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N-body Models of Open Clusters

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Abstract. N-body simulations of open cluster evolution with primordial binaries are reviewed. In particular, recent results arising from models with initial N in the range of 20 000–100 000 bodies are compared to earlier idealized models with $N \sim 2\,000$. Efforts to model real clusters are discussed, including how limitations of the models such as simplified initial conditions will be addressed in the near future.

Keywords. stellar dynamics, methods: n-body simulations, binaries: close, open clusters and associations: general

1. Introduction

From the time of the first N-body simulation of star cluster evolution recorded by von Hoerner (1960) the number of particles, N, that can be followed in a reasonable timeframe has risen from ten to of order 100 000 (Baumgardt & Makino 2003; Hurley, Aarseth & Shara 2007). This is due to increased hardware performance, such as the introduction of the GRAPE-4 special-purpose computers (Makino *et al.* 1997), as well as the development of improved computational algorithms. Further developments are required before direct models of globular clusters with the N-body method become feasible. In the meantime, much can be learnt from understanding the evolution of the open cluster type models performed to date.

In the pre-GRAPE era of the early 1990's the N limit was of the order of 2 000 stars. However, a major development at this time was the introduction of primordial binaries in the N-body models. This was important for understanding the evolution of real clusters as observations clearly indicate that open clusters contain a significant primordial binary population (e.g. Mermilliod & Mayor 1990). Two studies at this time by Heggie & Aarseth (1992) and McMillan & Hut (1994) stand out as landmarks because of their in-depth analysis of the effect of the primordial binaries on the cluster evolution and, in reverse, the effect of the evolution on the make-up of the binary population. These were idealized simulations in that only equal-mass stars were considered but they did include the effect of the tidal field of the Galaxy as well as primordial binary frequencies of 3–6%, in the case of Heggie & Aarseth (1992) and up to 20% for McMillan & Hut (1994). This work was extended by de la Fuente Marcos (1996) who looked at models with 33% primordial binaries and stellar masses distributed according to an initial mass function (but no stellar evolution). The models of Kroupa (1995) with $N = 400$ stars and 100% binaries also deserve mention†. The first model to move away from the idealized regime to what has become known as the 'kitchen-sink' regime was that of Aarseth (1996). This was a model starting with 10 000 stars and a 5% binary frequency evolved with the `NBODY4`

† This is by no means an extensive history of N-body simulations – for that the interested reader is referred to Aarseth (2003).

Table 1. Overview of N-body models used in this work.

Model	N_s	N_b	a_{max} [au][1]	M_i [$M_\odot$]	$r_{h,i}$ [pc]	t_f [Myr][2]
B1	9000	9000	200	14405	3.9	5770
B6	9000	9000	10	14010	4.0	5150
S7	30000	0	–	14570	4.2	8460

Notes:
[1] Maximum binary separation.
[2] Time when only 1 000 stars remain bound.

code that is still at the forefront of cluster modelling today. The tidal field of the Galaxy, an initial mass function (IMF) and stellar/binary evolution were all considered.

Recently, Heggie, Trenti & Hut (2006) have begun revisiting the idealized models of Heggie & Aarseth (1992) by extending the models to $N = 16\,000$ and including binary frequencies from 0–100%. This is supplemented in this paper by comparing the results from the pioneering work of Heggie & Aarseth (1992) and McMillan & Hut (1994) to recent 'kitchen-sink' models of binary-rich open clusters performed with NBODY4 (Aarseth 1999). Also included is a brief summary of ongoing efforts to understand observations of actual open clusters and a presentation of preliminary simulations aimed at improving the initial conditions of the cluster models.

2. Binary-rich models

To study the general evolution properties of binary-rich open clusters Hurley, Aarseth, Tout & Pols (in preparation) have evolved a series of N-body models with $N = 20$–$30\,000$ stars and 50% primordial binaries (some models with 40% and 10% binaries were also considered). Parameters varied between the models include the initial density profile of the stars and the distribution of binary binding energies. The aim is to complement the overview of the evolution of single-star open clusters provided in Hurley *et al.* (2004) using models with $N = 30\,000$ stars and 0% binaries.

Here two models from the binary-rich series will be used to make some early comparisons to the findings of Heggie & Aarseth (1992) and McMillan & Hut (1994). These are model B1 – the reference model – and model B6 which differs in setup from B1 only by the maximum orbital separation allowed for the primordial binaries. A single star model from Hurley *et al.* (2004), namely their Model 7 (labeled S7 here), is also used for comparison. An overview of the starting parameters of these three models is given in Table 1 including the model label, the number of single (N_s) and binary (N_b) stars, the cluster mass (M_i) and the half-mass radius ($r_{h,i}$). The maximum orbital separation (a_{max}) is also given, where applicable.

For each model the stellar masses were chosen from the IMF of Kroupa, Tout & Gilmore (1993) with a lower mass limit of $0.1M_\odot$ and an upper limit of $50M_\odot$. The component masses of binaries were set by choosing a mass-ratio, q, from a uniform distribution, $n(q) = 1$. A metallicity of $Z = 0.02$ was assumed in each case.

Orbital separations for the binaries were distributed according to the suggestion of Eggleton, Fitchett & Tout (1989, EFT) with a peak at 30 au. In model B1 an upper limit of $a_{max} = 200$ au was applied – safely in excess of the hard/soft boundary of approximately 70 au for the starting model. By comparison, model B6 took $a_{max} = 10$ au so that all primordial binaries were hard, i.e. tightly bound.

Each model was evolved using the NBODY4 code on a 32-chip GRAPE-6 board (Makino 2002). Stellar and binary evolution are included in NBODY4 as described in Hurley *et al.* (2001). The tidal field of the Galaxy was modeled by placing the model cluster on a

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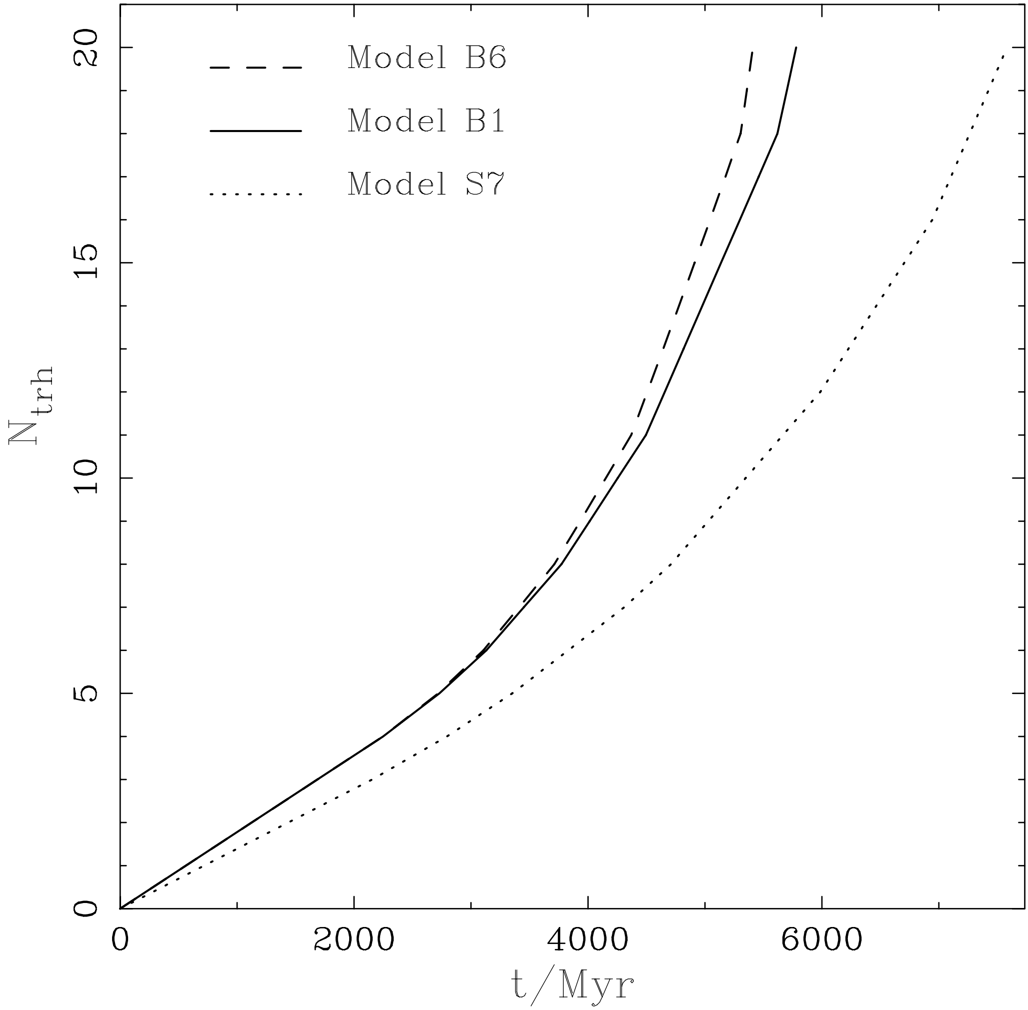

Figure 1. Number of half-mass relaxation times elapsed as a function of cluster age. This is calculated using the 'co-moving' instantaneous half-mass relaxation time which, for an evolved cluster, is typically a factor of 2-3 shorter than the initial half-mass relaxation time.

circular orbit at 8.5 kpc from the center of a point-mass galaxy. Full details of the setup and evolution of these simulations will be provided in Hurley, Aarseth, Tout & Pols (in preparation).

The lifetimes of models B1, B6 and S7 are given in Table 1 as $t_{\rm f}$, the age at which the cluster membership has been reduced to 1 000 stars. Lifetimes can also be compared by looking at Fig. 1 which demonstrates the relative dynamical ages of the models. The dissolution timescale clearly decreases when primordial binaries are included (comparing B1 to S7). This is not surprising as the presence of binaries in the center of the cluster leads to an increase in the escape of stars through velocity kicks imparted in three- and four-body interactions. The escape rate of single stars from B1 is typically 30% greater than for S7 at comparable ages. This result varies little if B6, or a model with 10% primordial binaries, is instead compared to S7, in agreement with the findings of Heggie & Aarseth (1992: see their Fig. 12). As the relaxation timescale of a tidally-limited cluster decreases with decreasing cluster mass, the presence of primordial binaries shortens the

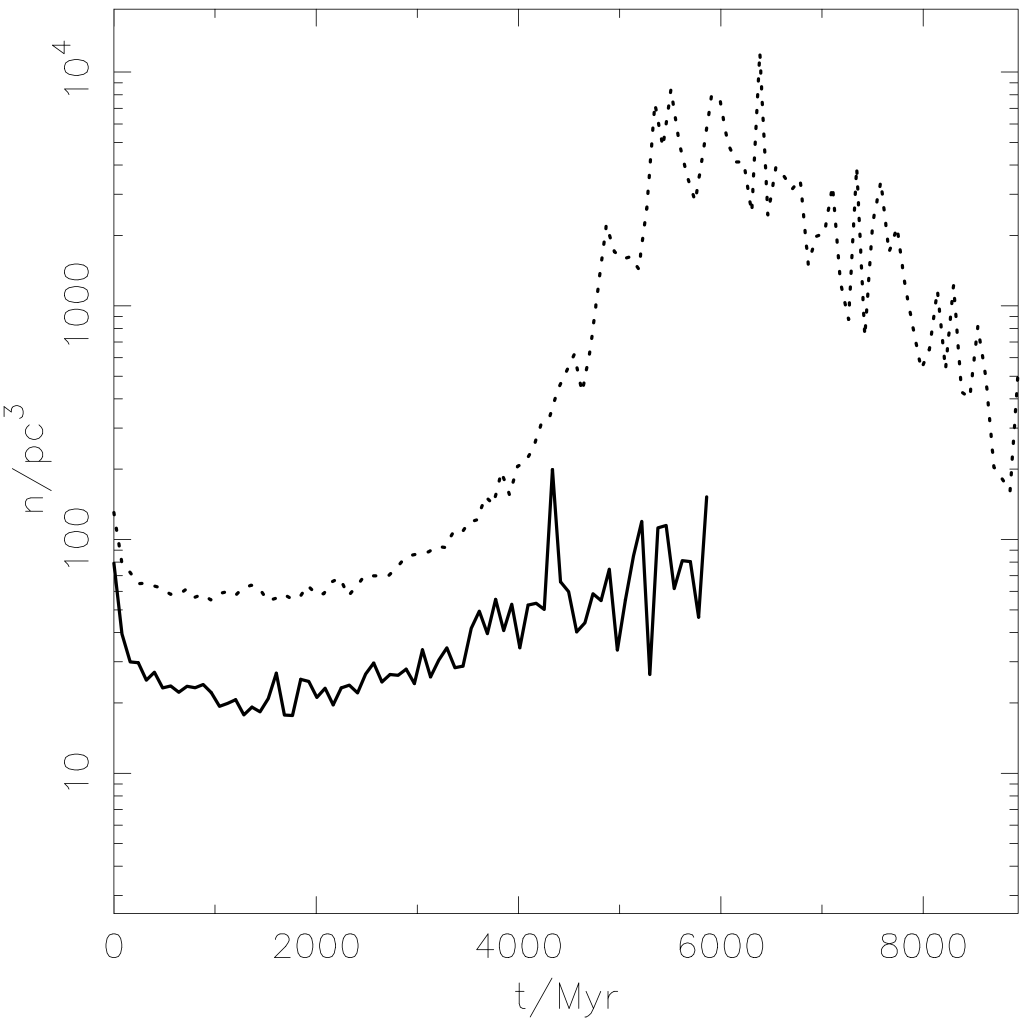

Figure 2. Core density evolution of models B1 (solid line) and S7 (dotted line).

relaxation timescale as the cluster evolves. Therefore, a cluster with primordial binaries is dynamically more evolved than a single star model at the same age (as exhibited in Fig. 1). Comparing the dissolution times of B1 and B6 the lifetime is shorter for the latter. However, the difference is small compared to that between B1/B6 and S7 and may be partly statistical. Analysis of the larger family of models will be able to confirm this. Previously, McMillan & Hut (1994) found that the dissolution timescale was insensitive to details of the primordial binary distribution (provided that the primordial fraction was non-zero). This was the result of mass-segregation saturating the core with 20–30% hard binaries regardless of the (non-zero) initial fraction or the relative distribution of initial binding energies.

Fig. 2 looks at the evolution of the number density of stars in the core of models with (B1) and without (S7) primordial binaries. There is an obvious difference with the single-star model able to achieve a much higher core density. This result is as expected from earlier models which showed that primordial binaries are efficient at reversing core-collapse and inflate the size of the core relative to single-star models (Heggie & Aarseth

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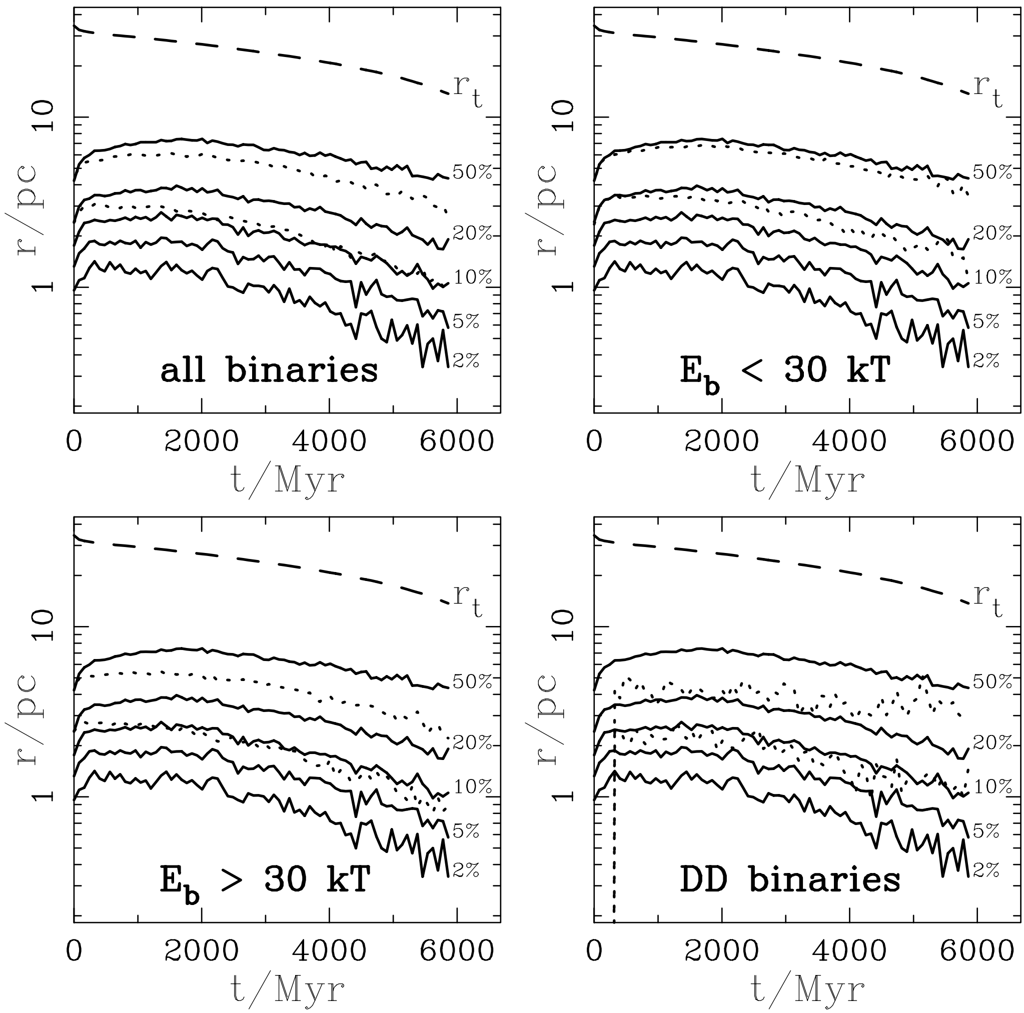

Figure 3. Time evolution of the Lagrangian radii in model B1. In each panel the five solid curves plot the Lagrangian radii containing the innermost 2, 5, 10, 20 and 50% of single stars, by mass. The dashed line denotes the tidal radius of the model cluster. Dotted lines are the 20 and 50% Lagrangian radii for: all binaries (upper-left panel); binaries with $E_{\rm b} < 30\,kT$ (upper-right panel); binaries with $E_{\rm b} > 30\,kT$ (lower-left panel); and, binaries containing two degenerate stars (lower-right panel).

1992; McMillan & Hut 1994). The age at which core-collapse is halted is about 4 000 Myr for B1 and 6 000 Myr for S7. Reference to Fig. 1 shows that this corresponds to a dynamical age of roughly ten half-mass relaxation times in both cases. This is similar to the dynamical age at core-collapse shown by the models of McMillan & Hut (1994) with $N = 1\,000$–$2\,000$ and 0–20% binaries (see their Fig. 2). However, the depth of core-collapse reached by the models presented here with 50% binaries appears to be greater than for the models of McMillan & Hut (1994) with 20% binaries or less, which is somewhat counter-intuitive. In models B1 and B6, as well as a wider range of recent `NBODY4` models with primordial binary fractions of 5% or more, the ratio of core-radius to half-mass radius at the point identified with the end of the core-collapse phase is in the 0.04–0.07 range. This is compared to a ratio of ~ 0.01 for single-star models such as S7. In the

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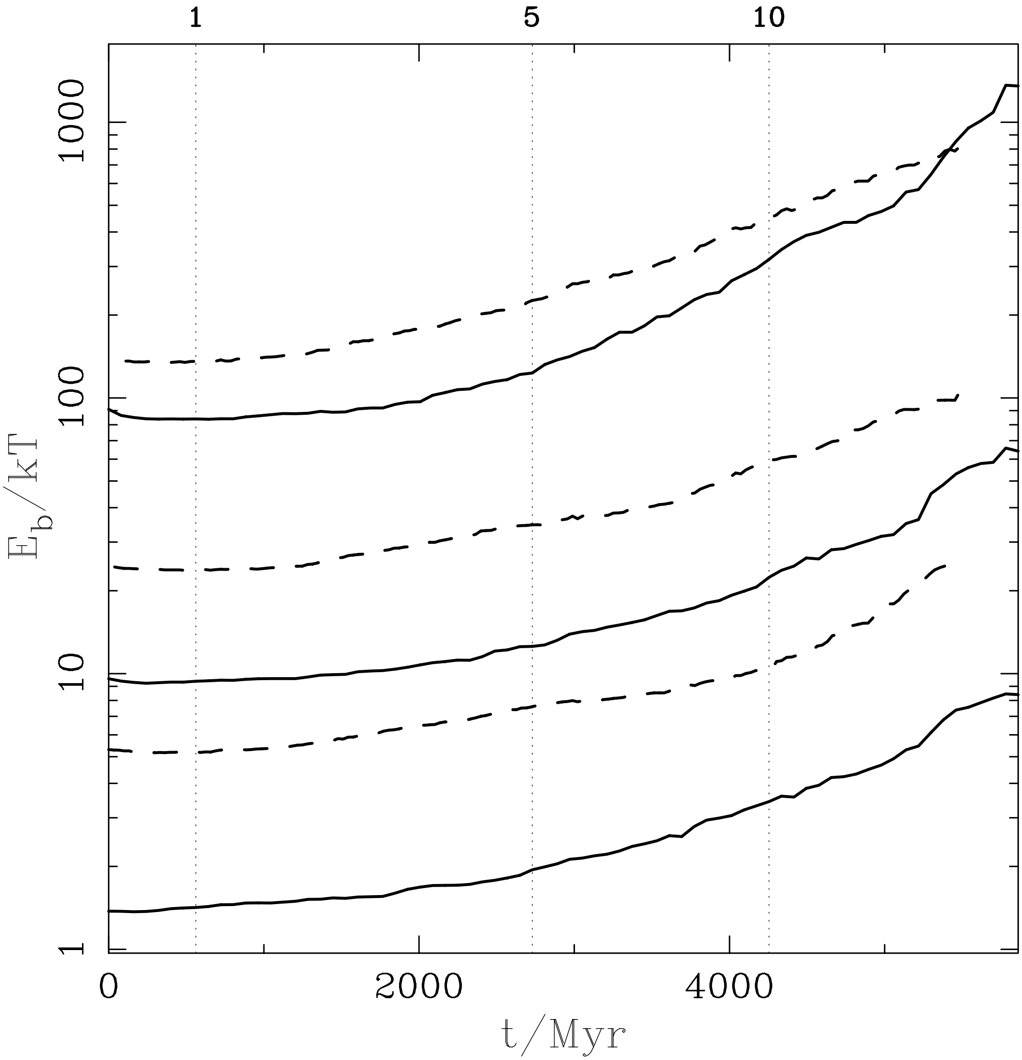

Figure 4. Energy quartiles for binary binding energies in models B1 (solid lines) and B6 (dashed lines). For each model, at any particular time, 25% of the binaries have binding energies below that of the lower curve, 50% have energies below the middle curve and 75% lie below the upper curve. So the hardest binaries lie above the upper curve. For reference, the times at which 1, 5 and 10 half-mass relaxation times have elapsed (for B1) are shown by the dotted lines.

primordial binary models of McMillan & Hut (1994) the core/half-mass radii ratios are typically 0.1 at core-collapse. Heggie & Aarseth (1992) find 0.03 for their model starting with $N = 2\,500$, 3% binaries and a tidal field.

The evolution of the spatial distribution of the binary population in model B1 is investigated in Fig. 3. This can be compared directly to Fig. 4 of McMillan & Hut (1994) and Fig. 8 of Heggie & Aarseth (1992). Note, however, that the latter is for a model without a tidal field. It should also be noted that both of these earlier studies were for models with equal-mass stars in which the mass of a binary was twice that of a single star. In the more recent models with an IMF the average binary mass is only slightly greater than the average single star mass so the effects of mass-segregation, as regards the binary population, will be exaggerated in the earlier models. Looking first at the

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spatial distribution of all binaries in Fig. 3 we see that the binary population is more concentrated towards the center of the cluster than are the single stars. This is clear evidence of mass-segregation and the effect increases with age. The binary population is then split into binaries with binding energies less than or greater than 30 kT, representing approximately the populations of loosely and tightly bound binaries, respectively†. This demonstrates that the spatial distribution of binaries depends upon their binding energy – the hardest binaries are more centrally concentrated (as shown by McMillan & Hut 1994). The distribution of double-degenerate binaries (primarily composed of two white dwarfs) is also shown in Fig. 3 for the sake of interest. These show strong signs of mass-segregation from early in the cluster evolution, being born from the most massive binaries, with the effect becoming weaker at late times as the cluster dissolves.

Related to the segregation of binaries towards the cluster center is the increase in core binary fraction with time highlighted by Hurley, Aarseth & Shara (2007). They looked at a range of models including a model similar to B1 and a model starting with $N = 100\,000$ stars and 5% binaries. Interestingly, the evolution of the core binary fraction in the latter model compares very well to that seen by Heggie & Aarseth (1992) in their model with $N = 2\,500$ stars and 3% binaries. In both the central binary fraction peaks at between 20–30% near the end of core-collapse and then steadily decreases back towards the primordial value thereafter. The results of the $N = 100\,000$ model also show that the critical binary fraction observed by McMillan & Hut (1994) – where clusters starting with less than 10% binaries exhausted their binary population before dissolution – does not scale to larger models.

Fig. 4 is a reproduction of Fig. 6 of McMillan & Hut (1994) showing the energy quartiles for the binary binding energies as the cluster evolves. Clearly visible is the hardening of binaries as the cluster evolves. This is true for both models B1 and B6 although the rate of hardening for the hardest binaries is less in the latter. Heggie & Aarseth (1992) reported a factor of 8 increase in the median binary binding energy as their model clusters evolved from zero-age towards dissolution. They also found that this factor decreased to 3–4 if they considered an extended initial energy range and a higher primordial binary fraction. Models B1 and B6 show a factor of 5–6 increase. In comparison some of the models presented by McMillan & Hut (1994) show an increase by a factor of 10 or more. Further interrogation of the series of binary-rich open clusters models will proceed and spatial and energy evolution of the binary populations to be looked at in detail.

3. Real Clusters

Much effort has been expended in the past decade to improve the realism of N-body codes such as `NBODY4`. Examples include the incorporation of stellar evolution, binary evolution, three- and four-body effects, and external tidal fields (see Aarseth 2003). This has paid off by allowing the generation of direct models that can be compared to observations of the stellar content and global properties of open clusters.

Young clusters such as the Pleiades and Hyades have been modelled: Kroupa, Aarseth & Hurley (2001) looked at the formation of the Pleiades and the consequences for its binary population while Portegies Zwart *et al.* (2001) looked more generally at the evolution of stellar content in young open clusters. Hurley *et al.* (2005) have presented a direct model of the old open cluster M67. They investigated in detail the formation of

† The unit of kT is a thermodynamic quantity commonly used to scale the binding energies of binaries. The mean stellar kinetic energy corresponds to $(3/2)kT$ which is used to determine the boundary between hard and soft binaries. Note that the binding energy $E_{\rm b}$ given throughout this work is the absolute value of this quantity.

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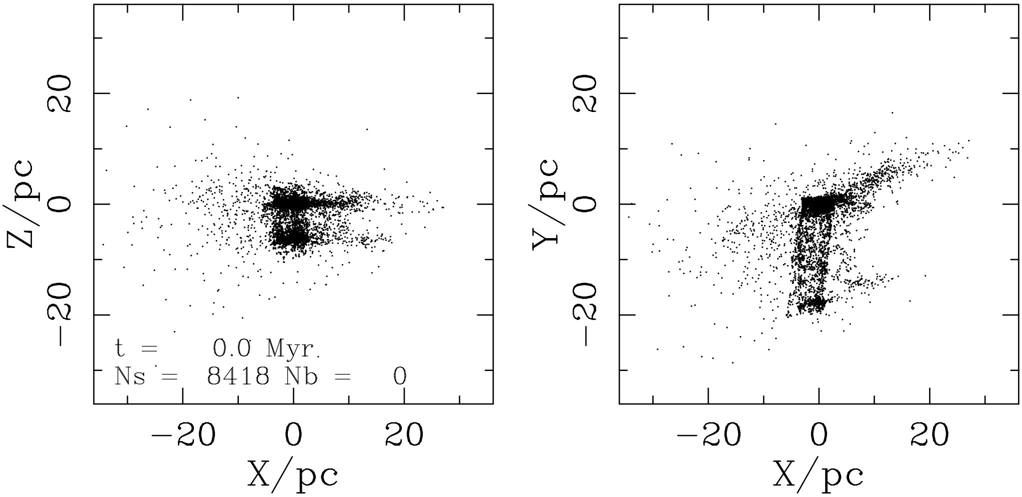

Figure 5. Spatial distribution in the X-Z (left panel) and X-Y (right panel) planes of stars in the proto-cluster input to `NBODY4` at a simulation age of 0 Myr.

blue straggler stars in M67 and also provided a census of the X-ray binaries expected and the white dwarf content. Comparison of the results of detailed models such as these with observations of particular clusters can teach us about the initial binary properties of open clusters as well as the intervening dynamical evolution. As such, this relatively new practice of targeting specific clusters should continue. Future candidates include NGC 6791, which is even older than M67, and NGC 6819 which has a well observed white dwarf sequence (Kalirai *et al.* 2001).

4. Initial Conditions

While it is true that the range of physical processes included in the cluster models has improved tremendously, as outlined in the previous section, there is still much that can be added. This is certainly the case for the initial conditions of the N-body models which remain somewhat naive. Typically the presence of gas is neglected, which will be important for young clusters, and pre-main-sequence stellar evolution, as well as the associated possibility of staggered star formation, is not included. In terms of the distributions of stars (positions and velocities) normal practice is to use a King or Plummer density profile and assume virial equilibrium (see Aarseth 2003). Such assumptions are based on observations of evolved clusters and are not necessarily correct for clusters at, or soon after, the formation stage. However, the error induced may be minimal if, for example, young clusters attain virial equilibrium on a timescale much shorter than their lifetime.

In an upcoming publication Hurley & Bekki (in preparation) aim to begin addressing some of these shortcomings by interfacing the results of galaxy-scale simulations of star cluster formation with the N-body codes that follow the long-term cluster evolution. A preliminary calculation along these lines is presented here. Fig. 5 shows the spatial characteristics of a proto-cluster formed from the collapse of a turbulent molecular cloud in a low-mass dwarf galaxy which in turn is embedded in a massive dark matter halo. This is output from the chemodynamical code of Bekki & Chiba (2007). The example proto-cluster contains $\sim 8\,400$ stars each with a mass close to $0.5 M_\odot$ and is used as input to the `NBODY4` code. It is found that the cluster, which was far from being in virial equilibrium

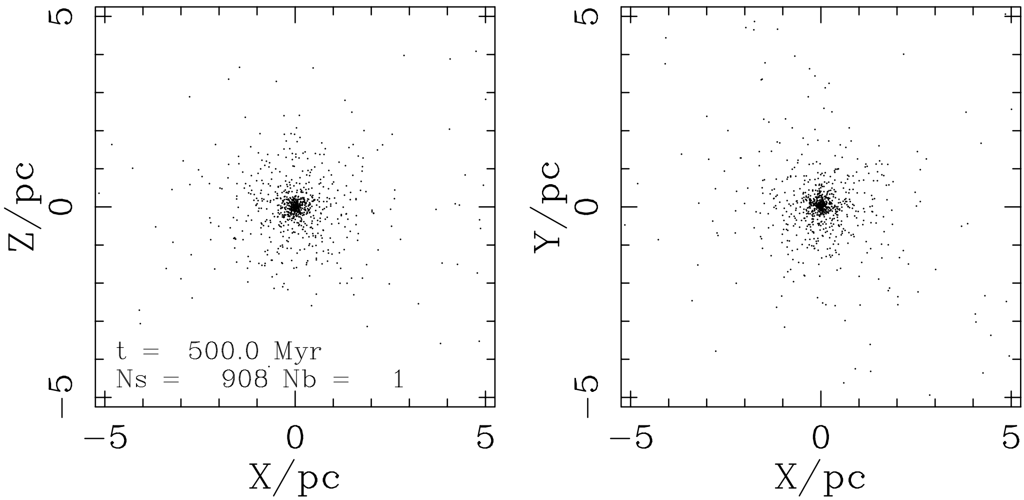

Figure 6. As for Fig. 5 but after 500 Myr of `NBODY4` evolution.

to begin with, reaches a state of virial equilibrium after ~ 50 Myr of evolution (with zero-age taken as the start of the `NBODY4` simulation). After 500 Myr approximately 900 stars remain bound in a relaxed and regular (in terms of appearance) cluster (as shown in Fig. 6). The results here are certainly promising and will be presented in more detail, and for a wider range of scenarios, in the upcoming publication.

5. Summary

The properties of early idealized models of open clusters with $N \sim 2\,000$ and primordial binaries generally scale well when compared to the new generation of more realistic models. A notable exception is the depth of core-collapse which warrants further investigation. Also, the critical primordial binary fraction below which the binary population of an open cluster is exhausted before cluster dissolution, found by McMillan & Hut (1994), is not observed in models with larger N. On a final point it is noted that Heggie & Aarseth (1992) demonstrated the effectiveness of comparing the results of cluster evolution models produced by complementary but differing simulation methods. This fine example needs to be continued using current statistical and N-body models (see Fregeau, these proceedings).

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