THE EQUALITY OF A MANIFOLD'S RANK AND DIMENSION

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T. J. Willmore has shown that if a differentiable manifold's rank (the maximum number of everywhere linearly independent commuting vector fields definable on it) equals the manifold's dimension, then the manifold is a torus of the appropriate dimension [1]. This theorem is proved more simply and without any differentiability hypothesis in the present note.

THEOREM. If M^n is a compact connected Hausdorff manifold of dimension n admitting a locally free action of R^n, then M^n is homeomorphic to an n-torus.

Proof. Let (M^n, R^n, π) be a topological transformation group (see [2] for terms), the action of R^n be locally free, and x be a point of M^n. Since the action is locally free, the orbit of x, xR^n, is an open set. Either $xR^n = M^n$ or it is a proper subset of M^n. In the latter case, xR^n has a boundary. Now the boundary is at most (n-1)-dimensional, but it contains the (open) orbit of each of its points. This is a contradiction and shows that $xR^n = M^n$. By [2; 3.09], x is a periodic point; but the local freedom of the action implies that the period of x is discrete. The period is therefore of the form Z^n, the only discrete syndetic subgroup of R^n, and M^n is homeomorphic to R^n/Z^n by [2; 3.08].

REFERENCES

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