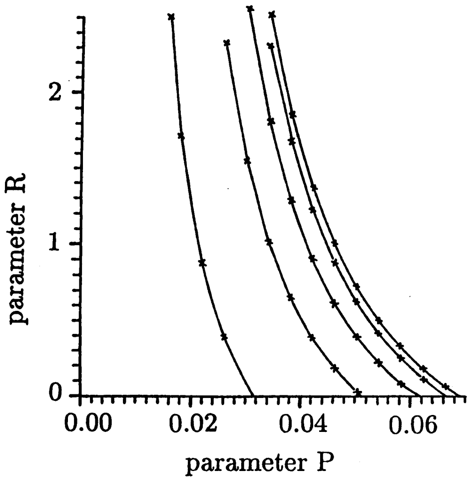

Figure 1. Each curve for fixed B (0.01, 0.03, 0.05, 0.07, and 0.29) and axes restrict the region of parameters in which the preferential excitation of non-axisymmetric magnetic structure over axisymmetric ones is possible.

The main restriction is imposed by the parameter P, which must be less than about 0.07. Besides, the change of the angular rotation rate gradient is necessary. Otherwise, parameter B is equal zero and the preferential excitation is impossible.

The main conclusion is that the excitation and especially the preferential excitation of non-axially symmetric magnetic structure is possible only in thin shell (less or of order 0.05 of typical scale), which is along the typical gradient of angular rotation rate. One can directly impose this restriction on the kinematic dynamo equation and construct the general solution in the approximation of thin generation shell. In this way the first general conclusion is that axisymmetric and non-axisymmetric magnetic fields modes can be excited with equal efficiency. For more details see (Ruzmaikin and Starchenko 1991).

5. Conclusion

In conclusion let us examine the conditions necessary for the generation of strong non-axisymmetric structures in different astrophysical objects. We should consider that the presence of such structures in real non-linear dynamo systems is possible only when the exitation of non-axisymmetric structures in the conditions of kinematic dynamo predominate. Besides, let us accept that the obtained local-asympotic solution (4) is suitable for the real situation too. We should note only that to prove the last thesis and to construct more detailed dynamo-models, detailed numerical investigations are necessary.

Let us start with strong non-axisymmetric galactic magnetic structures. For spiral galaxies the characteristic size along the axis of rotation is ten times less than in perpendicular direction along the radius (Ruzmaikin et al. 1988). When the characteristic gradient of the angular rotation rate along the axis is in the

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order of tenth or more than the radial gradient, the characteristic dimensionless half-thickness ρ_1 may be rather small (less than 0.07). Under such conditions the predominand excitation of non-axisymmetric magnetic structures is possible, as a kind of bisymmetric ($m = 1$) or more complex formation.

The observed data prove the presence of a rather strong change of the angular rotation rate along the axis. However, probably for the first time, the presence of such a gradient is used for the subtantiation of the preferred generation of a non-axisymmetric galactic magnetic structure.

The generation of the Sun's magnetic field happens in a rather thin shell. Modern helioseismological data (Proc. 1981) allow to divide additionally this zone of generation into two parts. In one of them, which is closer to the surface, the latitude gradient of angular rotation rate predominates. In the other one, near the bottom of the convection zone, the radial gradient predominates. Probably, in the latter one the generation of strong non-axisymmetric magnetic structures happens, and forms the corona holes and active longitudes.

The investigations of the inner structure of such planets as Mars, Earth, Mercury, Jupiter and Saturn (Habbard 1984) gives evidence that the generation of magnetic fields in this planets occur in rather stretched regions where $\rho_1 \approx 0.1$. Thus in these planets the axisymmetric magnetic structures dominate. On the contrary, in Uranus and Neptun the generation of magnetic field is possible in a comparatively narrow convective conducting shell. Perhaps, this shell is so thin that the dominating excitation of non-axisymmetric magnetic structures is possible.

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