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RESEARCH ARTICLE

Optimal Forest Management of Even-Aged Longleaf Pine Stands with Nontimber Benefits

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Abstract

We present an optimal control model to simultaneously determine the optimal planting density, thinning schedules, harvest age, and revenues of an even-aged longleaf pine (*Pinus palustris* Mill) stand, an iconic species in the Southeastern United States. We assume that the forest stand is managed for timber production and carbon sequestration under different site indexes—a measurement of potential forest productivity. Our simulation results show that the optimal planting density tends to increase when longleaf pine is managed in medium and high site indexes. Furthermore, the optimal harvest age tends to be extended with payments for carbon sequestration.

Keywords: land expectation value; longleaf pine; optimal control theory; planting density; thinning schedules

JEL classifications: Q23; Q57

1. Introduction

Longleaf pine (*Pinus palustris* Mill.) is an iconic tree species in the Southeastern United States (US). Longleaf pine ecosystems are critical for the biological diversity of the region by providing habitats for several endangered species, such as the red-cockaded woodpecker, gopher tortoise, and Florida pine snake, and by hosting more than 100 vascular endemic plants (Kirkman and Myers, 2018). Longleaf pine forests can produce higher-quality forest products and provide higher rates of carbon sequestration compared to other southern pines (McNulty et al., 2018; Raut et al., 2022). Furthermore, longleaf pine forests are more resilient to natural disturbances and fluctuating climatic conditions (Koontz et al., 2020; Sharma et al., 2020).

Despite the critical role that longleaf pine forests play in terms of ecological and economic benefits to the region, longleaf pine forests have been reduced to 3% of their historical area (1.7 million hectares) (Kirkman, Jack, and McIntyre, 2018). Some reasons for this decline were economical. The depletion of the northern pine forests and the adoption of rail technology after the Civil war meant that the lumber industry moved to the Southeast, where land was treated as an expendable commodity and longleaf pine forests were overharvested (Kirkman, Jack, and McIntyre, 2018). Furthermore, with a lack of technology to successfully regenerate longleaf pine forests after harvest, forest landowners considered forest management as inferior to other capital investments, switching the use of the land for agriculture and urbanization (Kirkman, Jack, and McIntyre, 2018). Other southern pines such as slash pine and loblolly pine also adapted to former longleaf pine sites and with their more rapid growth, became more profitable options for landowners in light of the nascent pulp and paper industry in the 20th century (McIntyre, McCall, and Wear, 2018). Additionally, the broad scale of exclusion of prescribed fires as a management tool in all US forests negatively impacted longleaf pine reproduction but benefited the growth of newly regenerated slash and loblolly pine (Jack and Pecot, 2018; McIntyre, McCall, and Wear, 2018).

Given the ongoing decline of this species, a diverse group of organizations interested in restoring longleaf pine formed the America's Longleaf Restoration Initiative in 2007. The objective of this initiative is to increase the longleaf pine area to 3.2 million hectares by 2025 and improve current conditions of existing longleaf pine forests for habitat conservation and wildlife (America's Longleaf, 2020). A main drawback for forest landowners to engage in such initiatives is the lack of economic management studies related to the optimal management of longleaf pine (Susaeta, Gong, and Adams, 2020). Only a few studies have determined the optimal forest management of even-aged longleaf pine forests considering the production of timber and forest amenities (Susaeta and Gong, 2019; Susaeta, Gong, and Adams, 2020; Stainback and Alavalapati, 2004). These studies have ascertained the optimal harvest age to regenerate the longleaf pine stand and the associated economic revenues, assuming a predetermined thinning program and a fixed initial planting density.

Some studies have analyzed the interdependence between optimal levels of planting density, thinning schedules, and harvest age in other southern species. Chang (1983) found unclear results between optimal planting density and harvest age of unthinned loblolly pine given their interactions with economic factors. When including financial risk, Taylor and Forston (1991) found that landowners should select high planting densities with long rotation ages for unthinned loblolly pine to maximize economic returns. Cao et al. (2006) determined that the number of thinnings increased and the harvest age of Norway spruce was longer with high initial planting density. Coordes (2014) postulated that the harvest age impacts the optimal thinning regimes for loblolly pine, but the conditions for the optimal harvest age cannot be generalized. Coordes (2013) found that the optimal planting density cannot be determined independently from the optimal harvest age for unthinned Norway spruce stands. Halbritter and Deegen (2015) developed a theoretical model for even-aged age stands to simultaneously determine the optimal planting density, thinning regimes, and length of the rotation period.

Despite these efforts, information about the interrelationships between the number of trees to be planted, and at what age longleaf pine forests should be thinned and harvested, are practically nonexistent. Similar to Cawrse, Betters, and Kent (1984), Clark and De Pree (1979), and Halbritter and Deegen (2015), we present an optimal control model in which optimal planting density, thinning schedules, and harvest age of even-aged longleaf pine stands are simultaneously determined for different levels of forest site productivity. Unlike these previous studies, we assume forest stands can also generate nontimber benefits. The rest of this paper is as follows. First, we lay out the theoretical model and the optimal conditions of thinning schedules, harvest age, and timber stock, considering timber and nontimber benefits. Second, we present the timber stock and carbon sequestration functions of longleaf pine, the economic parameters, and apply the model to a forest stand using carbon sequestration as an example of nontimber benefits. Third, we present the optimal planting density, thinning programs, harvest age, and economic returns for different site indices. Furthermore, we analyze the impact of changes of the discount rate and the rule of thinning on the optimal forest management of longleaf pine. Finally, we discuss the results and offer concluding remarks of our study.

2. Model Formulation

Let 'us define $V(t)$ as the timber stock of a longleaf pine stand at age t. The timber stock growth can be modeled as follows (Cawrse, Betters, and Kent, 1984; Clark and De Pree, 1979):

$$\frac{dV(t)}{dt} = g(t,\ V(t)) - h(t) \tag{1}$$

where $g(t,V(t))$ represents the annual volume increment dependent on timber stock and stand age t, and $h(t)$ and represents the thinning volume at age t, $0 \leq h(t) \leq V(t)$. We assume that the timber stock is a concave function with respect to time t, that is, $\partial V(t)/\partial t > 0$, $\partial^2 V(t)/\partial^2 t < 0$. Furthermore, we assume that the increment function is a positive decreasing function of stand age, $\partial g(t)/\partial t < 0$, and a concave positive function with respect to $V(t)$ with a unique maximum timber stock $V(t) = \hat{V}(t)$, such that $\partial g(t)/V(t) > 0$ when $V(t) < \hat{V}(t)$, $\partial g(t)/V(t) < 0$ when $V(t) > \hat{V}(t)$, and $\partial g(t)/V(t) = 0$ when $V(t) = \hat{V}(t)$.

We assume that the longleaf pine stand generates a flow of continuous forest amenities $f(t,V(t))$ that are increasing and concave in forest stock $V(t)$ and stand age t, such that $\partial f(t,V(t))/\partial t > 0$, $\partial^2 f(t,V(t))/\partial^2 t < 0$, $\partial f(t,V(t))/\partial V(t) > 0$, and $\partial^2 f(t,V(t))/\partial^2 V(t) < 0$. Furthermore, we denote V_0 ($V_0 > 0$) and $C(V_0)$ as the initial timber stock (volume) and regeneration costs. We assume that a forest landowner, starting from bare land, grows an even-aged longleaf pine stand for timber production and forest amenities. Once the forest stand reaches the harvest age T, the stand is clear-cut and immediately regenerated. The same process continues *ad infinitum*. The forest landowner's objective is thus to find the optimal initial timber volume $V_0{}^*$, thinning schedules $h^*(t)$, and harvest age T^* that maximizes the net present value of timber and nontimber benefits over perpetual forest rotations, also known as the land expectation value L. By defining $p(t)$ and r as the timber price at stand age t and discount rate, respectively, the L can be modeled as follows:

$$\max_{h(t),T,V_0} L = (1-e^{-rT})^{-1}\left[-C(V_0) + p(T)V(T)e^{-rT} + \int_0^T p(t)h(t)e^{-rt}dt + \int_0^T f(t,V(t))e^{-rt}dt\right] \tag{2}$$

s.t

$$\frac{dV(t)}{dt} = g(t,V(t)) - h(t)$$

$$0 \leq h(t) \leq V(t)$$

$$V_0 > 0, V(t) \geq 0$$

$$0 < t \leq T$$

The dynamic harvest optimal control problem present in equation (2) can be solved by using the Pontryagin's maximum principle, where $h(t)$ is the control variable and $V(t)$ is the state variable. Thus, the Hamiltonian H for this problem can be defined as:

$$H = (1-e^{-rt})^{-1}\left[p(t)h(t)e^{-rt} + f(t,V(t))e^{-rt}\right] + \lambda[g(t,V(t)) - h(t)] \tag{3}$$

where λ is the co-state variable or shadow price of the state variable $V(t)$. The conditions for the optimal thinning path are as follows:

$$\frac{\partial H}{\partial h(t)} = 0; \frac{\partial H}{\partial V(t)} = -\lambda'; \frac{\partial H}{\partial \lambda} = V'(t); V'(t) = g(t,V(t)) - h(t) \tag{4}$$

By solving the equations system presented in equation (4), we obtain the optimal condition for the timber stock path $V^*(t)$:

$$\frac{\partial g(t,V(t))}{\partial V(t)} = r - \left[\frac{p'(t) + f'(t,V(t))}{p(t)}\right] \tag{5}$$

Equation (5) differs from the optimal timber stock path obtained by Clark and De Pree (1979) due to the inclusion of the nontimber benefits in the ratio of total marginal benefits to timber price. The optimal path of timber stock $V^*(t)$ can be re-written as follows:

$$p(t)\frac{\partial g(t,V(t))}{\partial V(t)} + p'(t) + f'(t,V(t)) = rp(t) \tag{6}$$

In equation (6), the left-hand side represents the marginal value of increasing the timber stock given changes in growth and price and nontimber increments, while the right-hand side represents the marginal capital cost of an increment of the timber stock. The optimal timber stock path is achieved when both marginal revenue and marginal cost are equal. With the optimal timber stock path, the optimal thinning schedule $h^*(t)$ can be expressed as follows

$$h^*(t) = \begin{cases} 0 \quad \text{if } V(t) < V * (t) \\ V(t) - V^*(t) \quad \text{if } V(t) \geq V^*(t) \end{cases} \tag{7}$$

To determine the optimal harvest age T^*, the necessary condition is to maximize equation (2) such that $\frac{\partial L}{\partial T} = 0$. Thus, we have

$$p(t)g(t,V(t)) + p'(t)V(t) + f(t,V(t)) = rp(t)V(t) + rL \tag{8}$$

Equation (8) represents the optimal rule of harvest: the marginal revenue of delaying the harvest given marginal increases in timber price and timber stock plus nontimber benefits must equal the marginal costs of delaying the harvest given the capital costs of the timber benefits and the value of the land. Finally, we determine the optimal initial timber stock V_0^* in the same lines as Halbritter and Deegen (2015). We assume that the optimal harvest age and timber stock depend on the initial timber stock V_0, that is, $T^*(V_0)$ and $V^*(t,V_0)$. Furthermore, we define $t_1(V_0)$ as the time of the first thinning as a function of V_0. Equation (2) can be re-written as:

$$L = \left(1 - e^{-rT^*(V_0)}\right)^{-1}\Big[-C(V_0) + p(T^*)V^*(T^*(V_0),V_0)e^{-rT^*(V_0)} + \int_{t_1(V_0)}^{T^*(V_0)} p(t)h^*(t,V_0)e^{-rt}dt + \int_0^{T^*(V_0)} f(t,V^*(t,V_0))e^{-rt}dt\Big] \tag{9}$$

The optimal initial timber stock V_0^* can be found by maximizing equation (9) with respect to V_0, that is, $\partial L/\partial V_0 = 0$. Also, $T = T(V_0)$ and $t_1 = t_1(V_0)$. Thus,

$$\frac{\partial C(V_0)}{\partial V_0} = -e^{-rt_1}p(t_1)h^*(t_1,V_0)\frac{\partial t_1}{\partial V_0} + \int_{t_1}^{T} e^{-rt}\frac{\partial p(t)h^*(t_1,V_0)}{\partial V_0}dt + e^{-rt}\frac{\partial p(T)V^*(T,V_0)}{\partial V_0} + e^{-rT}f(T,V^*(T,V_0))\frac{\partial T}{\partial V_0} + \int_0^{T} e^{-rt}\frac{\partial f(t,V^*(t,V_0))}{\partial V_0}dt \tag{10}$$

Equation (10) represents a more general solution for the optimal initial timber stock V_0^* obtained by Halbritter and Deegen (2015) due to the inclusion of nontimber benefits. Here, V_0^* balances all the components of equation (10): (i) the positive changes in regeneration costs due to increases in the initial timber stock (left-hand side of equation (10)); (ii) the effect of changes in thinning benefits, the optimal level of timber stock V^*, and stumpage prices value (first, second, and third component on the right-hand side of equation (10)); and (iii) the impact of changes in nontimber benefits due to a change in the harvest age and the optimal marginal flow of nontimber benefits (fourth and fifth component on the right-hand side of equation (10)).

3. Model Specification and Data

We employ the growth and yield model developed by Gonzalez-Benecke et al. (2014) that generates estimates of commercial volume of the following forest products: sawtimber, chip-and-saw, and pulpwood. This model also allows for multiple thinnings. The functional form of the timber stock over bark ($m^3\,ha^{-1}$) is as follows:

$$V(t) = v(t)e^{\left(-1.0537\left(\frac{l}{\mathrm{qmd}}\right)^{4.2527} - 0.6546N^{-0.1356\left(\frac{d}{\mathrm{qmd}}\right)^{9.3108}}\right)} \tag{11}$$

$$v(t) = e^{\left(3.088 - 0.1943\,\mathrm{Ln}(N) + 1.2380\,\mathrm{Ln}(\mathrm{BA}) - 3.1281\frac{\mathrm{Ln}(\mathrm{BA})}{t} - 0.0982\,\mathrm{Ln}(\mathrm{SI})\right)} \tag{12}$$

where v is the inside bark timber stock at time t; d, l, and qmd represent (cm), respectively, the minimum diameter, merchantable diameter, and quadratic mean diameter—a particular measure of central tendency defined as the square root of the arithmetic mean of squared diameters; it can also be calculated using the following function: $\mathrm{qmd} = \sqrt{\mathrm{BA}/0.0000785N}$. BA is the basal area (the cross-sectional area of trees at breast height (1.3 m), $m^2\,ha^{-1}$), and N is the number of trees per hectare, H is the dominant height (m), and SI is the stand index (m). The site index—an indicator of site productivity—is defined as the height of dominant and codominant trees at a base age (50 years in the case of longleaf pine). Site productivity represents the potential growth of the forest, and it is determined by soil quality and climatic conditions (Landsberg and Sands, 2011). We consider site indexes between 27 and 35 m, a typical range for longleaf pine in the region (Gonzalez-Benecke et al., 2014). We consider the following minimum diameter d for sawtimber, chip-and-saw, and pulpwood, respectively: 30.5, 20.3, and 12.7 cm, and the following merchantable diameter l for the same forest products: 20.3, 12.7, and 5.1 cm.

We employ the average regional stumpage prices p 2000–2021 (base 2021) in the Southern U.S. (Timber Mart South, 2021) to model timber benefits due to final harvest and thinning revenues: \$31.7 m^{-3} (sawtimber), \$20.1 m^{-3} (chip-and-saw), and \$8.9 m^{-3} pulpwood. The regeneration costs C (\$ ha^{-1}) are modeled as follows:

$$C = C_{sp} + C_f + C_{wc} + C_{wc} + C_{vc}N \tag{13}$$

where C_{sp} (\$418 ha^{-1}), C_f (\$86 ha^{-1}), C_{wc} (\$203 ha^{-1}) are, respectively, costs associated with site preparation, prescribed burning, weed control, and C_{vc} (\$0.13 $seedling^{-1}$) is planting costs (Forest Landowner Magazine, 2021); N is the initial planting density and acts as a proxy for the initial timber stock V_0. The discount rate r is assumed to be 0.03, a value in the range of discount rates typically used to assess the economic viability of forestry investments in the Southeastern US (McIntyre, McCall, and Wear, 2018). Furthermore, the thinning schedules are planned when basal area reaches 23 $m^2\,ha^{-1}$ and thinned back to a basal area of 18 $m^2\,ha^{-1}$, a widely employed thinning program in the southeastern US (Megalos, 2019).

We consider carbon sequestration as the nontimber benefits generated by longleaf pine forests. We use the van Kooten et al. model (1995) to model net carbon benefits F. Thus

$$F = \int_0^T \alpha P_c \frac{\partial}{\partial t} V(t) e^{-\mathrm{rt}} dt - \alpha P_c (1 - \beta) V(T) e^{-\mathrm{rt}} \tag{14}$$

where P_c is the carbon price (\$ metric ton^{-1}), α is the proportion of metric tons of carbon per cubic meter of timber, β is the proportion of carbon permanently stored in long-term forest products, also known as the pickling factor. This model states that a landowner is annually paid a subsidy for each metric ton of carbon sequestered by the forest stand as the forest grows—the first term on the right-hand side of equation (14)—f in equation (2). Similarly, a landowner will pay a tax per metric ton of carbon released to the atmosphere during timber harvest and due to forest product decay in timber products—the second term on the right-hand side of equation (14).

There are several forest carbon project types available for forest landowners, for example, afforestation/reforestation projects, avoided conversion projects, and improved forest management initiatives, where the eligibility requirements are additionality (carbon sequestration would not have happened without the development of the project), permanence (carbon removal should

Table 1. List of parameters

Parameter	Description	Value
V	Timber volume	n.a
l	Minimum diameter (cm)	30.5 (*s*), 20.3 (*cns*), 12.7 (*pw*)
d	Minimum diameter (cm)	n.a
qmd	Quadratic mean diameter (cm)	20.3 (*s*), 12.7 (*cns*), 5.1 (*pw*)
SI	Site index (m)	27, 28, 29, 30, 35
P	Timber price ($\$$ m^{-3})	31.7 (*s*), 20.1 (*cns*), 8.9 (*pw*)
r	Discount rate	0.03, 0.05
C_{sp}	Site preparation cost ($\$$ ha^{-1})	418
C_f	Prescribed burning cost ($\$$ ha^{-1})	86
C_{wc}	Weed control cost ($\$$ ha^{-1})	203
C_{vc}	Planting cost ($\$$ $seedling^{-1}$)	0.13
P_c	Carbon price ($\$$ metric ton^{-1})	50
α	Proportion of metric ton in one cubic meter of timber	0.27
β	Pickling factor	0.8 (*s*), 0.5 (*cns*), 0.1 (*pw*)

s = sawtimber, *cns* = chip-and-saw, *pw* = pulpwood; V and d changes as the forest stand ages; therefore, a unique value cannot be specified. Timber volume V over time can be found in Figure 1.

be maintained for up to 100 years), and non-leakage (carbon removals in one area should not result in increase in carbon increases in another location) (Parajuli et al., 2019). We use the price of carbon of an improved forest management project (California's cap and trade program) to model carbon benefits and taxes. As such, we set $P_c = \$50$ metric ton^{-1}—a plausible value of real auction settlement prices of carbon (California Environmental Protection Agency Air Resources, 2019)—$\alpha = 0.27$ (Turner et al., 1995) and $\beta = 0.8$, 0.5, and 0.1 for sawtimber, chip-and-saw, and pulpwood, respectively (Susaeta et al., 2014). A value of $\alpha = 0.27$ represents that there is 0.27 metric ton of carbon in one cubic meter of timber, and a $\beta = 0.8$ for sawtimber means that 80% of the carbon is retained in long-lived products and landfills, or conversely, 20% of the carbon is released to the atmosphere.

By using equations (1 and 2), the optimality conditions described in equations (6–8) and equation (10), and applying equations (11–14), we determine the optimal thinning program ($h^*(t)$) and harvest (T^*) and initial planting density (N^*) together the associated economic benefits of longleaf pine forests (L). Since several functions are nonlinear, we conduct a numerical optimization using MS Excel Solver to obtain the optimal values of these unknown parameters. We employ the generalized reduced gradient nonlinear method to conduct our analysis and set a threshold value of 0.0001 between iterations for the objective function; that is, an optimal solution is found when this convergence criterion is met. We also perform a sensitivity analysis to determine the impacts of changes in initial productivity conditions (site index, SI), carbon prices (P_c), discount rate (r,) and thinning rules on the optimal forest management of longleaf pine. Table 1 summarizes the different parameter values employed in our analysis.

4. Results

The optimal management of longleaf pine and associated timber and nontimber benefits are presented in Table 2. For example, at the hectare level and with a site index of 31 m, the forest landowner should plant 985 trees, thin 173, 127, and 91 trees at ages 23, 28, and 35 years,

Table 2. Optimal planting density, thinning schedules and removals, thinning benefits, land values, and harvest ages of longleaf pine managed for timber production and carbon sequestration under different site indices

S/SI	N_0*	Age	$N_r(N_{rm})$	$V(V_{rm})$	V_s	V_{cns}	V_{pw}	*R*	*L* Carbon	Timber	Total	*T**
m	trees ha^{-1}	years	trees ha^{-1}	m^3 ha^{-1}				$ ha^{-1}				years
27	895	29	146 (489)	50.4 (173.7)	0.0	33.2	16.4	341.2	1763.8	2194.8	3958.5	48
		37	102 (331)	55.8 (193.3)	10.1	38.8	6.8	382.0				
29	874	27	158 (472)	55.9 (170.5)	0.0	38.6	16.6	411.1	1919.3	2856.7	4776.0	44
		34	103 (321)	57.4 (189.9)	12.7	38.0	6.5	441.4				
31	985	23	173 (571)	47.9 (157.7)	0.0	24.0	22.6	342.3	2040.8	3523.0	5563.8	44
		28	127 (407)	52.5 (174.7)	1.0	40.4	10.8	405.4				
		35	91 (272)	60.9 (194.5)	27.7	28.5	4.6	522.5				
33	1002	22	206 (554)	58.1 (155.4)	0.0	31.4	25.3	442.3	2176.1	4225.6	6401.8	41
		27	132 (384)	57.9 (173.7)	2.7	44.7	10.2	478.3				
		33	85 (262)	58.6 (192.2)	29.5	25.0	4.0	547.3				
35	1059	20	221 (600)	55.5 (148.2)	0.0	24.7	28.9	413.2	2240.1	4899.6	7139.7	41
		24	140 (430)	52.8 (164.7)	0.4	39.5	12.4	446.4				
		29	100 (297)	58.3 (182.5)	19.2	33.5	5.4	558.2				
		35	63 (203)	57.8 (200.7)	43.9	11.7	2.2	576.0				

SI = site index; N_0* = optimal planting density; N_r = number of thinned trees; N_{rm} = number of residual trees after thinnings; *V* = total volume removal including nonmarketable timber; V_{rm} = total residual volume after thinnings; $V_{s,cns,p}$ = sawtimber (*s*), chip-and-saw (*cns*) and pulpwood (*pw*) volume removal; *R* = thinning benefits; *T** = optimal harvest age.

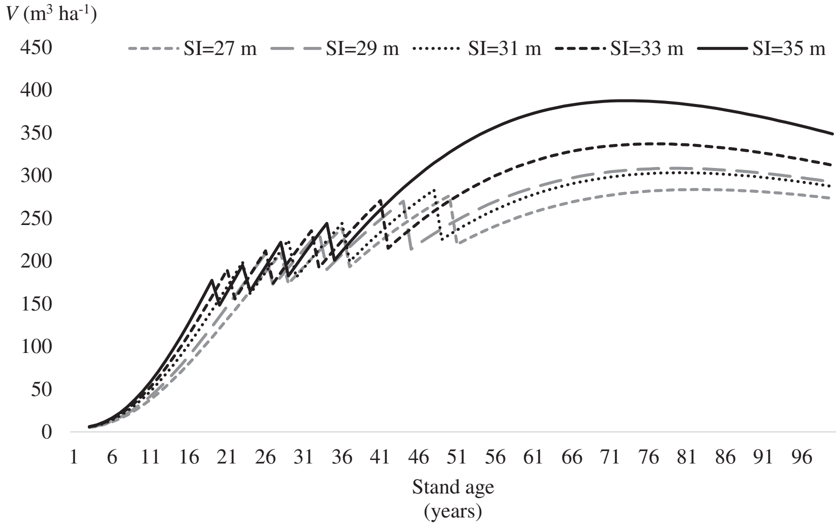

Figure 1. Optimal volume path of an even-aged longleaf pine stand for different site indexes.

Table 3. Optimal planting density, thinning schedules and removals, thinning benefits, land values, and harvest ages of longleaf pine managed for timber production under different site indices

S/SI	N_0*	Age	N_r	V	V_s	V_{cns}	V_p	R	L	T*
m	trees ha^{-1}	years	trees ha^{-1}	m^3 ha^{-1}				\$ ha^{-1}		years
27	1065	27	180	49.4	0.0	22.0	25.6	298.6	2223.1	48
		34	130	55.7	0.9	42.7	11.6	357.4		
		44	82	58.1	28.1	25.8	4.1	386.5		
29	1050	25	193	53.1	0.0	25.1	26.3	348.7	2896.1	44
		31	131	55.6	1.1	42.7	11.4	392.3		
		39	85	58.3	26.7	27.2	4.3	444.0		
31	1086	23	217	56.5	0.0	24.8	29.7	382.5	3565.2	42
		28	136	53.7	0.4	40.4	12.4	404.5		
		35	97	62.4	25.0	32.1	5.1	519.5		
33	1104	21	22	53.6	0.0	20.6	30.8	366.1	4245.0	41
		26	158	60.7	0.5	45.5	14.2	484.2		
		32	101	62.0	23.0	33.4	5.4	555.1		
		40	62	63.6	51.8	9.8	2.0	559.3		
35	1093	20	237	58.1	0.0	24.7	31.3	425.0	4943.9	38
		24	143	53.1	0.3	39.4	12.9	445.8		
		29	102	58.7	18.3	34.7	5.6	555.8		
		35	65	58.2	43.7	12.2	2.3	577.7		

SI = site index; N_0* = optimal planting density; N_r = number of thinned trees; N_{rm} = number of residual trees after thinnings; V = total volume removal including nonmarketable timber; V_{rm} = total residual volume after thinnings; $V_{s,cns,p}$ = sawtimber (*s*), chip-and-saw (*cns*) and pulpwood (*pw*) volume removal; R = thinning benefits; T* = optimal harvest age. Number of residual trees N_{rm} and total residual volume V_{rm} after thinnings are available from the author upon request.

respectively—which implies a removal of 58.5, 60.6, and 56.6 m^3—and harvest the longleaf pine stand at age 44 years. As such, the forest landowner will generate thinning benefits of \$434.0, \$477.5, and \$488.1 and yield a total land expectation value of \$5570.4. Also illustrated in Figure 1, each thinning is scheduled earlier as the site index increases. The first and second thinnings occur at ages 24, 22, and 20 years, and at ages 30, 27, and 24 years, respectively, with site indexes 31, 33, and 35 m.

As shown in Table 2, the optimal planting density of longleaf pine shows a positive trend as medium-low site indexes increase. For example, the planting density increases from 874 to 977 trees ha^{-1} as the site index increases from 29 to 31 m, until reaching an optimal planting density of 1059 trees ha^{-1} with site index of 35 m.

Not surprisingly, the proportion of sawtimber removals to the total removals grows when longleaf pine is managed under higher productivity conditions—at an average rate of 37% between site indexes 27 and 35 m. The proportion of chip-and-saw removals decreases at a rate of 8% while the proportion of pulpwood removals remained stable between site indexes 27 and 35 m.

The optimal harvest age of longleaf pine tends to reduce somewhat as site indexes increase (Table 2). Consistent with expectations, the total land expectation values increase with higher site indexes, averaging a 16% increase in land values. As site index increases, the contribution of carbon benefit decreases while the contribution of timber benefits increases. For example, for a site index 27 m, the proportion of carbon and timber to the total land value is 45% and 55%, respectively, but it decreases to 31% and 69% for a site index 35 m. On average, the proportion of timber

Table 4. Optimal planting density, thinning schedules and removals, land values, thinning benefits, and optimal harvest ages of longleaf pine managed for timber production and carbon sequestration with (a) 5% discount rate and (b) 14 m^2 ha^{-1} basal area[1]

(a) SISI	N_0*	Age	N_r	V	V_s	V_{cns}	V_p	R	L Carbon	L Timber	L Total	T*
m	trees ha^{-1}	years	trees ha^{-1}	m^3 ha^{-1}					\$ ha^{-1}			years
31	802	25	139	51.3	0.1	37.2	13.4	249.7	1198	891	2089	39
		31	98	55.9	14.8	35.2	5.8	260.5				
33	898	22	160	48.5	0.0	29.1	18.5	249.5	1267	1227	2495	39
		27	121	56.5	4.7	42.9	8.7	282.0				
		33	79	57.1	31.9	21.7	3.5	283.7				

(b) SISI	N_0*	Age	N_r	V	V_s	V_{cns}	V_p	R	L Carbon	L Timber	L Total	T*
m	trees ha^{-1}	years	trees ha^{-1}	m^3 ha^{-1}					\$ ha^{-1}			years
31	959	24	92	0.0	55.5	35.1	92.3	694.5	1695.7	3477.9	5173.5	44
		35	100	52.4	40.4	6.7	99.6	885.8				
33	948	22	88	0.0	50.8	34.9	87.5	688.4	1827.4	4087.0	5914.4	42
		32	103	53.1	43.1	7.1	103.5	1000.9				

SI = site index; N_0* = optimal planting density; N_r = number of thinned trees; N_{rm} = number of residual trees after thinnings; V = total volume removal including nonmarketable timber; V_{rm} = total residual volume after thinnings; $V_{s,cns,p}$ = sawtimber (*s*), chip-and-saw (*cns*) and pulpwood (*pw*) volume removal; R = thinning benefits; T* = optimal harvest age. Number of residual trees N_{rm} and total residual volume V_{rm} after thinnings, and full results for all site indexes are available from the author upon request.

and carbon benefits to the total land values increases and decreases, respectively, at a 5% and 9% rate, between site indexes 27 and 25 m. Timber benefits are the main driver of the total economic revenues, accounting for, on average, 63% of the total land values for all site indexes.

Compared to growing the stand for timber and nontimber benefits, the optimal planting density and the number of thinnings increase when the longleaf pine stand is grown exclusively for timber production ($P_c = 0$) (Table 3). For site index 31 m, the landowner will have to plant 1086 trees per ha and conduct three thinnings at ages 23, 28, and 35 years, generating thinning revenues of \$2476.1. The landowner will harvest the stand at age 42 years and obtain a land value of \$3565.2.

The optimal planting density, harvest age, and profitability of the forest stand are reduced when the discount rate is increased from 3% to 5% (Table 4a). For example, for a site index 31 m, the optimal planting density is reduced from 977 trees ha^{-1} to 802 trees ha^{-1}, and the forest stand is harvested at age 39 years. On average, the land values decrease by 62% for site indexes 31 and 33 m—with a greater impact on timber benefits which are reduced by 73%.

The optimal planting density and the economic revenues are also reduced when the minimum basal area is decreased from 18 m^2 ha^{-1} to 14 m^2 ha^{-1} (Table 4b). Carbon benefits are mainly impacted—showing a reduction of 16%—and the total land values are decreased by 7%. The optimal harvest age only tends to be reduced when longleaf pine is managed under medium-high productivity conditions.

5. Discussion and Conclusion

This paper presents an economic model to determine the optimal management of an even-aged longleaf pine stand for timber and nontimber benefits. This model focuses on providing the optimal planting density, thinning schedules, harvest age, and revenues of a longleaf pine stand when it is managed for timber production and carbon sequestration under different levels of site productivity. Our findings show that the optimal planting density tends to increase when longleaf pine is managed in medium and high productivity conditions. In this situation, the longleaf pine stand reaches the maximum upper limit of the basal area earlier, and consequently, the thinnings are scheduled earlier. Thus, the combination of greater planting density and earlier thinnings generates a higher proportion of sawtimber production and rates of carbon sequestration, and more thinning removals, leading to greater land values.

Although studies have suggested that carbon benefits are the main contributor to land values (Stainback and Alavalapati, 2002; Susaeta, Gong, and Adams, 2020), our results show that timber production is the main driver of economic revenues associated with longleaf pine management. This is due to including only carbon benefits from commercial timber and not considering other sources of carbon pools such as forest floor, foliage, and branches. The absence of carbon estimates over time in other sources of carbon pools in current growth and yields of longleaf pine was the reason for not incorporating them into our analysis.

Unsurprisingly, we find that the optimal harvest age tends to be extended with payments for carbon sequestration. Forest landowners would be encouraged to delay the harvest of the stand to accrue the accumulated carbon benefits as the forest stand is growing. It should be noted that the harvest age is extended since carbon sequestration is an increasing function of the stand age. If the forest landowner would manage the forest stand for decreasing nontimber benefits such as wildlife forage, we would expect a reduction of the harvest age.

Our findings suggest that the reduction of the lower limit of the basal area for thinning purposes causes a decrease in the land values. The forest stand is more heavily thinned; consequently, timber production and carbon sequestration are reduced, leading to lower economic returns for forest landowners. Therefore, the design of the thinning schedules together with the generation of timber and nontimber benefits are critical for landowners to establish longleaf pine forests, particularly on private lands, and accomplish the objective of restoring 3.2 million hectares by 2025 (America’s Longleaf, 2020). We must emphasize that our model does not solve the question of the optimal thinning intensity—the percentage of tree removal. Instead, the forest landowner decides the maximum and upper limits of the basal area as the thinning rule. This is an interesting question that should be further investigated.

Our analysis does not compare the economic feasibility of managing longleaf pine with other southern pines. However, it advances a deeper understanding of the economic and management implications of growing longleaf pine in the Southeastern US. As opposed to traditional economic studies, our economic analysis allows forest landowners to make flexible management decisions about planting densities and thinning regimens that maximize economic revenues. Our findings show that managing longleaf pine is a profitable option in the region. Several studies have shown the superiority of slash pine and loblolly pine over longleaf pine when these species are managed for timber production (Susaeta and Gong, 2019). However, there are other important factors primarily associated with longleaf pine management that can make this species a competitive alternative to other southern pines—for example, market factors such as high-quality timber (poles) and pinestraw production. Some forest landowners in the region also give more value to some factors other than profit-oriented goals, for example, wildlife diversity, esthetics, recreation, resistance to natural disturbance, and development of structurally complex longleaf pine forests in terms of composition and age (Gagnon and Jokela, 2002; Susaeta et al., 2023). These timber and nontimber benefits, and the risk of natural disturbances, could be also included in future analyses and guide forest landowners to optimally manage longleaf pine forests.

There are several other ways to extend our analysis. Thinnings can be employed to switch from current even-aged management to a stand with multiple age classes. These types of systems are thought to be more resilient and provide richer habitat diversity (Brockway et al., 2005). As such, our model can determine the optimal stand age to convert the current management structure to uneven-aged management. We assume that all parameters in the model are known with certainty. As such, it would be reasonable to extend this model considering a stochastic environment such as timber prices and forest growth. Although our numerical simulations can give an idea of the impacts of changes of parameters of the model, these results are dependent on the timber stock function of longleaf pine and should not be generalized. Therefore, the application of this model to other relevant southern pines is also a potential extension of this study.

Data availability statement. The data employed in this study and all the links to data sources have been provided in the reference section. The code used in this study is available from the corresponding author upon request.

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