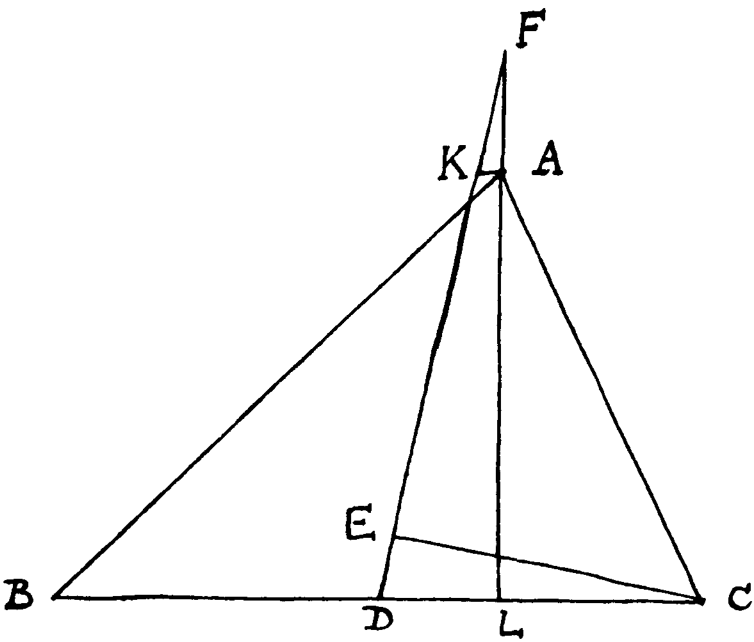
Note I.—Hero's Formula and Construction of Triangles.

1. ABC is a triangle ($AB > AC$); D the middle point of BC and AL perpendicular to BC. A point F is taken in LA produced such that $DF = \dfrac{b+c}{2}$ and AK drawn parallel to BC to meet DF at K. CE is drawn perpendicular to DF.



Now, since F, E, L, C are concyclic,

$$DE \, . \, DF = DC \, . \, DL = \frac{BL^2 - LC^2}{4} = \frac{c^2 - b^2}{4}$$

$$\therefore \quad DE = \frac{c-b}{2}$$

Hence $EF = AC$ and $EC = \sqrt{(s-b)(s-c)}$ .... (i)

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Again, from similar triangles,

$$\frac{AL^2}{DK^2}=\frac{FL^2}{DF^2}=\frac{CE^2}{CD^2}=\frac{FL^2-CE^2}{DF^2-CD^2}=\frac{FE^2-LC^2}{DF^2-CD^2}$$

$$=\frac{AC^2-LC^2}{DF^2-CD^2}$$

$$\left.\begin{aligned}\therefore\quad DK^2&=DF^2-CD^2\\ i.e.,\ DK&=\sqrt{s(s-a)}\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ldots\ldots\quad \text{(ii)}$$

From (i) and (ii) it is clear that

$$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$$

2. $$AD^2+CD^2=\frac{b^2+c^2}{2}=DE^2+DF^2$$

$$\therefore\quad DK^2=DF^2-CD^2=AD^2-DE^2$$

3. The above analysis enables us to devise simple geometrical constructions for triangles, given the sum or difference of two sides and any two of the three lengths: the base, the altitude perpendicular to the base, and the median bisecting the base.

Note II.—The Ambiguous Case.

The following two converses of the Ambiguous Case are noteworthy :—

Converse (1) If two triangles have two sides of the one equal to two sides of the other, each to each, and the angles opposite to one pair of equal sides supplementary, then the angles opposite to the other pair are equal.

The case of congruence of triangles which agree as to two sides and a right angle not included, is a particular case of the above.

The corresponding theorem in similarity may be enunciated thus :

Theorem: If two triangles have two sides of the one proportional to two sides of the other and the angles opposite to one pair of corresponding sides supplementary, then the angles opposite to the other pair are equal.

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