(i) $f(x)$ is a polynomial of degree $(m-1)(n-1)$ whose non-zero coefficients are alternately $+1$ and -1,

(ii) the number of non-zero coefficients is

$$Mm + Nn - 2MN$$

where M, N are integers defined by $Mm - Nn = 1$, $0 < M < n$.

J. D. Dixon
California Institute of Technology

P 58. (Conjecture) A graph of $\binom{k}{2} + t$ edges with $0 \leq t < k$ has at most $\binom{k}{3} + \binom{t}{2}$ triangles.

J. W. Moon and L. Moser,
University of Alberta

SOLUTIONS

P 43. (Corrected) Let G be a group generated by P and Q, and let H be the cyclic subgroup generated by T. If P and Q satisfy only the relations $P^2QP = Q^2$ and $Q^2PQ^{-4} = P^k$ for some k, then the index of H in G is 1 or 7.

N. S. Mendelsohn, University of Manitoba

Solution by F. A. Sherk, University of Toronto. Enumerating cosets of H by the Todd-Coxeter method (Coxeter and Moser, Generators and Relations for Discrete Groups, Ergebn. Math. 14 (1957) Chapter 2), we obtain the tables

P P P P...					
1	1	1	1	1	...
2	3	5	2	3	...
4	7	6	4	7	...

Q Q Q Q Q Q Q Q...									
1	2	3	4	5	6	7	1	2	...

$P^2\ Q\ P = Q^2$					
1	1	2	3	1	3
2	5	6	4	2	4
3	2	3	5	3	5
4	6	7	6	4	6
5	3	4	7	5	7
6	7	1	1	6	1
7	4	5	2	7	2

$Q^2\ P\ Q^{-4} = P^k$					
1	3	5	1	1	1
2	4	7	3	2	3
3	5	2	5	3	5
4	6	4	7	4	7
5	7	6	2	5	2
6	1	1	4	6	4
7	2	3	6	7	6

An examination of the tables makes it clear that the 7 cosets which are sufficient to close them up are distinct if and only if the three conditions

(i) the order of $\{P\}$ (=H) is infinite or $\equiv 0 \pmod 3$

(ii) the order of $\{Q\}$ is infinite or $\equiv 0 \pmod 7$

(iii) $k \equiv 1 \pmod 3$

all hold. In this case H is of index 7; otherwise the tables collapse and H is of index 1.

Also solved by the proposer.

<u>P 44.</u> Show that

$$\pi^2 = 10 - \sum_{n=1}^{\infty} \frac{1}{n^3(n+1)^3}$$

E. L. Whitney, University of Alberta

<u>Solution by W. J. Blundon, Memorial University of Newfoundland.</u> Using partial fractions and the well-known sum

$$\frac{\pi^2}{6} = \sum_{n-1}^{\infty} \frac{1}{n^2}, \quad \text{we have}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3(n+1)^3} = 6\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right) + \sum_{n=1}^{\infty}\left(\frac{1}{n^3} - \frac{1}{(n+1)^3}\right)$$

$$- 3\sum_{n=1}^{\infty}\frac{1}{n^2} - 3\sum_{n=1}^{\infty}\frac{1}{(n+1)^2}$$

$$= 6 + 1 - 3\left(\frac{\pi^2}{6}\right) - 3\left(\frac{\pi^2}{6} - 1\right)$$

$$= 10 - \pi^2 .$$

Also solved by M. Brisebois, J. L. Brown Jr., L. Carlitz, G. D. Chakerian, H. M. Gehman, A. Makowski, W. Moser, R. G. Stanton, E. H. Umberger, and the proposer.

Chakerian remarked that this is problem 127b on p. 272 of K. Knopp, Infinite Series.. Makowski pointed out that it also appears on pp. 79-80 of W. Sierpinski, Differential Calculus, Monografie Matematyczne 14 (Polish edition).

P 45. Show that

$$\sum_{i=0}^{n}\binom{n+1}{i}\int_0^1\binom{t}{i+2}\,dt = 0$$

for $n = 1, 3, 5, \ldots$, where

$$\binom{t}{k} = \frac{t(t-1)\ldots(t-k+1)}{k!}$$

B. Wolk, University of Manitoba

Solution by L. Carlitz, Duke University. We have

$$\sum_{i=0}^{n+1} \binom{n+1}{i} \binom{t}{i+2} = \sum_{i=2}^{n+3} \binom{n+1}{i-2} \binom{t}{i}$$

$$= \sum_{i=0}^{n+3} \binom{n+1}{n+3-i} \binom{t}{i}$$

$$= \binom{n+t+1}{n+3} ,$$

so that

$$\sum_{i=0}^{n} \binom{n+1}{i} \binom{t}{i+2} = \binom{n+t+1}{n+3} - \binom{t}{n+3} ,$$

$$\sum_{i=0}^{n} \binom{n+1}{i} \int_0^1 \binom{t}{i+2} dt = \int_0^1 \binom{n+t+1}{n+3} dt - \int_0^1 \binom{1-t}{n+3} dt .$$

Since $\binom{-t}{m} = (-1)^m \binom{t+m-1}{m}$ and n is odd it follows that

$$\binom{1-t}{n+3} = \binom{n+t+1}{n+3} .$$

Hence $\sum_{i=0}^{n} \binom{n+1}{i} \int_0^1 \binom{t}{i+2} dt = 0 .$

Also solved by R. G. Stanton, and the proposer.

P 47. Prove that for every surface of constant mean curvature there exists a parallel surface which also has constant mean curvature.

P. Scherk, University of Toronto

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Solution by R. Blum, University of Saskatchewan. The problem is an immediate consequence of formula (141) on p. 119 of Blaschke, Differentialgeometrie I (4th edition, 1945),

$$\bar{H} = \frac{H - nK}{1 - 2nH + n^2 K} .$$

The condition for $\bar{H}$ not to depend upon K is then

$$\begin{vmatrix} H & -n \\ 1-2nH & n^2 \end{vmatrix} = 0$$

or $n = 0$ (trivial) and $n = \frac{1}{H}$ $(H \neq 0)$, which gives the required solution with $\bar{H} = -H$. It is therefore clear that the formulation of the problem should be "...for every surface of (non-zero) constant mean curvature...".

Also solved by the proposer.