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Equity Trading Activity and Treasury Bond Risk Premia

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Abstract

We link equity and treasury bond markets via an informational channel. When macroeconomic state shifts are more probable, informed traders are more likely to have valid signals about fundamentals, so that uninformed traders are less willing to trade against informed ones. This implies low volume and high volatility, that is, a high volatility–volume ratio (VVR). Central banks react to state shifts, but their actions are uncertain. Therefore, a higher state shift likelihood implies larger bond risk premia. These arguments together imply that VVR should positively predict bond excess returns. We empirically test and confirm this prediction, both in- and out-of-sample.

I. Introduction

We link equity and treasury bond markets via an informational channel. When macroeconomic state shifts are more probable, informed traders are more likely to have valid signals about fundamentals, so that uninformed traders are less willing to trade against informed ones. This implies low volume and high volatility (i.e., a high volatility-volume ratio (VVR)). Central banks react to state shifts, but their actions are uncertain. Therefore, a higher state shift likelihood implies larger bond

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risk premia. These arguments together imply that VVR should positively predict bond excess returns. We empirically test and confirm this prediction, both in- and out-of-sample.

How asset markets link to each other is a central issue in finance. We develop and test a setting where an informational channel drives the link between equity and treasury bond prices. In our framework, high volatility and low volume in equities imply a greater likelihood of a shift in the macroeconomic state. Such an impending shift also implies an uncertain monetary intervention, which increases bond risk premia. These arguments suggest that the equity volume-volatility ratio is a state variable that proxies for the likelihood of a shift in macroeconomic conditions. The variable thus influences bond returns in equilibrium. We formalize the above arguments, and then empirically test the relation between the equity volatility-volume ratio and treasury bond premia.

Specifically, we consider a model with a dividend-paying stock and riskless bonds. These assets are traded by two sets of investors. The first set (the "informed") receives a signal which is informative about fundamentals prior to a macroeconomic state shift, and is pure noise otherwise. These investors, however, believe their signal is always valid.¹ The second set of investors (the "uninformed") does not receive this signal, and thus does not know whether their counterparty is informed or trading on noise. These latter investors infer the signal of the informed from the market price.

Now, a higher probability of a state shift (π_t) implies that the signal is more likely to represent valid information, so that the uninformed are less willing to trade with the informed. This results in lower volume and higher volatility in equilibrium. Thus, the ratio of volatility to volume proxies for the state variable π_t . A central bank intervenes to adjust the short-term interest rate, but it does so only when there is a state shift, and the magnitude of the adjustment is uncertain. Therefore, a high π_t is associated with low trading volume, high volatility, and increased bond premia.²

Our model generates the key prediction that equity market volume and increasing volatility–volume ratio (VVR) are respectively negatively and positively related to excess bond returns. Empirically, we provide support for this prediction. In particular, VVR displays strong forecasting power for excess returns across bonds of both long- and short-maturities. It explains up to 4%–7% of the 1-yearahead variation in the excess return for 2-, 3-, 4-, and 5-year bonds. A 1-standarddeviation increase in the aggregate VVR in the equity market leads to an increase of 84 bps in bond risk premia, where the average bond risk premium is 90 bps.

In our setting, both bond and equity VVR proxy for π_t . However, we focus on equity VVR for the most part because the bond counterpart is not available for much of our extensive sample period, and when it is available, the data only cover some maturities. Nevertheless, we do investigate the role of bond VVR over the shorter sample period corresponding to its availability. We find that the bond VVR is

¹This is a simplified version of overconfidence, wherein investors overestimate the precision of their information signal; see Odean [\(1998](#page-32-0)). The specific form of bias is not crucial to the argument, but it adds tractability. ²

 2 Du, Pflueger, and Schreger ([2020\)](#page-31-0) consider the effects of monetary policy uncertainty on sovereign debt. They show that such uncertainty causes sovereign bonds to command a risk premium and argue that such risk can be hedged by borrowing in other currencies.

statistically significant when included jointly with the equity VVR in predictive regressions, though equity VVR is an independent predictor of bond premia.

A clearly relevant concern is whether VVR captures cross-market liquidity premia and risk. That is, if equity and bond market liquidity comove, equity market VVR, which is the Amihud ([2002\)](#page-31-0) proxy for equity liquidity applied to the aggregate stock market, could capture market-wide liquidity premia in returns. We show that the aggregate equity market VVR predicts bond market premia even for the most liquid treasury securities with high reliability, and also predicts bond premia when the spillover across equity and bond liquidity is weak. The addition of stock market liquidity level controls does not qualitatively affect the VVR relation with excess bond returns. We also use the Pastor and Stambaugh [\(2003](#page-33-0)) equity market liquidity risk factor, as an additional control in the analysis. We find that equity liquidity risk is complementary to the VVR effect and does not affect the economic and statistical significance of VVR. These findings suggest that the liquidity spillover argument is not likely the driver of our results connecting aggregate equity volume to treasury bond premia.³

Tighter funding conditions may manifest as liquidity spillovers and be captured in VVR. We investigate the role of funding liquidity levels and risk for excess bond returns. We use the Fontaine and Garcia [\(2012](#page-32-0)) funding liquidity and the TED spread as measures of market-wide funding liquidity levels, and the Hu, Pan, and Wang [\(2013](#page-32-0)) funding liquidity risk measure as additional controls in our analysis. We find that these measures of illiquidity are important for excess bond returns, but they do not affect the statistical and economic importance of VVR. Additional robustness results show that the VVR continues to have strong predictive power after controlling for NBER recessions and the investor sentiment measure of Baker and Wurgler ([2006](#page-31-0)). Thus, these alternative explanations do not explain the connection between bond risk premia and VVR.

To assess the robustness of our results, we examine predictability in excess of the principal components of yields and the Cochrane and Piazzesi ([\(2005](#page-31-0)), CP) and Ludvigson and Ng (LN) ([2009\)](#page-32-0)) factors.⁴ The time series of the aggregate VVR contains additional information about bonds' expected returns beyond that captured in these factors. Our results are robust to accounting for small-sample properties of the data and to using different tests of forecasting accuracy. Finally, the VVR adds to the out-of-sample predictive power of the CP and LN factors.

This article contributes to several strands of the literature. First, Cochrane ([2011](#page-31-0)) points out the need for asset pricing studies to integrate stock and bond markets. Earlier literature, such as Fleming, Kirby, and Ostdiek [\(1998\)](#page-32-0), and Brenner, Pasquariello, and Subrahmanyam [\(2009\)](#page-31-0), indicates that these markets

³Akbas ([2016\)](#page-31-0) proposes that cross-sectionally, low volume forecasts bad news under short-selling constraints. He finds evidence for this idea using earnings announcements as news events. Such a phenomenon in the aggregate should imply lower returns after low volume, but we find that low volume forecasts higher bond returns. Thus, the Akbas ([2016\)](#page-31-0) argument also is more applicable in the cross-section of equities.

⁴While the CP factor subsumes variables like forward spreads, yield spreads, and yield factors, the LN factor focuses on variables outside the bond market and contains information from 132 measures of economic and financial activities, which include dividend yield, TED spread, credit spread, and S&P 500 returns.

are indeed at least partially integrated. We join a growing literature, including Baker and Wurgler ([2012](#page-31-0)), Koijen, Lustig, and Van Nieuwerburgh ([2017](#page-32-0)), and Campbell, Pflueger, and Viceira ([2020](#page-31-0)), which studies the joint pricing of equities and treasury bonds. Our setting also is related to the literature on uncertainty and/or disagreement about the information content of counterparties' trades. In Banerjee and Green ([2015\)](#page-31-0), some investors are unsure as to whether others are trading on information or noise. Further, in Gao, Song, and Wang ([2013\)](#page-32-0), and Easley, O'Hara, and Yang ([2016\)](#page-32-0), uncertainty about the extent of informed trading affects asset returns. We extend these settings to include both equity and bond prices in the presence of a central bank. We derive return predictability implications across markets, via equity market volatility and volume.

Campbell, Grossman, and Wang [\(1993\)](#page-31-0), Harris and Raviv ([1993\)](#page-32-0), Wang [\(1994\)](#page-33-0), and Kandel and Pearson ([1995\)](#page-32-0) consider the relation between equity market volume and prices. In these models, differences in beliefs generate trading volume. We build on these papers by combining informational considerations with trading volume and a monetary authority (central bank). Within our setting, since the bank only acts upon state shifts, the likelihood that the signal is informative about the shift affects both equity trading volume and volatility, as well as bond risk premia. David and Veronesi [\(2014](#page-31-0)) consider a setting where shocks to the option implied volatility signal changes in uncertainty and economic growth and thus cause central banks to react by changing the short-term rate. Complementing their work, we consider the economic connection between equity market trading activity and the pricing of long-term treasury bonds.

The article proceeds as follows: Section II presents our model and its empirical implications. [Section III](#page-10-0) presents the data and provides some preliminary analysis. [Section IV](#page-13-0) presents our central results. [Section V](#page-19-0) discusses whether our results capture liquidity premia, while [Section VI](#page-22-0) presents further robustness analyses. [Section VII](#page-24-0) concludes. There are three Appendices and a Supplementary Materials section. [Appendix A](#page-25-0) derives the model equilibrium, [Appendix B](#page-28-0) presents details of our bootstrap robustness procedure, and [Appendix C](#page-29-0) considers monthly bond returns, as opposed to overlapping annual observations. The Supplementary Material presents additional results. References to tables and figures not included in the main text are prefixed by the letter corresponding to the relevant appendix.

II. Model

We first develop a model which links equity market trading activity to the treasury bond market. Our argument goes as follows: Consider a setting where the macroeconomic state shifts stochastically. Some investors, call these "informed," receive a signal, which is informative about fundamentals prior to state shifts, and is pure noise otherwise. These investors, due to a form of overconfidence (Odean ([1998\)](#page-32-0)), believe that their signal is always valid. The uninformed act as Bayesians, as they infer the signal of the informed from the market price. When the probability of a state shift is high, the signal is more likely to represent valid information, which implies that uninformed investors are less willing to trade with informed ones. So volume is low and volatility is high when state shifts are more likely. Consider now a central bank that acts only during shifts in macroeconomic states.⁵ The central bank acts by changing the short-term discount rate, but its actions have a random element, so that there is uncertainty about how much the discount rate changes during a state shift. Thus, an increased likelihood of state shifts affects bond risk premia via increased monetary uncertainty, as well as equity volatility and volume via the expectations of market participants. These phenomena act to integrate the equity and treasury bond markets in our setting.

A. General Model Description

We consider a dynamic T-period model with three assets: a dividend-paying stock, a two-period (zero-coupon) bond, and a one-period risk-free bond. The twoperiod bond pays \$1 in 2 years' time and represents long-term bonds. The oneperiod bond pays \$1 in 1 year and represents short-term bonds. The price of the latter bond at a generic time t equals the short-term discount factor, β_t . The stock pays dividends D_t in every period and has an ex-dividend stock price of zero in the last period, $S_T = 0$. The discount factor β_t and the dividend process D_t are timevarying. The stock price S_t and the 2-year bond price B_t form in equilibrium.

1. Information Environment

Investors who bought shares at time t are paid dividends at time $t + 1$. Trading takes place in each period after dividends are paid out. The dividend D_{t+1} deviates from the average dividend δ due to macroeconomic shocks, $n \times m_{t+1}$, and an additional random shock $\sigma_t \times s_{t+1}$. additional random shock, $\sigma_d \times \varepsilon_{d,t+1}$,

(1)
$$
D_{t+1} = \delta + n \times m_{t+1} + \sigma_d \varepsilon_{d,t+1},
$$

with $\varepsilon_{d,t+1} \sim N(0,1)$ and

(2)
$$
m_{t+1} = \begin{cases} -1 & \text{with probability } \frac{\pi_t}{2}, \\ 0 & \text{with probability } 1 - \pi_t, \\ 1 & \text{with probability } \frac{\pi_t}{2}. \end{cases}
$$

The probability of macroeconomic changes π_t is a stochastic process and determined at time t.

In every period, there is a private signal i_t about the macroeconomic condition in the next period, m_{t+1} . Whenever the macroeconomic condition deviates from the normal state, $m_{t+1} = -1$ or $m_{t+1} = 1$, the signal correctly indicates the future condition. This state, M^I , occurs with probability π_t . In situations without deviations from the normal macroeconomic state, $m_{t+1} = 0$, the signal is based on noise. This normal state, M^N , occurs with probability $1 - \pi_t$. The signal distribution in this case is $P(i_t = 1 | m_{t+1} = 0) = P(i_t = -1 | m_{t+1} = 0) = 0.5$. Thus, the unconditional signal distribution is $P(i_t = 1) = P(i_t = -1) = 0.5$.⁶

⁵ See Greenwood and Vayanos ([2010\)](#page-32-0) and Pasquariello, Roush, and Vega ([2020\)](#page-32-0) for analyses of monetary interventions. ⁶

⁶This specification for i_t parsimoniously captures the notion that the probability of trading against an informed trader is higher when changes in the fundamental are more likely. In this specification, the

Because in state M^I the signal correctly predicts the deviation from the normal state, the true distribution of the next period's macro condition (conditional on the signal) is

(3)
$$
m_{t+1} = \begin{cases} i_t, & \text{in state } M^I & P(M^I) = \pi_t \\ 0, & \text{in state } M^N & P(M^N) = 1 - \pi_t. \end{cases}
$$

2. Investors

Two types of investors populate the economy: signal-receiving investors and rational nonsignal-receiving investors. As mentioned earlier, for convenience, we respectively term these investors "informed" and "uninformed." The informed receive a private signal i_t about shifts in the macroeconomic state, which is informative about fundamentals prior to a state shift, and is pure noise otherwise. Irrespective of whether the signal is true information or noise, the informed investors assume the signal is a true indicator of a deviation in the macroeconomic state, $m_{t+1} = i_t$. This assumption is made for simplicity and is a limiting version of the usual notion that overconfidence implies overestimating one's information quality (e.g., Odean [\(1998](#page-32-0))).

The informed investors can be interpreted as investors predicting or paying special attention to macroeconomic signals or forecasts, such as the Survey of Professional Forecasters or the Consensus Forecast (e.g., Ghysels and Wright ([2009\)](#page-32-0)). Another way to interpret the signal receiving agents is as market participants who have access to and analyze proprietary order flows, for example, foreign exchange order flows Evans and Lyons ([2008\)](#page-32-0), Rime, Sarno, and Sojli [\(2010](#page-33-0)), and Menkhoff, Sarno, Schmeling, and Schrimpf [\(2016](#page-32-0)), or marketwide equity order flows as in Albuquerque, De Francisco, and Marques [\(2008](#page-31-0)), to predict the state of the economy.

The uninformed investors do not receive the signal, but infer it through the price process. Moreover, these investors understand that the counterparty trades on true information with a probability π_t , so that π_t can be interpreted as the informational content of the signal.

3. Central Bank

The informativeness of the signal becomes public knowledge at time $t + 1$ and the state of the economy realized. If the macroeconomic condition M is normal (in state M^N), the central bank leaves the discount factor at a level $b > 0$. If the state shifts, the central bank decides whether to change the short-term discount factor β_{t+1} from the normal discount factor $b > 0$ to the rate $b+q_{t+1}$ with $q_{t+1} \sim N\left(0, \sigma_q^2\right)$. The central bank changes the discount factor (in state B^C) with probability *γ*, and it keeps the normal rate (in state B^L) with probability $1 - \gamma$, as follows:

signal is informative only when there is an impending state shift in the macroeconomy. Other specifications, including one where the signal is also informative about the lack of a macroeconomic state shift, are possible, but with a significant loss of tractability.

(4)
$$
\beta_{t+1} = \begin{cases} b + q_{t+1}, & \text{in state } M^I \& B^C \quad P(M^I \& B^C) = \pi_t \times \gamma \\ b, & \text{in state } M^I \& B^L \quad P(M^I \& B^L) = \pi_t \times (1 - \gamma) \\ b, & \text{in state } M^N \quad P(M^N) = 1 - \pi_t. \end{cases}
$$

In our specification, the distribution of q_{t+1} does not depend on the direction of the deviation from the normal macroeconomic state. In other words, the distribution of q_{t+1} is the same whether m_{t+1} equals $+1$ or -1 . This specification is chosen to isolate the variance effect of the central bank's reaction from its direction effect.7 Thus, within our setting, the expected discount factor does not depend on the macroeconomic condition.

4. Optimization

Investors maximize their mean–variance expected utility of next-period wealth given their own expectations (e.g., Van Nieuwerburgh and Veldkamp ([2009\)](#page-33-0) and Banerjee and Green [\(2015](#page-31-0)), among others). We denote the different expectations by the superscripts $O(U)$ for informed (uninformed) investors.

(5)
\n
$$
\operatorname{argmax}_{x'_{S}, x'_{B}} E^{j}[w_{t+1}] - \frac{\alpha}{2} \text{VAR}(w_{t+1}) \ \forall j \in \{O, U\}
$$
\n
$$
= \operatorname{argmax}_{x'_{S}, x'_{B}} x'_{S} E^{j}[S_{t+1} + D_{t+1} - \frac{1}{\beta_{t}} S_{t}] + x'_{B} E^{j}[\beta_{t+1} - \frac{1}{\beta_{t}} B_{t}] + \frac{1}{\beta_{t}} w_{t}
$$
\n
$$
- \frac{\alpha}{2} ((x'_{S})^{2} \text{VAR}^{j}(S_{t+1} + D_{t+1}) + 2x'_{S} x'_{B} \text{COV}^{j}(S_{t+1} + D_{t+1}, \beta_{t+1})
$$
\n
$$
+ (x'_{B})^{2} \text{VAR}^{j}(\beta_{t+1})).
$$

The optimization leads to the first-order conditions for equity holdings, x_S^j , and bond holdings, x_B^j , $j \in \{O, U\}$.

5. Equilibrium

The stock and bond holdings of the investors are determined via market clearing. As the one-period risk-free discount rate is solely determined by the central bank, the corresponding one-period bond is in exogenous (perfectly elastic) supply. The two-period bond and the dividend-paying stock are in fixed supply, denoted by Z_B and Z_S , respectively, obtaining:

(6)
$$
x_{t,S}^O + x_{t,S}^U = Z_S,
$$

$$
(7) \t\t\t x_{t,B}^O + x_{t,B}^U = Z_B.
$$

We define the equilibrium as follows:

⁷We do not include the effect of central bank policies on the macroeconomic condition for expositional simplicity. We can interpret our model setup such that the economic state gets renormalized in each period. Therefore, our model implies a simplified version of Taylor's rule, which focuses on output deviation. To bring out our intuition clearly, we abstract from inflation.

Definition 1. At each point in time, an equilibrium consists of prices for the risky asset, S_t , and for the 2-year bond, B_t , such that investor demands for the stock and the two-period bond, $x_{S,t}$ and $x_{B,t}$, are optimal, given their beliefs and information (equation (5)), and the markets for the stock and the 2-year bond clear (equations (6)) and [\(7\)](#page-6-0)).

We derive the unique equilibrium via backward recursion. The derivation is analytically complex and is provided in [Appendix A.](#page-25-0)

As the closed-form results are long, recursive expressions, we perform numerical simulations to examine the characteristics of this equilibrium. First, we show that return volatility and trading volume in the stock market are linked to the signal informativeness π_t and thus work as proxies for this variable. Second, we show that the signal informativeness is a state variable in the conditional asset pricing sense, which we can measure through the stock market volatility to volume ratio.

B. Model Simulation Setup

For the simulation, we use a time horizon of $T = 10,000$ periods. We measure stock return volatility as the absolute value of the stock return deviation from the average return:

(8)
$$
|r_t| = \left| \frac{S_t - S_{t-1}}{S_{t-1}} - \overline{r} \right|.
$$

Trading volume is measured by the absolute value of the change in the portfolio holdings of the investors. In addition, we assume an extra trading volume c in every period, which results from liquidity traders exchanging a constant amount of shares. This avoids a division by zero in the calculation of the volatility-volume ratio (VVR). Thus, the total volume (TV_t) is given by

(9)
$$
TV_t = |x_{S,t}^U - x_{S,t-1}^U| + |x_{S,t}^O - x_{S,t-1}^O| + c.
$$

The next period's excess bond return is determined as

(10)
$$
r_{B,t}^e = \frac{\beta_t}{B_{t-1}} - \frac{1}{\beta_{t-1}}.
$$

We calibrate the model to match one period per year. The chosen parameters are $b = 0.97$, $\sigma_a = 0.02$, $\gamma = 0.5$, $\delta = 6$, $\sigma_d = 1$, $n = 1$, $Z_B = 100$, and $Z_S = 1$. The risk aversion parameter α standardizes the average stock price to 100. The values for b and σ_a correspond to a central bank interest rate of 3%, with a volatility of 2%, which are reasonable. Since the bond pays unity, the supply of the bond is 100 times that for stocks, which corresponds to equal-sized markets at a stock price level of 100. The value for δ corresponds to a dividend yield of 6%. The unity values for *n* and σ_d in [equation \(1\),](#page-4-0) and the 50% value for the probability γ , are normalizations and not crucial for the simulation. We assume that the extra trading volume from exogenous sources is $c = 0.1$.

We specify the stochastic process of the signal informativeness π_t as an H-state Markov switching process with transition matrix Ω :

(11)
$$
\Omega = \begin{pmatrix} p_{11} & \cdots & p_{1H} \\ \vdots & \ddots & \vdots \\ p_{H1} & \cdots & p_{HH} \end{pmatrix}.
$$

We use $H = 5$ equally spread π -value states between $1/H$ and 1 in the Markov chain. The probability of switching to an adjacent π -value state is 0.025 for each adjacent state. Further, the probability of staying in the current state is 0.95 (0.975) for inner (boundary) π -value states.

C. Model Implications

In our model, the main driving force of the relation between volatility, volume, and future excess bond returns is the probability of the state shift π_t . This parameter influences the equity market's volume and volatility. The probability π_t is also connected to central bank policy uncertainty and, in turn, the bond risk premium, since the central bank intervenes only during state shifts.

To establish the above channels, we first show that a higher value of π_t links to a higher volatility and a lower trading volume in the stock market. To illustrate this link, we consider different economies with different, but constant levels of the signal informativeness $\pi_t = \pi$. [Figure 1](#page-9-0) shows the relation between π and stock market trading volume and volatility. Graph A shows that trading volume decreases in π . If π is high, uninformed investors' willingness to trade against potentially informed investors is low. This results in a low trading volume. Volatility increases with π , as shown in Graph B. With high π , uninformed investors are not willing to trade actively and mitigate the volatility caused by informed investors, so that volatility is high. Graph C shows that π is positively related to a higher bond risk premium. A higher π implies greater interest rate uncertainty owing to an increased likelihood of the central bank's intervention (that has a random element). This potential change in next period's short rate is a priced risk for the 2-year bond relative to the 1-year bond.

In a second step, we show that π_t is a state variable in the conditional asset pricing sense. We can see from [equation \(A-2\)](#page-25-0) that the expected returns of the 2-period bond depend on this state variable: Even though the expected 1-period bond price at time $t + 1$ does not depend on the current (time t) value of π_t , the price of the 2-period bond (and thus the expected return) does.

In a simulated regression with stochastic π_t , we show that a higher volatility and a lower trading volume (indicating a higher π_t) predict higher excess bond returns in the subsequent period. Columns 1–2 of [Table 1](#page-9-0) illustrate this model result. Column 3 shows the results from the regression of excess bond returns $r_{B,t+1}^e$ on the equity volatility volume ratio VVP. We can see that a higher ratio VVP predicts equity volatility-volume ratio, VVR_t . We can see that a higher ratio VVR_t predicts higher bond premia. Finally, column 4 of [Table 1](#page-9-0) shows that bond and equity market VVR both predict bond premia as they both proxy for π_t . In our empirical tests we mostly focus on equity VVR due to greater data availability, but briefly discuss the role of bond VVR in [Section VI.A.](#page-22-0)

FIGURE 1

Volatility and Volume As a Function of the Information-to-Signal Ratio

Figure 1 shows trading volume, volatility, and 2-year bond price as a function of the information-to-signal ratio π. We use a time horizon of $T = 10,000$ and the parameters $b = 0.97$, $\sigma_a = 0.02$, $\gamma = 0.5$, $\delta = 6$, $\sigma_d = 1$, $n = 1$, $z_B = 100$, and $z_S = 1$. To avoid division by a trading volume of 0, we add an additional trading volume from exogenous sources of $c = 0.1$. Graphs A, B, and C present the relation between signal informativeness π and stock market trading volume, stock market return volatility, and average excess bond returns, respectively.

TABLE 1 Simulation-Based Bond Premium Regression

Table 1 presents the results of a regression of the bond premium on the volatility-to-volume ratio, volatility, and trading volume. We generate the data from our model with time-varying signal informativeness. We use a time horizon of $T = 10,000$. The parameters are b = 0.97, σ_q = 0.02, γ = 0.5, δ = 6, σ_d = 1, n = 1, z_B = 100, and z_S = 1. There are H = 5 equally spread π -value
states in the Markov chain. The probability of switching to an adjac staying in the current state is 0.95 (0.975) for inner (boundary) π-value states. The risk aversion parameter is chosen to standardize the average stock price to 100. To avoid division by a trading volume of 0, we add an additional trading volume from other sources of $c = 0.1$. The p-values are presented in square brackets. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

III. Data and Methodology

Following the literature, we use end-of-month data on U.S. treasury bonds from the Fama–Bliss data set available from the Center for Research in Security Prices (CRSP) database to construct excess bond returns and forward rates for the main analysis. In later sections, we also consider two other data sets developed by Gurkaynak, Sack, and Wright ([2007\)](#page-32-0) and Le and Singleton ([2013\)](#page-32-0). Our sample includes monthly data for the Jan. 1964 to Dec. 2018 period. The data set contains constant-maturity yields for the 1- to 5-year maturities. The calculations are as follows: Let $p_t^{(n)}$ denote the log-price in year $t = 1, ..., T$ of an *n*-year zero-coupon bond. The log yield on this bond is $y_t^{(n)} = -\frac{1}{n}p_t^{(n)}$. The log 1-year forward rate at time t of a loan from time $t + n - 1$ to $t + n$ is then $f_t^{(n)} = p_t^{(n-1)} - p_t^{(n)}$. The log excess return of holding an *n*-year zero-coupon bond from time t to $t + 1$ is given as $r_{t+1}^{e,(n)} = p_t^{(n-1)} - p_t^{(n)} - y_t^{(1)}$. The average premium, \overline{r}_{t+1}^e , is the equal-weighted pre-
mium across all maturities. The predictable component in the excess bond return mium across all maturities. The predictable component in the excess bond return reflects a bond risk premium.

A. Aggregate Volatility-Volume Ratio

We first construct the volatility-volume ratio at the individual stock level as $\frac{1}{N} \sum_{t=1}^{N} (|r_t|/TV_t)$, where $|r_t|$ is the daily absolute return, TV_t is the daily total dollar trading volume, and N is the number of trading days in a month. We use stock returns and trading volume from CRSP. Our sample includes only common stocks (Common Stock Indicator Type $= 0$), common shares (Share Code 10 and 11), and stocks not trading on a "when issued" basis. Stocks that change primary exchange, ticker symbol, or CUSIP during our sample period are removed from the sample (Chordia, Roll, and Subrahmanyam ([2000\)](#page-31-0), Goyenko, Holden, and Trzcinka ([2009\)](#page-32-0), and Hasbrouck [\(2009](#page-32-0))). To avoid very small or illiquid stock-related issues, we also remove stocks that have a price lower than \$1 and have less than 15 trading days in a month. 8 Atkins and Dyl [\(1997](#page-31-0)) show that there is double-counting of the volume in Nasdaq. We correct for this double-counting by following Anderson and Dyl [\(2005](#page-31-0)); specifically, for the periods prior to Feb. 2001, Feb. to Dec. 2001, and 2002 to 2004, we divide the Nasdaq trading volume by 2, 1.8, and 1.6, respectively. Our market VVR is calculated by taking the market capitalization-weighted average of the individual stock volatility-volume ratios across stocks to create a marketwide measure every month.⁹

We recognize that VVR is basically the Amihud ([2002\)](#page-31-0) measure of illiquidity, calculated for the aggregate market, raising the issue that our results might capture

⁸The results remain qualitatively unchanged if we remove the change in exchange, ticker, and CUSIP filters (see Section 2 in the Supplementary Material), as well as the stock price and trading day filters. The latter results are available from the authors upon request.

 $⁹$ There are other methods one can use to aggregate VVR at the market level. The results remain</sup> qualitatively unchanged when using the equal-weighted average VVR of the market value weighting of firms split into large and small stocks by median market capitalization, see results in Section 3 in the Supplementary Material. The results are also qualitatively unchanged when using a simple average of individual stock VVR. These results are available from the authors upon request.

TABLE 2 Data Characteristics

liquidity premia. We note, however, that Lou and Shu [\(2017](#page-32-0)) argue against the Amihud measure capturing a liquidity premium in the cross-section of equities and propose that it captures mispricing. While we do not comment on this debate at the cross-sectional level, in [Section V,](#page-19-0) we show that our results are likely not due to liquidity spillovers or liquidity premia at the aggregate level (i.e., across stocks and treasury bonds).

Panel A of Table 2 presents the characteristics of the level of VVR, and Figure IA.1 of the Supplementary Material depicts volume, volatility, and VVR levels. Our model in [Section II](#page-3-0) is built in a stationary environment; however, Figure IA.1 of the Supplementary Material shows that VVR at the monthly level exhibits a unit root.¹⁰ Therefore, our empirical tests consider a version of VVR that accounts for nonstationarity. Specifically, following Hamilton ([2018\)](#page-32-0), we construct a stationary difference, the yearly change in ln(VVR), that matches our forecast maturity (see Footnote 13 in Hamilton [\(2018](#page-32-0))).¹¹ Letting VVR (t) denote VVR in month t, we redefine: $VVR_t = \ln [VVR(t)] - \ln [VVR(t-12)]$. For convenience, we drop the time subscript from VVR_t in the ensuing exposition, so that, henceforth, VVR denotes the transformed value as above. Graph A of [Figure 2](#page-12-0) shows that VVR does not exhibit nonstationarity, and it co-moves substantially with the average excess bond return.¹²

¹⁰Standard nonstationarity tests confirm this visual observation.

¹¹There are several ways to deal with nonstationarity and the method that we use is only one way to transform the data. We conduct robustness checks using the 1-month differences in VVR, a version of VVR that is filtered through an AR(12) process, as well as using trend and exponential smoothing to transform VVR and find qualitatively similar results. The results are available from the authors upon request.

¹²Unreported test results readily reject the hypothesis that VVR has a unit root.

FIGURE 2

Average Annual Excess Bond Returns and Explanatory Factors

Figure 2 presents the average annual excess bond return, $\overline{r^e}_{t+12}$ return, and forecasts from explanatory factors. The explanatory factors are the stock market volatility-to-volume ratio VVR, in Graph A, the Cochrane–Piazzesi factor CP, in Graph B, and the Ludvigson-Ng factor LN_t in Graph C.

B. Forecasting Bond Premia In-Sample

We adopt the standard approach to uncover predictable variation in excess bond returns by regressing these returns on a vector of predictor variables, X_t :

(12)
$$
r_{t+1}^{e,(n)} = \beta_0 + \beta'_1 X_t + \varepsilon_{t+1}^{(n)}.
$$

The bond return regressions are estimated over a sample of monthly data, which includes overlapping 1-year excess return observations. The presence of overlap introduces serial correlation in the prediction errors (i.e., autocorrelated residuals, e.g., Stambaugh ([1999](#page-33-0)), Amihud and Hurvich ([2004](#page-31-0))). Following Cochrane and Piazzesi ([2005](#page-31-0)), we compute standard errors using the Newey–West procedure with 18 lags to account for heteroscedasticity and autocorrelation in the residuals.

The Newey–West standard errors are based on asymptotic approximations that might be inadequate in finite samples with overlapping observations. In addition, the dependent and independent variables are highly persistent, which may lead to substantial standard error bias in small samples with low standard errors (Bauer and Hamilton [\(2017\)](#page-31-0)). Thus, we use a parametric VAR block-bootstrap analysis to check for the robustness of our inference in finite samples. For this procedure, we first estimate a restricted vector autoregressive (VAR) for monthly excess returns $(r_{t+1}^{e,(n)})$ and explanatory variables (X_t) under the null hypothesis of no predictability.
We draw 10,000 samples from this estimated process to obtain the bootstranned We draw 10,000 samples from this estimated process to obtain the bootstrapped p-values. We then test for the significance of our variables of interest in the bond premia regressions in [equation \(12\).](#page-12-0) The bootstrap procedure is described in detail within [Appendix B.](#page-29-0)¹³

IV. Results

In this section, we present the in- and out-of-sample forecasting performance of the equity market VVR for bond risk premia. We use a comprehensive set of control variables and techniques.

A. In-Sample Predictions

Panel B of [Table 2](#page-11-0) presents the correlations between the change in VVR (the independent variable in the regressions) and bond premia. Higher VVR in the equity market is associated with higher bond premia. The correlation between average excess bond returns and VVR is 23% and ranges between 22% and 26% for 2–5 year maturities.

[Table 3](#page-14-0) presents the results of the regression of bond premia on the equity market VVR. We present results for the equal-weighted bond premium and the 2-, 3-, 4-, and 5-year log excess bond returns. We find that VVR has a large and positive impact on the average excess bond returns (coeff. $= 1.731$; *p*-value $= 0.01$). The VVR explains 5% of the variation of the yearly average excess bond return.

Specifically, a 1-standard-deviation change in VVR increases average expected excess bond returns by about 84 bps, where the average excess bond return is 90 bps, which represents 23% of the volatility of the average bond premium. The estimated coefficient for VVR is stable and at least 2-standard-deviations away from zero, as shown in the recursive, expanding window, estimate in Figure IA.2 of the Supplementary Material for the regression results in column 2 of [Table 3](#page-14-0).

The results on individual excess bond returns in columns 3–6 in [Table 3](#page-14-0) confirm the average bond premium results. VVR is statistically significant at the 1% level, and the adjusted R^2 for next year's 2- to 5-year log excess bond returns are 7%, 6%, 5%, and 4%, respectively. We also find that the estimated coefficients for VVR increase monotonically with bond maturity. The estimated coefficient for the 5-year excess bond returns regression is 2.383, more than twice the magnitude of the estimated coefficient for the 2-year note. The economic significance of VVR for

¹³Another approach to correcting the standard errors for overlapping observations is to use the reverse regression (RR) approach in Wei and Wright ([2013\)](#page-33-0) based on Hodrick [\(1992](#page-32-0)). In unreported analysis, we use the RR approach and find qualitatively similar results.

TABLE 3 Volatility-to-Volume Ratio and Bond Premia

longer term maturities is much larger. The bootstrapped p-values do not change our conclusions.

B. VVR and Yield-Based Predictors

We are interested in evaluating how much of the variation in bond risk premia is captured by VVR. In this subsection, we use as a benchmark the spanning hypothesis, according to which the yield curve spans all information relevant for forecasting future yields and returns. We evaluate whether VVR conveys additional information about bond risk premia beyond the yield curve. We use zero-coupon bond yields on U.S. treasury bonds constructed from daily data on a large cross-section of coupon yields following Le and Singleton (LS) [\(2013](#page-32-0)). LS argue that the Fama–Bliss CRSP data set only considers treasury bonds with maturities up to 5 years, which might be a limited sample for studying bond risk premia. They, therefore, use a set of unsmoothed zero-coupon Fama–Bliss yields with maturities up to 10 years to construct excess bond returns. In unreported analysis, we also use the Gurkaynak, Sack, and Wright ([2007\)](#page-32-0) data, which is constructed based on the smoothed fitted yields using the extended Nelson and Siegel model, and find qualitatively similar results. For brevity, we only present results based on the LS data in this section.

We consider the first three and, in turn, five principal components (PC) of treasury yields of maturities from 1 to 10 years. [Table 4](#page-16-0) presents the results for the spanning test for the average excess bond return based on 2- to 10-year bonds, as well as four different individual maturities (2–5 years) for comparability with the Fama–Bliss treasuries used in the rest of the analysis. The results show that PCs have strong statistical and economic significance, with high R^2 s for average and individual maturity bond premia. VVR is highly statistically significant after controlling for bond yields.¹⁴ The addition of VVR does not affect the relevance of the

¹⁴The coefficient of VVR is different from that in Table 3, because the average bond premia here include bonds of longer maturities, up to 10 years. From Table 3, the impact of VVR increases with bond

PC variables, and VVR adds more than 4% –6% to the explanatory power of these variables. In the bottom row of [Table 4](#page-16-0), we use the bias-corrected Bauer and Hamilton ([2017\)](#page-31-0) standard errors. The results are qualitatively unaltered.

We also investigate the predictability of VVR in addition to the five PCs for monthly excess bond returns based on nonoverlapping observations of bond returns. These results are presented in [Appendix C.](#page-29-0) [Table C1](#page-30-0) therein shows that the results are robust to the use of monthly returns.

C. Controlling for Bond Factors

Cochrane and Piazzesi ([2005\)](#page-31-0) regress the excess returns of 2- to 5-year bonds on a constant and five forward rates and find that a single tent-shaped linear combination of the five forward rates, the CP-factor, explains between 30% and 35% of the variation in excess bond returns. Ludvigson and $Ng(2009)$ $Ng(2009)$ examine the link between bond risk premia and macroeconomic fundamentals by regressing excess bond returns on macro factors. Instead of selecting specific macro variables, they use dynamic factor analysis to extract nine macroeconomic factors (LNfactors) from a panel of 132 measures of economic activity. We now control for the CP and LN factors in our analysis.

We calculate forward prices from the Fama–Bliss data set, as described in [Section III.](#page-10-0) Data on the macro factors of Ludvigson and Ng ([2009](#page-32-0)) are obtained from the website of Sydney Ludvigson.¹⁵ Table IA.1 of the Supplementary Material presents the characteristics for all the variables and their correlations with VVR and the average bond premium. We find that VVR is positively correlated with all the forward rates, while it has varying signs and correlation magnitudes with the Ludvigson and Ng (LN) factors. The correlation of VVR with the factors varies between 21% with LNF₇ and $-25%$ with LNF₂, implying that VVR might contain information additional to these factors. [Figure 2](#page-12-0) plots the equal-weighted excess bond returns 1 year ahead, the CP and LN factors, and the VVR factor. We find that the CP and LN factors substantially co-move with the average excess bond return.

We control for the predictive information in the forward rates and in macro variables by including these variables as control variables in the bond prediction regressions, in and out of the sample. The 9 macro factors of Ludvigson and Ng and the forward rates of Cochrane and Piazzesi collectively explain 40% of the monthly variation in future excess bond returns (see Table IA.2 of the Supplementary Material). We construct the CP factor by pooling the regressions for the individual maturities as follows:

(13)
$$
\overline{r^e}_{t+1} = \gamma_0^{\text{CP}} + \gamma_1^{\text{CP}} X_t^{\text{CP}} + \overline{\epsilon}_{t+1}^{\text{CP}},
$$

maturity, so the VVR coefficient should be higher for average premia calculated over 2- to 10-year bonds than over 2- to 5-year bonds. For bonds with 2- to 5-year maturity, the VVR coefficients are comparable to the VVR coefficients in [Table 3.](#page-14-0)
¹⁵Ludvigson and Ng [\(2009](#page-32-0)) describe the factor construction and the data are available at [https://](https://www.sydneyludvigson.com/data-and-appendixes)

www.sydneyludvigson.com/data-and-appendixes.

TABLE 4Volatility-to-Volume Ratio and Spanning Hypothesis Tests

Table 4 presents the monthly in-sample forecasting regression for excess bond returns, VVR, and yields: $r_{i+12}^p = \beta_0 + \beta_i \mathbf{X}_i + \varepsilon_{t+12}$. Returns are from Le and Singleton ([2013](#page-32-0)). $r^{\rm e}$ are yearly excess bond retu principal components. *, **, and *** denote significance at the 10%, 5%, and 1% levels using the Newey–West p-values, respectively.

where $\overline{r}_{t+1} = \frac{1}{4} \sum_{n=1}^{5} r_{t+1}^{e_n(n)}$ and $X_t^{CP} = \left[y_t^{(1)} \cdot f_t^{(2)} \cdot \dots \cdot f_t^{(5)} \right]$. The factor combines the information in all forward rates and is defined as $\overrightarrow{CP}_t = \hat{\gamma}_0^{\text{CP}} + \hat{\gamma}_1^{\text{CP}} X_t^{\text{CP}}$. We also
combine the nine macro factors into a single forecasting factor by using the combine the nine macro factors into a single forecasting factor by using the regression:

(14)
$$
\overline{r^e}_{t+1} = \gamma_0^{\text{LN}} + \gamma_1^{\text{LN}'} \mathbf{X}_t^{\text{LN}} + \overline{e}_{t+1}^{\text{LN}},
$$

where $X_t^{LN} = [LNF_{1,t},...,LNF_{9,t}]$ contains the macro factors of Ludvigson and
Ng (2009) We define the single forecasting factor, the IN factor as Ng ([2009](#page-32-0)). We define the single forecasting factor, the LN factor, as $LN_t = \hat{\gamma}_0^{LN} + \hat{\gamma}_1^{LN'} X_t^{LN}$. Table IA.2 in the Supplementary Material reports the regres-
sion results for the CP and I N factors. The results remain qualitatively similar to sion results for the CP and LN factors. The results remain qualitatively similar to using the individual variables. To provide a more compact representation of the results, we use factor controls for the rest of the analyses.

Next, we examine whether the equity VVR has predictive power for excess bond returns, using the CP and LN factors as a benchmark. [Table 5](#page-18-0) reports the results from the in-sample forecasting regression for the average bond premia and 2-, 3-, 4-, and 5-year log excess bond returns. The results show that the CP and LN factors on their own are statistically significant at the 1% level, and the adjusted R^2 for next year's 2-, 3-, 4-, and 5-year log excess bond returns are 30%, 33%, 36%, and 34%, respectively. Our results are close to those reported in Table 2 of Ludvigson and Ng $(2009).¹⁶$ $(2009).¹⁶$ $(2009).¹⁶$

More importantly, the equity market VVR is still statistically and economically significant with the inclusion of CP and LN factors across all maturities. The adjusted R^2 s increase by 4%–7% across the board with the addition of VVR. The 6% increase in adjusted R^2 with a single return forecasting factor for all maturities suggests that equity market VVR contains additional information not encompassed in the CP and LN factors. VVR also remains highly economically important. Specifically, a 1-standard-deviation change in VVR increases the average expected excess returns by about 83 bps, which represents 23% of the volatility of the average bond premium, even in the presence of the CP and LN factors. Accounting for small sample biases by bootstrapping the p-values does not change our conclusions.

Subsample Analysis

The sample period we investigate is long with many changes both in the term structure of interest rates and in monetary policy. Several subsamples are of particular interest: the period from the beginning of the sample up to the end of the great moderation (1964–2008), the period of high inflation and high interest rates (1964– 1984), the period post-Volcker and post-high-inflationary pressures, including the great moderation (1985–2018), and the period during and post the great recession characterized by very low interest rates and large quantitative easing (2009–2018). Table IA.3 of the Supplementary Material shows the results or the sample divided into the above subsamples. Column 1 shows the full sample results (as in [Table 5\)](#page-18-0),

¹⁶This alleviates potential concerns about the use of the CP and LN combined factors and the longer sample.

TABLE 5 Volatility-to-Volume Ratio, Bond Premia, CP, and LN Factors

Table 5 presents monthly in-sample forecasting regression results of bond premia, VVR, and the Cochrane–Piazessi and Ludvigson and Ng factors. We estimate the regression $r_{t+12}^{\rho} = \beta_0 + \beta_1' \mathbf{X}_t + \varepsilon_{t+12}$, where r^{ρ} is the excess bond risk return: average (equal-weighted) and included the principle of the property factors and the incompar individual maturities (2- to 5-years). CP denotes the Cochrane–Piazzesi factor, and LN is the linear combination of the nine macro factors of Ludvigson and Ng, as described in [Section IV.C.](#page-15-0) VVR is the yearly change in log volatility-to-volume ratio as described in [Section III.A](#page-10-0). The VVR coefficient is presented in % points. The sample period is Jan. 1964 to Dec. 2018. The p-values calculated using the Newey– West correction for autocorrelation and heteroscedasticity with 18 lags are presented in square brackets. The *p*-values based on the
parametric VAR block-bootstrap analysis, as described in [Appendix B,](#page-29-0) are presented in pa the 10%, 5%, and 1% levels, respectively.

and columns 2–5 show the results for the four subsamples. The impact of VVR is significant in all periods, though the coefficients vary across subsamples.¹⁷

D. Out-of-Sample Prediction

We augment our in-sample analysis with out-of-sample forecasting evidence; see Welch and Goyal ([2008\)](#page-33-0) for a discussion on predicting returns out of sample. We continue to use the CP and LN factors in the benchmark model for the forecast exercise. We use a rolling window of 15 years (i.e., 180 monthly observations) to construct our out-of-sample forecasts. To avoid lookahead bias, we first estimate the CP and LN factors for this window. Next, we estimate the regressions over the sample window using 180 observations and obtain forecasts of the 1-year ahead excess returns. For the next observation, the window is shifted 1 month ahead. For example, the first window runs from Mar. 1961 to Mar. 1975 and is used to forecast the excess bond return for the Apr. 1975 to Apr. 1976 period. The second window runs from Apr. 1961 to Apr. 1975 and is used to forecast the excess bond return for May 1975 to May 1976 period, and so on.

Using the forecasts, we compute the one-step-ahead prediction errors that would prevail under competing models and test which model makes larger errors on average. We compare the out-of-sample forecasting ability of the model with VVR as a predictor, in addition to the CP and LN factors to the benchmark forecasting model that contains only the CP and LN factors.

We compare the prediction errors of the forecasting models using the ratio of Root Mean Squared Errors (RMSEs), the Clark and West ([2007\)](#page-31-0) test, and the

¹⁷The VVR coefficient remains qualitatively similar across subsamples when conditioning on the first five principal components rather than the CP and LN factors (as in Table IA.3 in the Supplementary Material). Results are available from the authors.

TABLE 6

Giacomini and White ([2006\)](#page-32-0) test for predictive ability. Using the Clark–West (CW) test, we evaluate the null hypothesis of equal predictive ability by comparing the mean squared prediction errors of two forecasting models, applied to nested models. The Giacomini–White (GW) test is also a test of equal predictive ability that compares mean squared prediction errors like CW, but it explicitly accounts for parameter uncertainty in the formulation of the null hypothesis.

Table 6 presents the out-of-sample forecasting results for the equal-weighted and the 2-, 3-, 4-, and 5-year excess bond returns. The benchmark model only includes the LN and CP factors. The forecasting models that include VVR generate lower RMSEs than the benchmark model (i.e., RMSE ratios less than 1). The VVR appears to add the most forecasting power for bonds with shorter maturities. This is in line with the in-sample results, where VVR leads to larger increases in R^2 s for bonds with shorter maturities.

The difference in the out-of-sample forecasting power between the model that includes VVR and the benchmark model with only the CP and LN factors is statistically significant. The CW test results show that the model that includes VVR has superior predictive ability compared to the benchmark model. These results are confirmed when using the stricter Giacomini and White ([2006\)](#page-32-0) test.

V. Do Our Results Capture Liquidity Premia?

An alternative explanation for our result that VVR positively predicts bond premia is that equity VVR captures liquidity premia in the bond market, via a spillover in liquidity from the equity to the bond market; see Chordia, Sarkar, and Subrahmanyam [\(2005](#page-31-0)) and Goyenko, Subrahmanyam, and Ukhov [\(2011\)](#page-32-0). VVR could be highly correlated with liquidity risk. This issue is important, and we discuss it from several perspectives: liquidity levels, liquidity risk, and funding liquidity levels and risk.

We start with the effect of liquidity level spillovers. First, liquidity premia are more likely to manifest in illiquid treasury bonds. Amihud and Mendelson [\(1991](#page-31-0)) show that treasury bills (with less than a 1-year maturity) are the most liquid U.S. treasuries. The results using monthly Fama treasury returns in [Table C2](#page-30-0) show that the VVR effect is economically and statistically significant at all maturities, from less than 1 year up to 10 years. Adrian, Fleming, and Vogt ([2017\)](#page-31-0) and Nguyen,

Engle, Fleming, and Ghysels ([2020\)](#page-32-0) show that among treasury notes, the main dependent variable in our analysis, the 2-year note is the most liquid. Results in [Tables 3](#page-14-0)–[5](#page-18-0) show that the equity market VVR effect is economically and statistically significant at all maturities from 2 to 5-years, and its explanatory power (R^2) is higher at shorter maturities. The large economic and statistical effect of VVR at the more liquid maturities is contrary to a liquidity premium.

Second, we directly control for the level of illiquidity in the equity market in our main regression. We construct an equity market aggregate bid–ask spread measure, illiquidity level proxy, as the value-weighted average of the individual stocks' average monthly bid–ask spread. Results in column 1 of Table IA.4 of the Supplementary Material show that controlling for average equity market liquidity does not qualitatively affect the relation between VVR and excess bond returns. The spread-based illiquidity measure has the opposite sign of VVR (i.e., higher spreads in the equity market predict lower average excess bond returns). The coefficient is statistically insignificant when using bootstrapped standard errors.¹⁸

Finally, we distinguish between the VVR effect and bond liquidity premia by investigating whether contemporaneous changes in bond and equity VVR accord with liquidity spillovers. In unreported analysis, we observe that changes in bond and equity VVR are not significantly correlated. The correlation varies between 0.01 for the monthly difference to -0.03 for the annual difference, none of which are statistically significant. Furthermore, we find that the predictability of VVR is mainly derived from periods when equity VVR moves in the opposite direction of bond VVR (i.e., when there is no spillover across the markets).

A. Liquidity Risk

Our evidence in the previous section is on liquidity levels and does not preclude the existence of liquidity risk spillovers across markets. To address this issue, we use the Pastor and Stambaugh ([2003\)](#page-33-0) factor as an additional control in the main analysis. We focus on the traded liquidity factor (LIQ_V) measured as the value-weighted return on the 10-1 portfolio from a sort on historical liquidity betas Pastor and Stambaugh [\(2003](#page-33-0)).¹⁹ Results in Panel A of [Table 7](#page-21-0) show that there indeed is evidence of a liquidity risk spillover from the equity to the treasury bond market. LIQ_V is statistically significant on its own and when controlling for other factors (CP and LN), as well as VVR. However, the liquidity risk effect complements the VVR effect and does not affect the economic and statistical significance of VVR as compared to the results in [Table 5](#page-18-0).

In addition, we separate the VVR level effect from any effect deriving from the volatility in VVR. Specifically, we control for the variation in VVR, measured as the standard deviation of VVR over a rolling window of 12 months. The results reported

¹⁸The results remain qualitatively similar when using an equal-weighted measure for median separated spread of large and small stock portfolios, which are market value-weighted spread averages within the portfolio, and when using the levels of aggregated liquidity from Pastor and Stambaugh [\(2003](#page-33-0)). These results are available upon request. 19 The data starting from Jan. 1968 are available from Rob Stambaugh's website at [http://finance.](http://finance.wharton.upenn.edu/~stambaug/liq_data_1962_2020.txt)

[wharton.upenn.edu/~stambaug/liq_data_1962_2020.txt.](http://finance.wharton.upenn.edu/~stambaug/liq_data_1962_2020.txt) The results, available upon request, are qualitatively similar when using the innovation in aggregated liquidity (i.e., the nontraded liquidity factor).

TABLE 7 VVR and Liquidity Premia

Table 7 presents monthly forecasting regression results for excess bond returns of the form $\overline{r^e}_{t+12} = \beta_0 + \beta_1' \mathbf{X}_t + \overline{\epsilon}_{t+12}$ and
liquidity premia $\overline{r^e}$ is the equal-weighted vearly excess bond return, an liquidity premia. \overline{r} is the equal-weighted yearly excess bond return, and VVR is the yearly change in log volatility-to-volume ratio as described in [Section III.A.](#page-10-0) Panel A presents the relation with equity market liquidity risk. LIQ_V is the traded liquidity factor measured as the value-weighted return on the 10–1 portfolio from a sort on historical liquidity betas (Pastor and Stambaugh [\(2003](#page-33-0))). The sample period is Jan. 1968 to Dec. 2017 as per data availability in [http://finance.wharton.upenn.](http://finance.wharton.upenn.edu/~stambaug/liq_data_1962_2020.txt) [edu/~stambaug/liq_data_1962_2020.txt](http://finance.wharton.upenn.edu/~stambaug/liq_data_1962_2020.txt). Panel B presents the relation with funding liquidity premia. L_t is the funding liquidity measure from Fontaine and Garcia ([2012](#page-32-0)) based on mispricing of bonds with similar characteristics but different ages. The sample period is Jan. 1987 to Dec. 2016. Panel C presents the relation with funding constraints. TED is the Ted spread, and HPW is the Hu et al. [\(2013](#page-32-0)) funding risk measure constructed using yield errors or differences between observed market yields and model-implied yields based on Svensson ([1994](#page-33-0)). The sample period Jan. 1987 to Dec. 2016 as per data availability. The VVR and L_t coefficients are presented in % points. p-values are calculated using the Newey–West correction for
heteroscedasticity and autocorrelation with 18 lags. *, **, and *** denote significance at the 10%, 5%, and 1 respectively.

in column 2 of Table IA.4 of the Supplementary Material show that controlling for the variability in VVR does not affect the economic significance of VVR.

B. Funding Liquidity

A final concern is the role of funding liquidity, which is an important driver of the transaction-cost-based notion of liquidity; see Brunnermeier and Pedersen ([2009\)](#page-31-0) and Kiyotaki and Moore [\(2019](#page-32-0)). We now assess whether our results proxy for funding constraints.

Fontaine and Garcia [\(2012](#page-32-0)) measure a latent liquidity premium in treasury securities by estimating price differentials between pairs of bonds with similar cash flows but different ages. Based on the argument that older bonds are less liquid, they use this quantity as a proxy for funding liquidity.²⁰ We use the Fontaine and Garcia ([2012\)](#page-32-0) liquidity factor L_t as a conditioning variable in the bond premium

²⁰The phenomenon of different yields between bonds with similar cash flows but different maturities is widely documented (e.g., Amihud and Mendelson ([1991\)](#page-31-0), Pasquariello and Vega ([2009\)](#page-33-0)).

regressions, which is only available from 1987 to 2016. Panel B of [Table 7](#page-21-0) shows the relation between the average premium, VVR, and the funding liquidity level of Fontaine and Garcia [\(2012\)](#page-32-0). Higher L_t implies higher liquidity levels. The results are consistent with the findings of Fontaine and Garcia ([2012\)](#page-32-0). However, the addition of the funding liquidity factor variable does not affect the economic and statistical significance of the equity market VVR.

We use two additional measures of funding illiquidity as controls in our main regression for robustness. Specifically, we include the TED spread and the market funding liquidity factor suggested by Hu et al. [\(2013](#page-32-0)). The latter considers the connection between the amount of arbitrage capital in the market and observed price deviations in treasury bonds. The measure is constructed using yield errors, that is, differences between observed market yields and model-implied yields based on Svensson ([1994\)](#page-33-0).²¹ Panel C of [Table 7](#page-21-0) includes these measures in the predictability regressions $\overline{r^e}_{t+12} = \beta_0 + \beta'_1 X_t + \overline{\varepsilon}_{t+12}$. The additional measures have varying effects on excess bond returns, but they do not affect the predictive ability of the equity market VVR.

Overall, the above results indicate that our findings on the link between aggregate equity VVR and treasury bond returns are distinct from funding liquidity and liquidity risk. Note, however, that we do not rule out the possibility of liquidity premia in the cross-section of equity and bond returns.

VI. Other Robustness Checks

In this section, we investigate the robustness of our results along additional dimensions. First, we investigate the role of the volatility-volume ratio of the treasury market itself. Second, we investigate the effect of VVR above macroeconomic variables, like NBER recessions, that might account for our VVR-based predictability. Finally, we also consider how investor sentiment interacts with equity VVR in forecasting bond risk premia.

A. Bond Market Volatility-Volume Ratio

As shown in [Section II.C](#page-8-0), not only the equity VVR, but also the bond VVR can be used to measure the state variable π_t . Thus, in addition to the equity VVR, we include the treasury bond VVR in our regression. To maintain comparability with the equity VVR, we construct a VVR for the treasury bond market. Michael Fleming has provided the monthly trading volumes we use for the on-the-run treasury bonds traded via GovPX and BrokerTec for 2-, 5-, and 10-year maturities for Jan. 1991 to Dec. 2017 period.²² Adrian et al. [\(2017](#page-31-0)) provide a detailed description of the treasury bond market structure, as well as of the trading data. We use monthly bond returns from the Fama–Bliss zero-coupon 2- and 5-year bonds, CRSP monthly data for the 10-year note returns, and the monthly trading

²¹The Hu et al. ([2013\)](#page-32-0) measure, starting from Jan. 1987, is available from Jun Pan's website at [https://](https://en.saif.sjtu.edu.cn/junpan/) [en.saif.sjtu.edu.cn/junpan/.](https://en.saif.sjtu.edu.cn/junpan/)
²²Once issued, the security is considered as on-the-run and the older issues are off-the-run. The bulk

of the trading occurs in on-the-run bonds.

Table 8 presents in-sample forecasting regressions for excess bond returns, using equity and bond market VVRs. The table
presents yearly excess bond return regressions: "वि:+12 = //b +/x/ + /z.+ iz... re is the equal-weigh return. Bond VVR is the treasury market illiquidity measured as the yearly change in the average log volatility-to-volume ratio for treasury bonds of 2-, 5-, and 10-year maturity, CP is the Cochrane Piazzesi factor, LN is the linear combination of the Ludvigson and Ng factors, VVR is the yearly change in the log volatility-to-volume ratio as described in [Section III.A](#page-10-0). The VVR coefficients are presented in % points. The sample period is Jun. 1991 to Dec. 2017. *p*-value are calculated using the Newey–
West correction for heteroscedasticity and autocorrelation with 18 lags. *, **, and *** denote 1% levels, respectively.

volume for each of these three maturities. We construct the monthly VVR for each maturity and then aggregate it to the bond market VVR to match the equal-weighted risk premium. Thus, bond VVR is the equal-weighted VVR from that for the 2-, 5-, and 10-year bonds.

The bond VVR series exhibits a unit root, therefore we take the yearly log difference, in the same way as for the equity market VVR in [Section III.A.](#page-10-0) Table 8 presents the regression results for the average yearly bond returns. Note that the sample period is more than 30 years shorter than that used for the baseline analysis. The findings in Table 8 show that bond market VVR is marginally significant (at the 10% level) on its own in explaining the average yearly risk premium, even though its economic significance is very low. The statistical and economic significance of equity market VVR over the shorter sample period is not affected by the introduction of the bond counterpart. Furthermore, we attempt to control the effects of a treasury bond liquidity risk factor. While we do not have daily data on treasury trading volume, we use the method of Li, Wang, Wu, and He [\(2009](#page-32-0)) to estimate a monthly version of the Pastor and Stambaugh [\(2003](#page-33-0)) liquidity risk measure for bonds. Controlling for this measure leaves the results unchanged. Details are available on request.

B. Macroeconomic States

The state of the macroeconomy is a logical conditioning variable for excess bond returns. One may argue that recessions drive term premia, and VVR provides an early signal on the state of the economy and recessions. Accordingly, we use an NBER recession dummy as a conditioning variable in the predictability regression (12): $\overline{r^e}_{t+12} = \beta_0 + \beta' X_t + \overline{\varepsilon}_{t+12}$. The results in the second and third columns of Panel
A of Table IA 5 in the Supplementary Material show that NBER recessions indeed A of Table IA.5 in the Supplementary Material show that NBER recessions indeed predict bond premia. However, the dummy variable's effect is subsumed by the addition of the CP and LN factors in subsequent columns, and including this variable does not affect the significance of VVR. In addition, our model suggests

that VVR should have higher predictive ability during recessions, since these events represent macroeconomic state shifts. We introduce an interaction term $(VVR \times Rec.)$ between NBER recessions and VVR. The interaction term is positive and highly statistically significant, implying a stronger effect of VVR during recesand highly statistically significant, implying a stronger effect of VVR during recessions, in line with our prediction. This result is also consistent with the findings of Næs, Skjeltorp, and Ødegaard [\(2011](#page-32-0)), who show that VVR predicts turning points in NBER recessions (see their Figure 1 and associated discussion).

C. Sentiment

In our final robustness exercise, we analyze the relation between investor sentiment and VVR. Various papers have shown the importance of investor sentiment for both equity and bond premia, see Baker and Wurgler ([2006\)](#page-31-0) and Laborda and Olmo ([2014\)](#page-32-0). We use measures of investor sentiment provided by Jeff Wurgler as conditioning variables in the bond predictability regression (12) for Jan. 1965 to Dec. 2018 period. The results in Panel B of Table IA.5 in the Supplementary Material show that in accordance with prior research, investor sentiment predicts bond premia in the same direction as VVR. The interaction term between VVR and investor sentiment is also highly statistically and economically significant. The results suggest complementarity between investor sentiment and VVR, as the inclusion of the sentiment variables increases the statistical and economic magnitude of the VVR effect.

VII. Conclusions

We use an informational channel to theoretically and empirically link equity and treasury bond markets. In our setting, some investors receive a potentially informative signal. The signal is valid only when there is an impending shift in the macroeconomic state; otherwise, it is pure noise. When the probability of a state shift is high, the uninformed investors are less willing to trade against the informed, as it is more likely that the informed signal represents fundamental information. A central bank influences the short-term interest rate through random interventions in the event of a state shift. A high probability of a state shift also makes it more likely that the central bank will intervene, which reliably increases bond risk premia. As a result, we predict that a decreasing equity market volume and an increasing equity market volatility (and the equity volatility–volume ratio) predict higher future treasury bond returns. This implication is reliably supported by the data.

Our analysis brings out the notion that high volatility and low volume in the equity market proxy for a greater likelihood of informed trading, and such trading has implications for bond risk premia via a monetary policy channel. It would be interesting to see if these quantities in markets other than equities (such as commodities and foreign currencies) also influence the treasury bond market. This issue is left for future research.

Appendix A. Proof

We state and then prove the following proposition:

Proposition A1. The unique equilibrium in a general finite horizon model with T periods, where the signal informativeness π_t follows an H-state Markov process with transition matrix given by [equation \(11\),](#page-8-0) is signal-revealing with

• The stock price at time $t \in \{1,..,T\}$ in state $h \in \{1,..,H\}$ is

$$
(A-1) \t S_{t,h} = \beta_t \left(\overline{f}_{t,h} + \sigma_{f,t,h} i_t \right).
$$

The coefficients are $\overline{f}_{t,h} = \left(b\hat{f}_{t+1,h} + \delta \right) - \hat{f}_{t+1,h} \frac{\alpha_7 \sigma_q^2 \pi_{t,h}}{1 + \pi_{t,h}} \left[Z_B + \hat{f}_{t+1,h} Z_S \right] ...$ $...-\alpha \times \left(\frac{1}{\gamma \sigma_q^2 \sigma_{f,t+1,h}^2 + b^2 \sigma_{f,t+1,h}^2 + \sigma_d^2} + \frac{1}{\pi_{t,h} \gamma \sigma_q^2 \sigma_{f,t+1,h}^2 + b^2 \sigma_{f,t+1,h}^2 + \pi_{t,h} (1-\pi_{t,h}) n^2 + \sigma_d^2}\right)$ ${}^{-1}Z_{S}$, and $\sigma_{f,t,h} = n \frac{(2\pi_{t,h}\gamma\sigma_q^2 + (1+\pi_{t,h})b^2)\hat{\sigma}_{f,t+1,h}^2 + (1+\pi_{t,h})\sigma_d^2 + \pi_{t,h}(1-\pi_{t,h})n^2 + 2\sigma_t^2)}{(1+\pi_{t,h})\gamma\sigma_t^2 + 2b^2}\hat{\sigma}_{t+1,h}^2 + \frac{1}{2}\pi_{t,h}^2\hat{\sigma}_{t+1,h}^2$ $\int_{\frac{1}{2}}^{\frac{1}{2}} \frac{f(t+\pi_{t,h})b^2}{\sigma_{f,t+1,h}^2 + (1+\pi_{t,h})\sigma_d^2 + \pi_{t,h}(1-\pi_{t,h})n^2}}$, with $\hat{f}_{t+1,h} = \sum_{l=1}^H p_{h,l} \overline{f}_{t+1,l}$, and
 $\int_{\frac{1}{2}}^{\frac{1}{2}} f(t+\pi_{t,h})\sigma_q^2 + 2b^2 \frac{\partial^2 f_{t+1,h} + \pi_{t,h}(1-\pi_{t,h})n^2 + 2\sigma_d^2}{\sigma_d^2}}$, with $\hat{\sigma}_{f,t+1,h} = \sum_{l=1}^{H} p_{h,l} \left(\overline{f}_{t+1,l}^2 + \sigma_{f,t+1,l}^2 \right)$ $\left(\overline{f}_{t+1,l}^2 + \sigma_{f,t+1,l}^2\right) - \hat{f}_{t+1,h}^2$.

• The two-period bond price at time $t \in \{1, ..., T\}$ in state $h \in \{1, ..., H\}$ is

(A-2)
$$
B_{t,h} = \beta_t \times \left(b - \frac{\alpha \pi_{t,h} \gamma \sigma_q^2}{1 + \pi_{t,h}} \left(Z_B + \hat{f}_{t+1,h} Z_S\right)\right).
$$

• The portfolio holdings are specified as in equations $(A-6)$ – $(A-9)$.

A.1. To Show

We show, via mathematical induction, that the unique equilibrium price process has the following functional form

$$
(A-3) \t St,h = \beta_t (\overline{f}_{t,h} + \sigma_{f,t,h} i_t), \quad \forall h \in \{1,...,H\}.
$$

We derive the backward recursion for the coefficients f_{th} and $\sigma_{f,t,h}$, $\forall h \in \{1, ..., H\}.$

A.2. Derivation

A.2.1. Starting Case

The terminal period $t = T$ is the starting case for the backward induction. The assumption $S_T = 0$ implies $\sigma_{f,T,h} = 0$ and $\overline{f}_{T,h} = 0$. Thus, the base case satisfies equation (A-3).

A.2.1.1. Recursion

In the recursion, we assume that the unique equilibrium price function at time $t + 1$ has the form $S_{t+1,l} = \beta_t (\bar{f}_{t+1,l} + \sigma_{f,t+1,l} i_{t+1})$, for all $l \in \{1, ..., H\}$. Investors maximize their expected utility at time t in Markov state h

$$
\text{maxE}_{t,h}^{j}[w_{t+1}^{j}] - \frac{\alpha}{2} \text{VAR}_{t,h}^{j}(w_{t+1}^{j}),
$$

subject to the budget constraint

$$
w_{t+1}^j = x_{S,t,h}^j \left(S_{t+1} + D_{t+1} - \frac{1}{\beta_t} S_{t,h} \right) + x_{B,t,h}^j \left(\beta_{t+1} - \frac{1}{\beta_t} B_{t,h} \right) + \frac{1}{\beta_t} w_t.
$$

Optimizing over $x_{S,t,h}^j$ and $x_{B,t,h}^j$ for uninformed $(j = U)$ and overconfident $(j = O)$ investors, we get the first-order conditions

(A-4)
$$
E_{t,h}^j\bigg[S_{t+1}+D_{t+1}-\frac{1}{\beta_t}S_{t,h}\bigg]=\frac{\alpha x_{S,t,h}^j \text{VAR}_{t,h}^j(S_{t+1}+D_{t+1})}{+\alpha x_{B,t,h}^j \text{COV}_{t,h}^j(S_{t+1}+D_{t+1},\beta_{t+1}),
$$

$$
(A-5) \quad E'_{t,h} \left[\beta_{t+1} - \frac{1}{\beta_t} B_{t,h} \right] = \alpha x'_{B,t,h} \nabla A R_{t,h}' \left(\beta_{t+1} \right) + \alpha x'_{S,t,h} \nabla O V_{t,h}' \left(S_{t+1} + D_{t+1,h} \beta_{t+1} \right).
$$

Overconfident investors have the following subjective expectations and variances:

$$
E_{t,h}^{O}[S_{t+1} + D_{t+1}] = b \times \hat{f}_{t+1,h} + \delta + n \times i_t
$$

\n
$$
E_{t,h}^{O}[\beta_{t+1}] = b
$$

\n
$$
VAR_{t,h}^{O}(\beta_{t+1}) = \gamma \sigma_q^2
$$

\n
$$
COV_{t,h}^{O}(S_{t+1} + D_{t+1}, \beta_{t+1}) = \gamma \sigma_q^2 \times \hat{f}_{t+1,h}
$$

\n
$$
VAR_{t,h}^{O}(S_{t+1} + D_{t+1}) = \gamma \sigma_q^2 \hat{\sigma}_{f,t+1,h}^2 + \hat{f}_{t+1,h}^2 \gamma \sigma_q^2 + b^2 \hat{\sigma}_{f,t+1,h}^2 + \sigma_d^2,
$$

with $\hat{f}_{t+1,h} = \sum_{l=1}^{H} p_{h,l} \overline{f}_{t+1,l}$ and $\hat{\sigma}_{f,t+1,h} = \sum_{l=1}^{H} p_{h,l} (\overline{f}_{t+1,l}^2 + \sigma_{f,t+1,h}^2)$ with $\hat{f}_{t+1,h} = \sum_{l=1}^{H} p_{h,l} \vec{f}_{t+1,l}$ and $\hat{\sigma}_{f,t+1,h} = \sum_{l=1}^{H} p_{h,l} (\vec{f}_{t+1,l}^2 + \sigma_{f,t+1,l}^2) - \hat{f}_{t+1,h}^2$. Inserting the subjective expectations and variances into equations (A-4) and (A-5), we obtain the optimal portfolio holdings of overconfident, informed investors as

(A-6)
$$
x_{B,t}^{O} = \frac{\gamma \sigma_q^2 \hat{\sigma}_{f,t+1}^2 + \hat{f}_{t+1}^2 \gamma \sigma_q^2 + b^2 \hat{\sigma}_{f,t+1}^2 + \sigma_d^2}{\left(\alpha \gamma \sigma_q^2\right) \left(\gamma \sigma_q^2 \hat{\sigma}_{f,t+1}^2 + b^2 \hat{\sigma}_{f,t+1}^2 + \sigma_d^2\right)} \left(b - \frac{1}{\beta_t} B_t\right) - \frac{\hat{f}_{t+1}}{\alpha} \frac{1}{\gamma \sigma_q^2 \hat{\sigma}_{f,t+1}^2 + b^2 \hat{\sigma}_{f,t+1}^2 + \sigma_d^2} \left(b \hat{f}_{t+1} + \delta + n i_t - \frac{1}{\beta_t} S_t\right),
$$

(A-7)
$$
x_{S,t}^{O} = \frac{1}{\alpha \gamma \sigma_q^2 \hat{\sigma}_{f,t+1}^2 + b^2 \hat{\sigma}_{f,t+1}^2 + \sigma_d^2} \times \left(b \hat{f}_{t+1} + \delta + n i_t - \frac{1}{\beta_t} S_t \right) - \frac{\hat{f}_{t+1}}{\alpha \gamma \sigma_q^2 \hat{\sigma}_{f,t+1}^2 + b^2 \hat{\sigma}_{f,t+1}^2 + \sigma_d^2} \left(b - \frac{1}{\beta_t} B_t \right).
$$

Uninformed investors can infer the signal from the observation of the market prices and the residual demand. Their expectations and variances of the next period's realizations are

$$
E_{t,h}^U[S_{t+1} + D_{t+1}] = b \times \hat{f}_{t+1,h} + \delta + \pi_{t,h} n \times i_t
$$

\n
$$
E_{t,h}^U[\beta_{t+1}] = b
$$

\n
$$
VAR_{t,h}^U(\beta_{t+1}) = \pi_{t,h} \gamma \sigma_q^2
$$

\n
$$
COV_{t,h}^U(S_{t+1} + D_{t+1}, \beta_{t+1}) = \pi_{t,h} \gamma \sigma_q^2 \times \hat{f}_{t+1,h}
$$

\n
$$
VAR_{t,h}^U(S_{t+1} + D_{t+1}) = \pi_{t,h} \gamma \sigma_q^2 \hat{\sigma}_{f,t+1,h}^2 + \hat{f}_{t+1,h}^2 \pi_{t,h} \gamma \sigma_q^2 + b^2 \hat{\sigma}_{f,t+1,h}^2 + \pi_{t,h} (1 - \pi_{t,h}) n^2 + \sigma_d^2.
$$

This leads to the following optimal portfolio holdings for uninformed investors:

$$
(A-8) \t x_{B,t}^U = \frac{\pi_{t,h}\gamma\sigma_q^2\hat{\sigma}_{f,t+1}^2 + \hat{f}_{t+1}^2\pi_{t,h}\gamma\sigma_q^2 + b^2\hat{\sigma}_{f,t+1}^2 + \pi_{t,h}(1-\pi_{t,h})n^2 + \sigma_d^2}{\left(\alpha\pi_{t,h}\gamma\sigma_q^2\right)\left(\pi_{t,h}\gamma\sigma_q^2\hat{\sigma}_{f,t+1}^2 + b^2\hat{\sigma}_{f,t+1}^2 + \pi_{t,h}(1-\pi_{t,h})n^2 + \sigma_d^2\right)} \left(b - \frac{1}{\beta_t}B_t\right) - \frac{\hat{f}_{t+1}}{\alpha} \frac{1}{\pi_{t,h}\gamma\sigma_q^2\hat{\sigma}_{f,t+1}^2 + b^2\hat{\sigma}_{f,t+1}^2 + \pi_{t,h}(1-\pi_{t,h})n^2 + \sigma_d^2}\left(b\hat{f}_{t+1} + \delta + n\pi_{t,h}i_t - \frac{1}{\beta_t}S_t\right),
$$

$$
\begin{split} \text{(A-9)} \quad x_{S,t}^U &= \frac{1}{\alpha \pi_{t,h} \gamma \sigma_q^2 \hat{\sigma}_{f,t+1}^2 + b^2 \hat{\sigma}_{f,t+1}^2 + \pi_{t,h} (1 - \pi_{t,h}) n^2 + \sigma_d^2} \times \left(b \hat{f}_{t+1} + \delta + n \pi_{t,h} i_t - \frac{1}{\beta_t} S_t \right) \\ &- \frac{\hat{f}_{t+1}}{\alpha} \frac{1}{\pi_{t,h} \gamma \sigma_q^2 \hat{\sigma}_{f,t+1}^2 + b^2 \hat{\sigma}_{f,t+1}^2 + \pi_{t,h} (1 - \pi_{t,h}) n^2 + \sigma_d^2} \left(b - \frac{1}{\beta_t} B_t \right). \end{split}
$$

Using the market clearing conditions $x_{S,t,h}^O + x_{S,t,h}^U = Z_S$ and $x_{B,t,h}^O + x_{B,t,h}^U = Z_B$, we the following unique equilibrium stock price equation for time t. obtain the following unique equilibrium stock price equation for time t:

$$
S_{t,h} = \beta_t \left(\overline{f}_{t,h} + \sigma_{f,t,h} i_t \right),
$$

where

$$
\sigma_{f,t,h} = n \frac{\left(2\pi_{t,h}\gamma\sigma_q^2 + (1+\pi_{t,h})b^2\right)\hat{\sigma}_{f,t+1,h}^2 + (1+\pi_{t,h})\sigma_d^2 + \pi_{t,h}(1-\pi_{t,h})n^2}{\left((1+\pi_{t,h})\gamma\sigma_q^2 + 2b^2\right)\hat{\sigma}_{f,t+1,h}^2 + \pi_{t,h}(1-\pi_{t,h})n^2 + 2\sigma_d^2}
$$

and

$$
\overline{f}_{t,h} = \left(b\hat{f}_{t+1,h} + \delta\right) - \hat{f}_{t+1,h} \frac{\alpha y \sigma_q^2 \pi_{t,h}}{1 + \pi_{t,h}} \left[Z_B + \hat{f}_{t+1,h} Z_S\right] - \frac{1}{\frac{1}{\gamma \sigma_q^2 \hat{\sigma}_{f,t+1,h}^2 + b^2 \hat{\sigma}_{f,t+1,h}^2 + \sigma_d^2} + \frac{1}{\pi_{t,h} \gamma \sigma_q^2 \hat{\sigma}_{f,t+1,h}^2 + b^2 \hat{\sigma}_{f,t+1,h}^2 + \pi_{t,h} (1 - \pi_{t,h}) n^2 + \sigma_d^2}}
$$

A.2.2. Induction Conclusion

From the induction, we conclude that the unique equilibrium stock pricing function is linear in the signal i_t with the functional form $S_{t,h} = \beta_t (\overline{f}_{t,h} + \sigma_{f,t,h} i_t)$. The above
equations provide the formulas for the backward requision equations provide the formulas for the backward recursion.

A.3. Bond Price

From the market clearing conditions, we also obtain the bond price at a generic time t , when the stock price has the structural form as shown in the mathematical backward induction:

$$
B_t = \beta_t \left(b - \frac{\left(\hat{f}_{t+1,h} Z_S + Z_B \right) \pi_{t,h} \alpha y \sigma_q^2}{\left(1 + \pi_{t,h} \right)} \right).
$$

A.4. Portfolio Holdings

We obtain the optimal portfolio holdings by inserting the bond and stock prices into the portfolio holdings in [equations \(A-6\)](#page-26-0)–[\(A-9\)](#page-27-0).

Appendix B. Bootstrap Procedure

We use a bootstrap procedure to conduct small sample inference on the stock market illiquidity variables in the bond return regressions. In particular, we test the significance of variables of interest in the following regression:

$$
(B-1) \t\t y_t = \alpha_0 + \alpha'_1 X_t + \eta_t,
$$

by constructing bootstrap samples of (y_t^*, X_t^*) generated under the null hypothesis that the variable of interest has a regression coefficient equal to zero. To assess its significance, the actual regression coefficient is compared to the distribution of regression coefficients obtained for the bootstrap samples.

Our bootstrap procedure has two important features. First, we sample blocks of 12 subsequent regression residuals n_t to accommodate the autocorrelation in the residuals. Second, our procedure accounts for the endogeneity of the regressors X_t by sampling new sample paths based on a VAR process. The procedure has the following steps:

• Estimate the first-order VAR by OLS on the regressors X_t in

$$
X_{t+1} = \phi_0 + \boldsymbol{\Phi}_1 X_t + \boldsymbol{\zeta}_{t+1}, \quad \boldsymbol{\eta}_t \sim \text{IIDN}(0, \boldsymbol{\Sigma}_{\zeta}).
$$

Store the estimates $\hat{\phi}_0$, $\hat{\Phi}_1$, and $\hat{\Sigma}_{\zeta}$ and calculate the time series of the residuals v_t . Let L denote the Cholesky factorization of $\hat{\Sigma}_{\zeta}$ such that $\hat{\Sigma}_{\zeta} = LL'$. Store the orthogonalized residuals calculated by $w_t = L^{-1}v_t$.

- Run the restricted regression in (B-1) under the null hypothesis. Store the estimates $\hat{\alpha}_0^o$, $\hat{\alpha}_1^o$ and the residuals n_t .
- Generate an artificial sample w_t^* by randomly sampling individual elements $w_{i,t}$ with replacement. Subsequently simulate a new sample path X_t^* of the same length as X_t by starting with $X_1^* = X_1$ and generating subsequent values by: $X_{t+1}^* = \hat{\phi}_0 + \hat{\Phi}_1 X_t^* + L w_t^*.$
- Generate an artificial sample of regression residuals n_t^* by randomly drawing with replacement blocks of 12 subsequent residuals of n_t . Construct an artificial sample of the dependent variable under the null hypothesis as follows: $y_t^* = \hat{a}_0^o + \hat{a}_0^o' \mathbf{Y}^* + \mathbf{P}^*$ $\hat{\alpha}_1^{o'}\overline{\mathbf{X}}_t^* + n_t^*.$
- • Run the full regression [\(B-1\)](#page-28-0) on the artificial sample (y_t^*, X_t^*) and store the coefficient of interest $\hat{\alpha}^*_{1,i}$.
- Repeat steps 3–5 10,000 times.
- Calculate the one-sided bootstrapped p-value of $\hat{a}_{1,i}$ by comparing it to the distribution of the $\hat{\alpha}_{1,i}^*$ for the artificial samples. The *p*-value is calculated as the fraction of $\hat{\alpha}_{1,i}^*$ s that exceeds $\hat{\alpha}_{1,i}$.

Appendix C. Monthly Bond Portfolio Returns

An extensive literature highlights the importance of addressing spurious regression bias in predictive regressions with persistent variables (e.g., Stambaugh [\(1999](#page-33-0)), Amihud and Hurvich [\(2004](#page-31-0))). The overlapping scheme we adopt in the bond return regressions in [Section IV](#page-13-0) might induce strong autocorrelation. Although we HACcorrect and VAR-bootstrap the standard errors, concerns may remain. Therefore, we investigate the validity and robustness of our results using monthly returns for portfolios of treasury bills and bonds. For this analysis, we also use the monthly change in $ln(VVR)$ to match the maturity of the right-hand-side variable. For time t in monthly units, we define $VVR_{m,t} = \ln(VVR_t) - \ln(VVR_{t-1})$.

While this setup is different from Cochrane and Piazzesi ([2005\)](#page-31-0) and our earlier exercise in studying annual returns, Duffee ([2012\)](#page-31-0) argues that predicting monthly excess returns of bond portfolios provides an alternative test to the statistical significance of predictive variables. The use of monthly bond returns and monthly changes in the volatility-to-volume ratio reduces any concerns related to overlapping observations and an autoregressive error structure. We repeat the spanning hypothesis analysis in [Section IV.B](#page-14-0) using the monthly LS bond portfolio returns as the dependent variable and monthly log changes in the volatility-to-volume ratio (VVR_m) as the independent variable.²³

[Table C1](#page-30-0) presents the [Table 4](#page-16-0)-equivalent results for the monthly bond portfolio returns:

(C-1)
$$
r^e{}_{m,t+1} = O_0 + \theta'_1 X_t + \varepsilon_{m,t},
$$

where r_{m}^{e} is the average bond portfolio return, averaged over maturities from 1 to 10 years. As before, there is a positive relation between VVR and excess bond returns, beyond 3 and 5 principal components. The bias-corrected bootstrap (Bauer and Hamilton [\(2017\)](#page-31-0)) *p*-value continues to reject the spanning hypothesis. The stock VVR is highly significant, and the coefficient implies that an increase in VVR of 1-standarddeviation increases monthly excess bond returns by 11 bps.

Duffee [\(2012](#page-31-0)) proposes the use of Fama constant maturity CRSP bond portfolio returns with maturities up to 1 year, between 1 and 2 years, 2 and 3 years, 3 and 4 years, 4 and 5 years, and 5 and 10 years. We also investigate the validity and robustness of our results using monthly returns for portfolios of treasury bills and bonds, as in Duffee ([2012\)](#page-31-0). We obtain excess returns by subtracting the 1-month T-bill rate from the

 23 Gargano, Pettenuzzo, and Timmermann [\(2019](#page-32-0)) use a different method to calculate monthly excess returns based on a tradable strategy by market participants. We also use the Gargano et al. [\(2019\)](#page-32-0) monthly excess bond returns and find qualitatively similar results to those in [Table C1](#page-30-0). We thank the authors for very kindly sharing the tradable portfolio data with us. Results are available upon demand from the authors.

TABLE C1

Volatility-to-Volume Ratio and Monthly Returns

Table C1 presents the monthly forecasting regression results for equal-weighted bond portfolio returns: $\overline{r^e}_{m,t+1}$ =
 $O_0 + \theta^2 \mathbf{X}_t + \overline{\varepsilon}_{m,t}$. $\overline{r^e}_{m,t}$ the equal-weighted monthly bond portfolio return from the five principal components (PCs) of the term structure. VVR^m is the monthly change in log volatility-to-volume ratio as described in [Appendix C](#page-29-0). VVR_m VW in Panel A is the value-weighted market VVR and VVR_m MDW in Panel B is the equally weighted average of the market value weighting of stocks' VVR split into large and small stocks by median market capitalization. All coefficients are presented in % points. The sample period is Jan. 1964 to Dec. 2017. *p*-val is the *p*-value calculated using the Newey–West correction for heteroscedasticity and autocorrelation with 18 lags. The *p-*values based on
the bias-corrected bootstrap analysis (Bauer and Hamilton ([2017\)](#page-31-0)) are presented in parentheses. *, components.

TABLE C2

Volatility-to-Volume Ratio and Monthly Bond Portfolio Returns

Table C2 presents monthly regressions for bond portfolios with different maturities, $r_{m,t+1}^{\theta,(n)} = O_0 + \theta'_1 \mathbf{X}_t + \epsilon_{t+1}^{(n)}$, where $r_m^{\theta,(n)}$ is the
average bond risk premium and the bond risk premium of maturity p average bond risk premium and the bond risk premium of maturity n. Bond premia are calculated from Fama constant maturity portfolios from CRSP. VVR_m is the monthly change in log volatility-to-volume ratio as described in [Appendix C](#page-29-0). VVR_m VW in Panel A is the value-weighted market VVR and VVR_m MDW in Panel B is the equal-weighted average of the market value weighting of stocks' VVR split into large and small stocks by median market capitalization. The VVR_m coefficient is presented in % points. The sample period is Jan. 1964 to Dec. 2018. The p-value calculated using the Newey–West correction for autocorrelation and heteroscedasticity is presented in square brackets. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

portfolio returns. The results in [Table C2](#page-30-0) are qualitatively similar to those reported in [Table C1.](#page-30-0) In addition, the statistical significance of VVR is high for each of the 6 individual portfolio return regressions. The addition of the stock market VVR to the CP and LN factors increases the adjusted R^2 by 1% for all maturities.

Supplementary Material

Supplementary Material for this article is available at [https://doi.org/10.1017/](https://doi.org/10.1017/S0022109022000497) [S0022109022000497.](https://doi.org/10.1017/S0022109022000497)

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