

FORUM

Graphical Methods of Finding the Sun's Azimuth and Elevation

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LIKE Mr. R. S. Blicq (*Journal* 15, 456) we have also been concerned in supplying details of the Sun's position to engineers on radar sites who use radiation from the Sun for, at present, two purposes:

- (a) Measuring or checking the radiation pattern of the aerial in the vertical plane.
- (b) Lining up the aerial in azimuth or elevation.

For these purposes the required accuracy of computation is usually about $\pm 0.1^\circ$ in azimuth and elevation. Greater accuracy is never required, but less accuracy is sometimes adequate, up to $\pm 1^\circ$.

We have found that graphical methods appeal most to engineers, and that even when a full table of values is supplied to them they prefer to display this information in graphical form. We have accordingly evolved a simple method of constructing graphs of elevation against time, and azimuth against time.

In order to achieve the required accuracy of $\pm 0.1^\circ$ in elevation we have constructed a graticule of size 21 in. \times 40 in. A skeleton version of this is shown in Fig. 1. The engineer has to know his latitude, and to find, from the Nautical Almanac or other source, the time of local noon (i.e. when the Sun is due south) and the mean declination, δ , of the Sun during the period of observation, which usually lasts up to three hours. In order to be able to draw a graph of elevation against time, he has to compute some or all of the following quantities:

$$E_1 = 90^\circ - \lambda - \delta$$

$$E_5 = 90^\circ - \lambda + \delta$$

$$P = \sin E_1$$

$$Q = \sin E_5$$

$$M = P + Q$$

$$E_2 = -4P + M$$

$$E_3 = -4P + 2M$$

$$E_4 = -4P + 3M$$

He then draws a straight line on the graph through E_1 , E_2 , E_3 , E_4 and E_5 . This is a graph of θ_t , the true elevation of the Sun, against apparent solar time (i.e. time adjusted so that the Sun is due south at 12h. 00m.) The abscissa has been marked in Fig. 1 with time during the morning; for afternoon observations the diagram must be turned upside down.

If the Nautical Almanac (or similar publication) is not available to the engineer at the radar station, it will be necessary to tell him the declination of the Sun and the time of local noon for the period of observation. He can then, however,

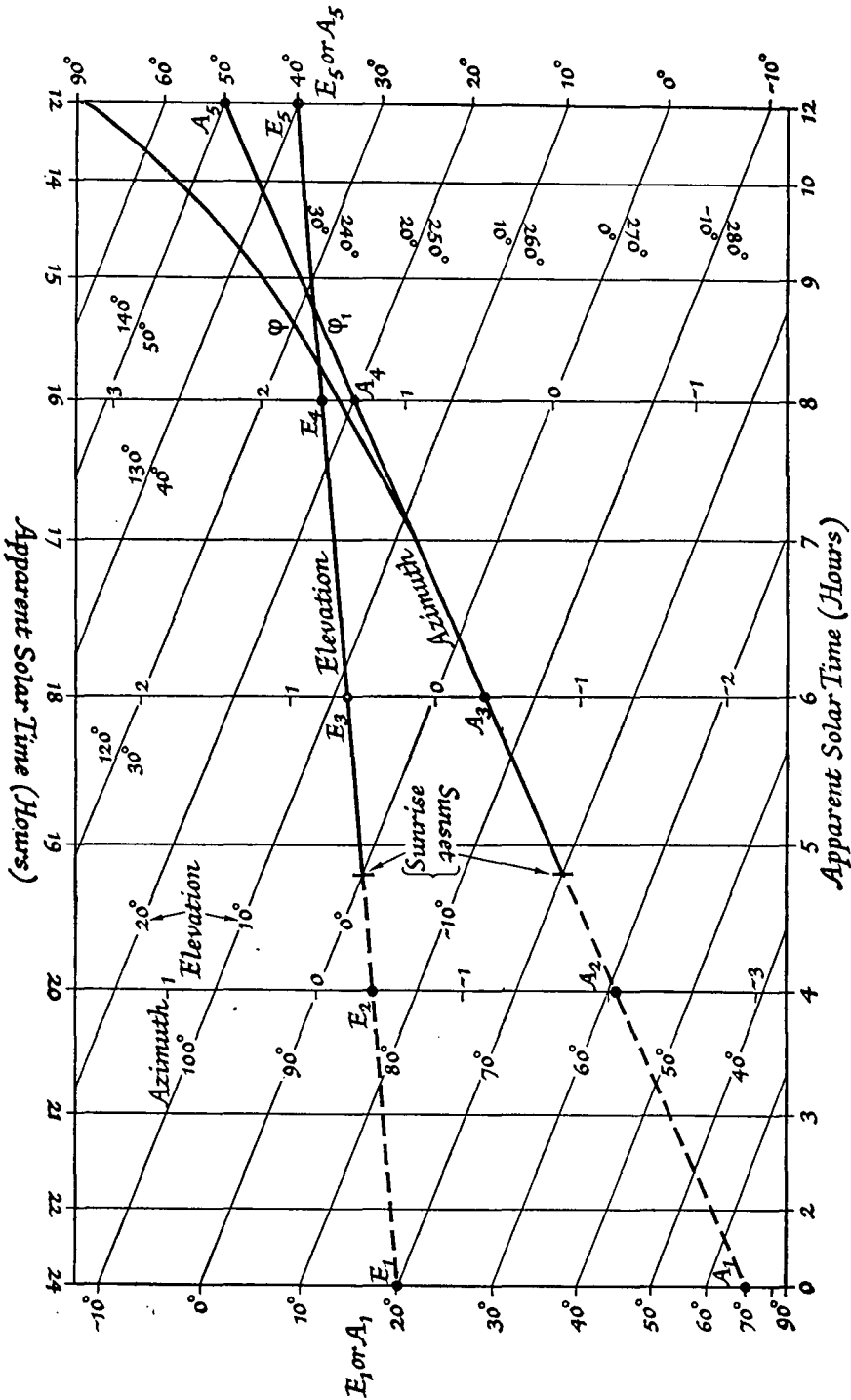


FIG. 1. Graticule for finding Sun's elevation and azimuth, given latitude λ , declination δ and time of local noon. The straight lines shown are graphs of true elevation against time and approximate azimuth, ϕ_1 , against time, for $\lambda = 60^\circ N$ and $\delta = 10^\circ N$. The curved line gives the correct azimuth, ϕ . To obtain graphs for the afternoon, turn Fig. 1 upside down.

carry out the computation and construction as above, very simply on his own. The only apparatus required is the graticule, a table of sines and a long straight edge; the computation consists entirely of addition, subtraction and one simple multiplication.

The way in which the method works can be explained by stating that E_1 gives the elevation of the Sun below the horizon at midnight, E_5 gives the elevation above the horizon at midday, and the scales of the graticule have been adjusted so that the graph between E_1 and E_5 is a straight line; the other points, E_2, E_3 and E_4 , are merely the points a quarter, a half and three-quarters of the way along the straight line; it is not strictly necessary to compute and enter these three points, but they do provide a graphical check, and some improvement in accuracy.

The construction is based on the equation

$$\sin \theta_t = \sin \delta \sin \lambda + \cos \delta \cos \lambda \cos H \tag{1}$$

where θ_t = true elevation

and H = local hour angle ($= 15^\circ \times$ (apparent solar time - 12 hours)).

It follows from this equation that the graph of $\sin \theta_t$ against $\cos H$ is a straight line. We therefore choose $x = 1 + \cos H$ and $y = \sin \theta_t$ as oblique Cartesian coordinates and construct the graticule of Fig. 1 accordingly.

At the point E_1 , $H = 180^\circ$, $\cos H = -1$, hence $\sin \theta_t = -\cos(\lambda + \delta)$ and $\theta_t = -(90^\circ - \lambda - \delta)$.

At the point E_5 , $H = 0^\circ$, $\cos H = 1$, hence $\sin \theta_t = \cos(\lambda - \delta)$ and $\theta_t = 90^\circ - \lambda + \delta$. Since the graph of $\sin \theta_t$ against $\cos H$ is a straight line, and these two points lie on it, this graph must be the line E_1E_5 .

Generally, the radio engineer wishes to know the apparent elevation, θ_a , which exceeds the true elevation, θ_t , by the amount shown in Fig. 2. If desired,

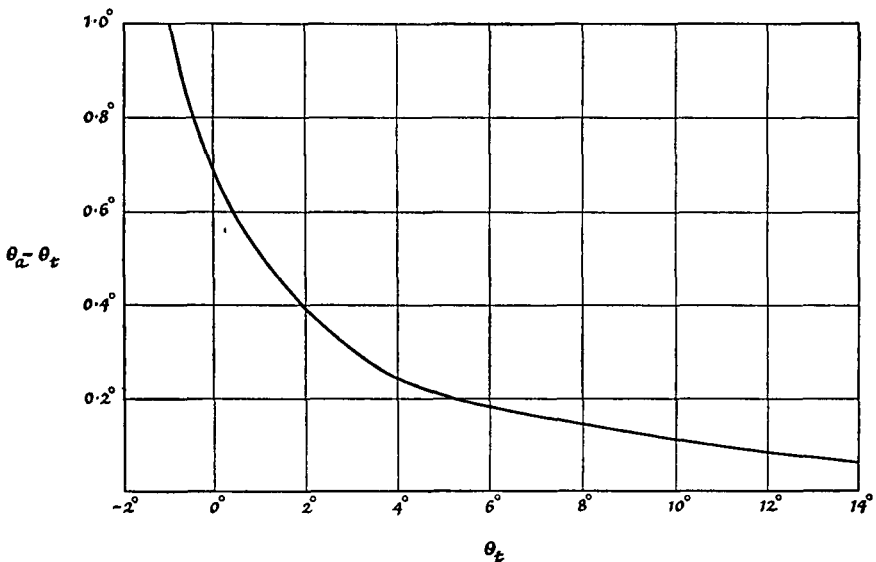


FIG. 2. Mean refraction of radio waves by the atmosphere, θ_t = true elevation, θ_a = apparent elevation.

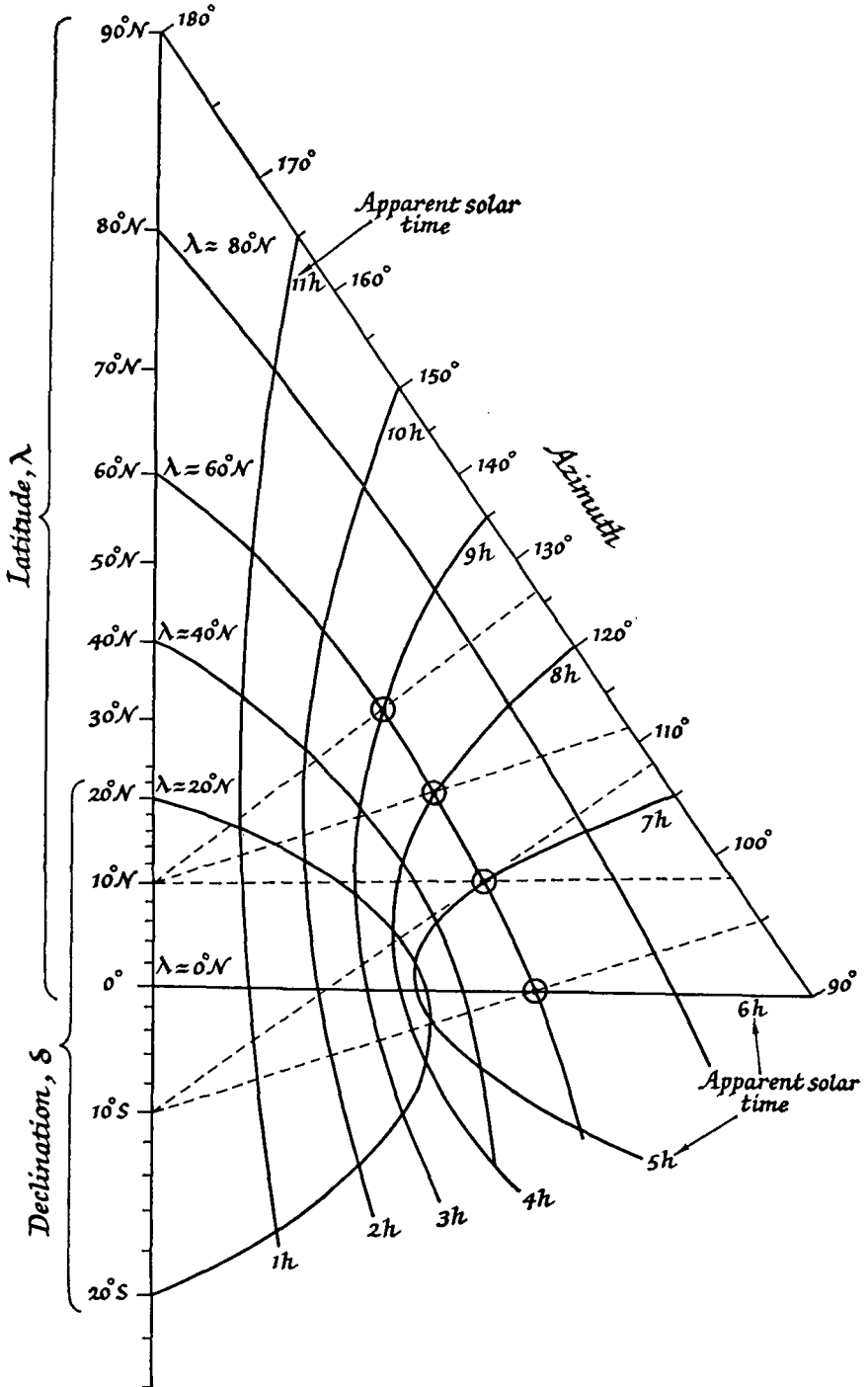


FIG. 3. Nomogram for finding the azimuth, ϕ , in the morning in the northern hemisphere. When $\phi < 90^\circ$, enter the contrary declination and 12 hours—apparent solar time; the intercept is then 180° —azimuth. The dashed lines indicate the following azimuths for $\lambda = 60^\circ\text{N}$, $\delta = 10^\circ\text{N}$.

Time	05.00	06.00	07.00	08.00	09.00
Azimuth	72°	85°	98°	111½°	126½°

a graticule showing θ_a can be superimposed on Fig. 1, and from this graticule the apparent elevation can be read directly.

There are two alternative graphical methods of finding azimuth. The nomogram shown in Fig. 3 is universally applicable, but its accuracy varies with latitude and declination and nowhere exceeds about $\pm 0.3^\circ$ (for a nomogram 6 in. wide). It is based on the equation

$$\cot \phi = \sin \lambda \cot H - \tan \delta \cos \lambda \operatorname{cosec} H \quad (2)$$

where ϕ = azimuth.

It is possible to insert on Fig. 1 a graph giving azimuth. This method is based on the equations

$$\cos \phi = \cos \phi_1 \sec \theta_t \quad (3)$$

and
$$\cos \phi_1 = \cos \lambda \sin \delta - \sin \lambda \cos \delta \cos H \quad (4)$$

It follows from the first of these equations that when θ_t is small the value of ϕ is nearly the same as that of ϕ_1 . It also follows from the second equation that the graph of ϕ_1 on the graticule of Fig. 1 is a straight line (by putting y on the graticule equal to $-\cos \phi_1$). The points A_1 to A_5 on this straight line are computed as follows:

$$\begin{aligned} A_1 &= \lambda + \delta \\ A_5 &= \lambda - \delta \\ C &= \sin A_1 \\ D &= \sin A_5 \\ N &= C + D \\ A_2 &= -4C + N \\ A_3 &= -4C + 2N \text{ or } 2(D - C) \\ A_4 &= -4C + 3N \end{aligned}$$

The value of ϕ can be found from that of ϕ_1 and θ_t either from equation (3) or by means of a diagram such as Fig. 4. The ringed point in this diagram indicates, for example, that when $\phi_1 = 60^\circ$ and $\theta_t = 10^\circ$, then $\epsilon (= |\phi - \phi_1|)$ is 0.5° . It is clear from equation (3) that ϕ always gives a point nearer to north or south than ϕ_1 . Hence if $\phi_1 = 60^\circ 0'$, then $\phi = 59^\circ 5'$; if $\phi_1 = 120^\circ 0'$, then $\phi = 120^\circ 5'$, and so on.

It is noteworthy that the difference between ϕ and ϕ_1 is small over a considerable range of values of ϕ_1 and θ_t . This can be seen clearly in Fig. 1. For example, if an error in azimuth of $\pm 1^\circ$ is tolerable, then the graph of ϕ_1 in Fig. 1 may be taken to give azimuth directly at all times between sunrise and 07h. 30m. (elevation 20°); this may be sufficient for all radio purposes.

The graphical methods described above are, like all other graphical methods, of limited accuracy, but their simplicity may well prove advantageous whenever the azimuth and/or elevation of an astronomical body is required to an accuracy no better than $\pm 0.1^\circ$. In fact, provided that sufficiently accurate graticules and nomograms like Figs. 1, 3 and 4 are available, these graphical methods may well prove to be the most straightforward of all possible methods for engineers to use in the field. As an alternative method, a programme has now been written for an automatic computer, which finds azimuth and elevation at equal intervals of time (e.g. every 20 minutes) for a given location and declination.

In order to use this programme, it is necessary to punch input cards, to put these and the programme cards into the machine, to transfer the output cards to

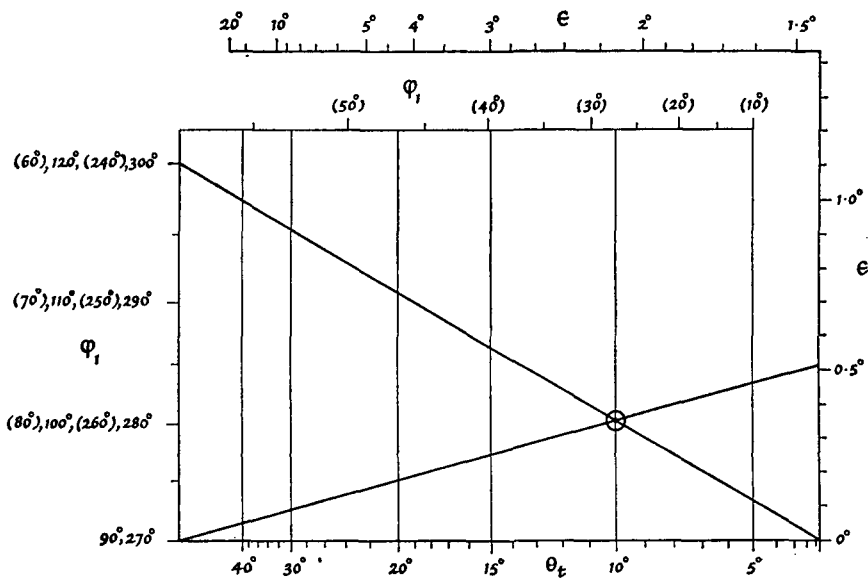


FIG. 4. Graticule for finding $\epsilon = |\phi - \phi_1|$

a tabulator, and to rewrite the tabulated results in a form suitable for the engineer. The total labour involved in this procedure is actually greater than the total labour required for a graphical computation. It appears therefore that the graphical methods will normally be used by engineers, and that machine or other methods will be used only for special purposes. For example, if many values are required to an accuracy *better* than ± 0.1 , it is probably best to use the machine; if only one or two accurate values are required, then it may be advisable to use ordinary hand computation, with or without the aid of tables of computed azimuth and altitude. The graphical methods will, however, always have the advantage of being simple to use and self-checking.

The construction can also be used for solving other problems in spherical trigonometry. For example, the elevation and azimuth graphs in Fig. 1 give, respectively, the latitude and longitude of all places at which the elevation of a certain star at declination 60°N . is 10° (e.g. the star's elevation is 10° when observed from a point at latitude 20°N ., longitude 105° different from that of the star). The same figures give the locus of all points on the Earth which are 80° from a certain point at latitude 60° . This type of information can be used to construct sunset or twilight contours, or contours of constant elevation of an artificial satellite. The elevation and azimuth graphs in Fig. 1 can also be used to give the distance and bearing respectively of points at latitude 10°N . from points at latitude 60°N ., e.g. since Trinidad is 72° west of Oslo, its distance from Oslo is 72° ($= 4300$ n.m. or 8000 km.) and its bearing is 259° .

I would like to thank the Chief of Research of Marconi's Wireless Telegraph Co. Ltd. for permission to publish this note.