PULSAR ELECTRODYNAMICS

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This paper briefly discusses the salient features of quasi-steady electrodynamics; that is to say it considers certain general properties of Maxwell's equations and their solutions when these equations are subjected to the constraint

$$\frac{\partial}{\partial t} = -\omega \frac{\partial}{\partial \phi}, \tag{1}$$

where ω is a constant angular frequency, ϕ and t are the azimuthal angle and time, and the operators are applied only to quantities which are scalars or the cylindrical components of a vector (Mestel 1971).

When formulated in a coordinate-independent manner (Ardavan 1980a), the quasi-static constraint emerges as a statement of symmetry that has a dual character, a symmetry which is with respect to time in $r\omega/c < 1$ and with respect to space in $r\omega/c > 1$ (c and r are the speed of light and the cylindrical radius, respectively). From the point of view of a local observer whose instantaneous speed in the azimuthal direction is $r\omega$, equation (1) is a statement of time-independence, whereas from the standpoint of a similar observer beyond the light cylinder whose speed is $c^2/(r\omega)$, this equation is a statement of azimuthal symmetry. In other words, the quasi-static Maxwell's equations describe static electromagnetism in a rotating frame behind the light cylinder, and axisymmetric electrodynamics in a non-inertial frame beyond the light cylinder. A mathematical manifestation of this change in the character of the symmetry across the light cylinder is that the quasi-static field equations are elliptic in $r\omega/c < 1$ and hyperbolic in $r\omega/c > 1$. The wave equation governing the time component A^o of the electromagnetic potential, for instance, appears as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A^o}{\partial r}\right) + \frac{\partial^2 A^o}{\partial z^2} + \left(\frac{1}{r^2} - \frac{\omega^2}{c^2}\right)\frac{\partial^2 A^o}{\partial \phi^2} = -4\pi j^o \tag{2}$$

which is an equation of the mixed type (j^o is the electric charge density). Thus the light cylinder represents the surface of parabolic

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degeneracy of the field equations and as such plays a special role in quasi-steady electrodynamics.

On the basis of equation (2), it can be shown that in the absence of boundaries the electromagnetic fields arising from a quasi-static source possess a singularity at the light cylinder, unless at all points of this surface the densities of both the electric charge and the electric current are zero (Ardavan 1980a). Physically, this is due to the fact that under the constraint of quasi-steadiness the source distribution has a rigidly rotating pattern; even though the particles which supply the charge and the current are not themselves constrained to be corotating. The charged particles in the plasma are only required to move in such a way as to create a pattern - of charge separation, for instance - which propagates around the rotation axis at the local speed $r\omega$. The electromagnetic potential arising from an element of such a source which is located beyond the light cylinder, however, turns out to be mathematically analogous to that of a particle which is moving at a superluminous velocity. In the same way as in Cherenkov radiation, the spherical field wavelets emanating at successive instants of time from an element of the quasi-static source in $r\omega/c > 1$ lag behind their source inside a corresponding Cherenkov cone. There is no Cherenkov shock in the present case because the source is extended. But when the source element is located right at the light cylinder, i.e., when its speed equals the speed of propagation of the wavelets emanating from it, the accumulation and the resulting constructive interference of the wavelets at the source gives rise to a singularity.

Another feature of quasi-steady electrodynamics which pertains to the solution of equation (2) in the far zone is the following: radiation of frequency $\nu \gg \omega/2\pi$ emitted by a quasi-static current distribution whose length scale of density variation is c/ω, arises predominantly from the vicinity of the light cylinder (Ardavan 1980b). This result is again a consequence of the fact that the source distribution has a co-rotating pattern. Just as in the case of radiation from a particle in circular orbit, so here the expression for the power emitted by a quasi-static source into the mth multiple of ω turns out to entail Bessel functions of the form $J_m(mr\omega/c)$ whose main contributions to the values of the integrals over the source distribution arise from the neighbourhood of $r = c/\omega$ when $m \gg 1$.

In connection with the relevance of the above results to pulsars, it should here be added that a quasi-static domain cannot be confined to a localized region of the magnetosphere. Once the relevance of the quasi-static constraint to the emitting region is acknowledged on observational grounds, it can be shown that the plasma-electromagnetic fields underlying the entire magnetosphere must comply with equation (1) (Ardavan 1980a).

Ardavan, H. : 1980a, Astrophys. J., submitted.
Ardavan, H. : 1980b, Nature, submitted.
Mestel, L. : 1971, Nature Phys. Sci. 223, p. 149.

DISCUSSION

KUNDT: I question your result that the 4-current density must vanish at the speed-of-light cylinder in corotating pulsar-wind solutions. In analogy to Cerenkov radiation, your singularity of the field strength stemming from sources at the speed-of-light cylinder will probably go away when you evaluate your integrals for source distributions of finite extent.

ARDAVAN: The analogy with Cerenkov radiation is a purely formal one. I have in fact evaluated the integrals for an extended source and not a point source.

KAHN: It is possible to improve the output of radiation by a current system located beyond the light cylinder. Instead of considering a single filament of current, take an assembly of such currents lying in a thin sheet. If the sheet is such that its normal advances with the speed of light, then it will feed energy into an electromagnetic wave. There is constructive interference between the signal from different parts of the sheet, and this enhances the power output.

ARDAVAN: I have assumed that the length scale of variation of the current density is c/ω.

D.F. SMITH: Is there anything wrong with particles lagging field lines giving rise to finite charge densities beyond the light cylinder?

ARDAVAN: Particles can lag field lines even when you impose the quasi-static constraint.