Two-integral distribution functions for axisymmetric galaxies

C. HUNTER[1,2], E. QIAN[1]
[1] *Florida State University* & [2] *Sterrewacht Leiden*

Abstract. We present a new method for finding a distribution function f, which depends only on the two classical integrals of energy E and angular momentum J about the axis of symmetry, for a stellar system with a known axisymmetric potential and density.

Our method requires an analytical axisymmetric potential Ψ. This potential, together with the radial distance R from the axis of symmetry, define a coordinate system in terms of which we express the analytic density as $\rho(\Psi, R^2)$. The part of f that is even in J is then given by the contour integral

$$f(E, J^2) = \frac{1}{4\pi^2 i\sqrt{2}} \frac{\partial}{\partial E} \oint \frac{d\Psi}{(\Psi - E)^{\frac{1}{2}}} \rho_1 \left[\Psi, \frac{J^2}{2(\Psi - E)}\right].$$

The path of this integral starts and ends at the value of Ψ at large distances, and is determined from properties of the potential. The subscript 1 on ρ denotes a partial derivative with respect to its first argument. Our formula is the analogue for the axisymmetric case of Eddington's (1916) classical solution for the isotropic distribution function $f(E)$ for a known spherical density, and reduces to his solution when ρ has no R–dependence.

A numerical quadrature is generally required to evaluate this solution. Contour integrals can be computed simply and accurately by numerical quadrature even for complicated densities. This is a simpler and much more accurate procedure than direct solution of the integral equation that determines f. It may also be preferable to an explicit evaluation of f when the latter is an infinite or doubly infinite series obtained using Fricke's (1952) expansion method.

The figures display contours of the density and distribution function of one of Satoh's (1980) models. In both cases, only the region below the solid curve is physically relevant. The density of Fig.1 is in fact infinite on the dashed curve, so that the Laplace transform of it required for Lynden-Bell's (1962) method of determining f can not be taken. Generally, contours of f reflect the trends of those of ρ in exaggerated form.

Acknowledgements

CH acknowledges support from the NSF, and a visitor's grant in the Netherlands from the NWO.

References

Eddington, A.S., 1916, *Mon. Not. Roy. Astr. Soc.*, **76** 572
Fricke, W., 1952, *Astr. Nachr.*, **280** 193

H. Dejonghe and H. J. Habing (eds.), Galactic Bulges, 357–358.

https://doi.org/10.1017/S0074180900123599 Published online by Cambridge University Press

Lynden-Bell, D., 1962, *Mon. Not. Roy. Astr. Soc.*, **123** 447
Satoh, C., 1980, *Publ. Astr. Soc. Japan*, **32** 41

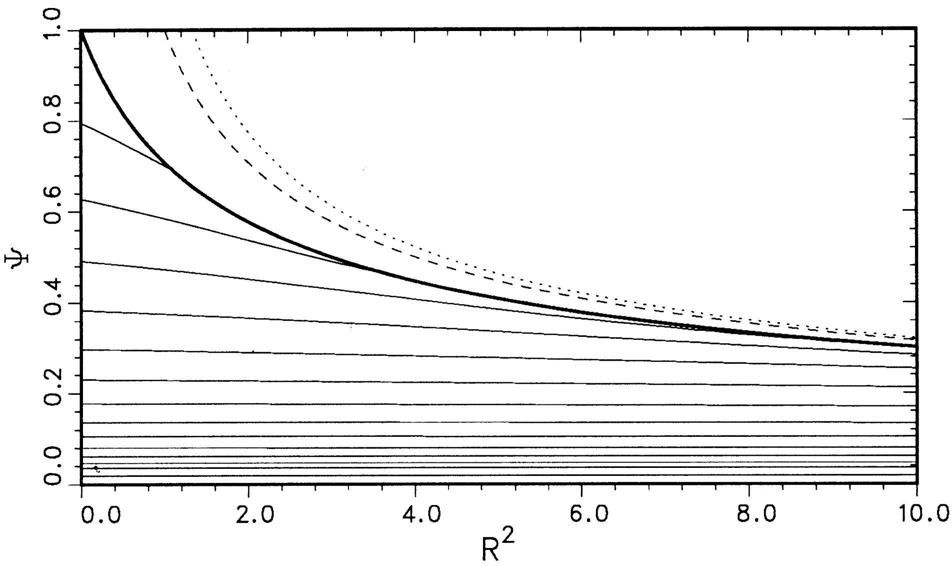

Fig. 1. The thin solid curves are equidensity contours, within the physically relevant domain, of a Satoh model with parameters $b/a = 1$. Outside this domain, $\rho(\Psi, R^2)$ is infinite on the dashed curve, and complex above the dotted curve.

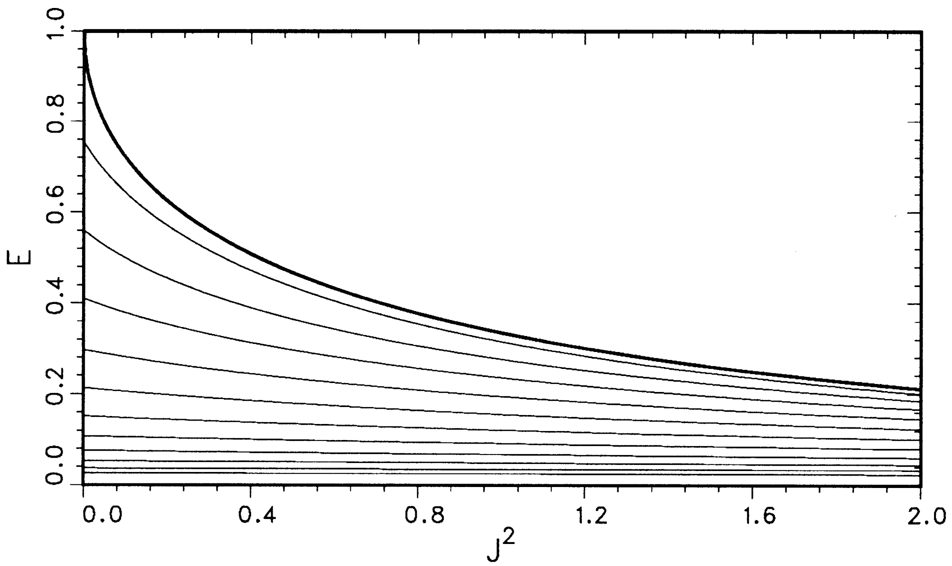

Fig. 2. Contours of $f(E, J^2)$ for the Satoh model of Fig.1. Successive contour levels of both figures differ by factors of 0.2.

https://doi.org/10.1017/S0074180900123599 Published online by Cambridge University Press