RESISTIVE INSTABILITIES IN ASTROPHYSICAL CONDITIONS: A CRITICAL DISCUSSION.

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Abstract. Resistive instabilities have often been indicated as the possible cause of rapid release of energy in astrophysical situations. A correct assessment of the validity of this idea requires a detailed analysis of the theory of resistive instabilities in the regimes of astrophysical interest. In particular, effects as the presence of asymmetries due to current gradients, the influence of geometry and of shear flows must be explicitly evaluated. We have started a program of investigation on this subject, whose preliminary results are reported here.

Magnetic fields are one of the commonest components of the astronomical universe. Their presence constitutes, among other things, a potential source of energy that could be utilized in a number of situations. In many instances they are the only conceivable energy source. However, the exploitation of this reserve is generally made difficult by the fact that astrophysical plasmas are, as a rule, almost ideal electrical conductors. The Alfvén theorem then prevents topological changes in the field structure and makes inaccessible to the system many states with low magnetic energy. The presence of a small, but finite, electrical resistivity relaxes this constraint and allows the transformation of magnetic energy into different forms, like heat or bulk particle motion.

The above considerations explain the interest in resistive instabilities, namely those that either disappear in the infinite conductivity limit, or strongly modify instabilities that may exist also in ideal conditions. Most of our knowledge of this class of instabilities derives from studies of laboratory plasmas and often results valid for fusion devices have been directly applied to astrophysical systems in spite of the vast difference in regimes, boundary conditions and geometrical constraints encountered in the two cases. Another problem that has often prevented a successful application of the theory of resistive instabilities in the astrophysical context is that of the

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timescale of magnetic energy release. In the case of the solar flares, for instances, the resistive timescales are substantially larger than those on which the phenomenon is observed to develop.

Considering the interest of the problem, the number of still unanswered questions and the fact that from our experience it is hard to predict the behaviour of a given system by simply transferring the results of a completely different one, we have felt the necessity of a detailed analysis of resistive instabilities in regimes of direct astrophysical interest. The first results of such a study are briefly reported here and will form the subject of subsequent papers. All these results refer to the linear stage of the instability.

The linear development of resistive instabilities in plane geometry has been extensively studied since the pioneering FKR paper (Furth, Killeen and Rosenbluth, 1963) both analytically and numerically. Analytic investigations generally rely on boundary layer techniques and often require further approximations, as for instance the so-called "constant-ψ". Numerical investigations have until recently been limited to relatively low S values, where S is the magnetic Reynolds number. An extensive numerical study has been published recently (Steinolfson and Van Hoven, 1983) where the range of S has been increased up to astrophysically relevant values. We have included in our numerical code the effects of gradients of the current density at the singular points that produce asymmetries in the F-profile ($F = \mathbf{k}.\mathbf{B}$). The reason for this inclusion derives from the fact that asymmetries are always present in geometries more realistic than the planar one, so that the entity of these effects can be only assessed in the plane case. The field profile was the familiar $\mathbf{B} = B_o$ (tgh x $\mathbf{e}_y$+ sech x $\mathbf{e}_z$), with $\mathbf{k} = k_y \mathbf{e}_y + k_z \mathbf{e}_z$. The equations were Fourier time-analyzed, unlike in Steinolfson and Van Hoven, 1983. Our analysis generally confirms all classical results on symmetric configurations along with the only analytical results for the asymmetric case (Bertin, 1982) valid in the framework of a generalized constant-ψ approach. Our growth rates are lower than those of Steinolfson and Van Hoven, 1983 at small and large values of the normalized wavenumber, α, for large S ($\gtrsim 10^{10}$). The presence of asymmetries modifies the dispersion curve (20-50%) especially at low α's but has little influence on the form of the eigenfunctions.

We have also examined the resistive behaviour of a system with cylindrical symmetry under modes possessing an m = 1 azimuthal symmetry. The equilibrium field represents a current channel surrounded by a potential field. This type of field had already been studied by Chiuderi and Einaudi, 1981 (hereafter referred to as CE) that showed that configurations exist that are absolutely (i.e. for all k's) stable to ideal perturbations. It is, however, also possible to find ideally unstable configurations for low k-values. A first study of the linear tearing-mode in these systems had also been given in CE, by using the same technique employed by Furth et al. (1973). The use of the constant-ψ approximation, implicit in CE, must be considered with caution in these configurations even at relatively large α's (unlike in

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the planar case) because of the possible existence of an ideal unstable region for $0 \le \alpha \le \alpha_c$. In fact, when this happens we may have sizable α's corresponding to a marginal behaviour. For $\alpha \gtrsim \alpha_c$ the resistive modes appear to be of the reconnecting (tearing) type, whereas for $\alpha \lesssim \alpha_c$ a different mode develops, quite similar to the m = 1, n = 1 internal resistive kink found in tokamaks and studied by Coppi et al. (1976).

The computational method used is essentially a numerical boundary layer with the internal solution computed exactly. A shooting technique is used to find the eigenvalues: this method appears to be fast and accurate enough up to $S = 10^9$, but is not reliable at higher S-values due to the extreme thinness of the resistive layer. There are several distinct aspects of the problem that must be considered. One is the spatial structure of the mode that is strongly influenced by the value of α, particularly for α close to α_c. Crossing α_c from the ideally stable side, we find that the perturbation corresponds to an essentially rigid radial displacement of the internal part of the plasma inside the current channel towards the singular surface. This type of perturbation is possible also in an ideal plasma (internal kink) but saturates at low amplitude due to the build-up of a magnetic counter-pressure. The presence of a finite conductivity allows the continuation of the process. In this situation the perturbed magnetic field (ψ) changes sign, but the displacement does not, unlike the usual reconnecting mode. This different behaviour may prove to be important in the subsequent non-linear stage.

A second aspect refers to the maximum growth rate attainable by these processes. Since the constant-ψ reconnecting and internal resistive kink modes scale differently with S, the concept of fast and slow reconnecting modes has been sometimes introduced in the literature. We would like to point out that, on the basis of different scaling properties only, it is not possible to assess the relative speed of the two processes. For a given S an unstable configuration attains a maximum growth rate for a well-defined value of α: this may correspond either to an internal resistive structure or to a $\psi \neq$ const tearing mode. To decide which is the configuration that evolves faster we must simply compare the maximum growth rates. From our preliminary results, that incidentally confirm the known S-scalings of the tearing and internal resistive modes, we find that at a given S different configurations corresponding to different spatial structures of the mode evolve on essentially the same timescale. This is due to the fact that the only meaningful comparison must involve the maximum growth rates that do not occur at the same value of α for different configurations and S. From our computations it turns out that at $S = 10^9$ the shortest timescale is $\simeq 10^5$ s, regardless of the topology of the operating mode. Since all timescales increase with S this result implies that resistive instabilities based on classical resistivity in a static configuration are not a viable mechanism for solar flares that occur in plasmas with $S \simeq 10^{12}$.

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The presence of shear flows in the unperturbed state can modifv considerably the growth rates of the resistive modes (Hoffmann, 1975, Pollard and Taylor, 1979, Dobrowolny et al. 1983). The presence of a fluid velocity considerably complicates the numerical problem since all the growing modes are generally overstable. We have produced a numerical code, based on finite different methods, that computes the (complex) eigenvalues by finding the zeros of the determinant of the coefficients of the homogeneous linear system to which the original differential system has been reduced. The eigenvalues, besides of k, depend on β and on the ratio of the spatial scales for the velocity field and the magnetic field. Initial tests of the code have reproduced known results in selected limiting cases.

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