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EDITOR'S INTRODUCTION

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The papers collected in this issue are united in a common view that it is rational to recognize that we have a poor perception of the constraints we face when making economic decisions and hence we employ decision rules that are robust. Robustness can be interpreted in different ways but generally it implies that our decision rules should not depend critically on an *exact* description of these constraints but they should perform well over a prespecified range of potential variations in the assumed economic environment. So, we are interested in deriving optimal and hence rational decisions where our utility or loss function incorporates the need for robustness in the face of a misspecified model. This misspecification can involve placing simple bounds on deviations from the parameters we assume for a nominal model, or misspecified dynamics, neglected nonlinearities, time variation, or quite general arbitrary misspecification in the transfer function between the input uncertainties and the output variables in which we are ultimately interested.

The same motivation can be seen in the work of a number of earlier economists such as Herbert Simon and William Baumol, but it has only been in the past 10 years or so that we have been in a position to formulate these problems mathematically and to derive robust decision rules. This process has been made possible by a dramatic paradigm shift in mathematical control theory in the development of what are known as H_∞ methods of robust control. Control theory has always recognized the need for robustness and the progress from open-loop decision rules to the use of feedback was the first step. Modern control theory, based on a state-space representation of the environment, advanced by employing a stochastic representation of uncertainty and developed optimal feedback rules derived from a Linear Quadratic Gaussian model. However, this framework was too restrictive since it implied through certainty equivalence that, although the cost would be altered, the characteristics of the noise process would not change the optimal decision rules, and yet, in practice, control engineers saw that they needed to account for the specific nature of the uncertainties they faced.

Control theory struggled with this reality and the fact that the stochastic formulation of uncertainty limited the range of problems for which analytic solutions could be found for many years. The release came in the work of Zames (1981), which introduced a deterministic normbounded description of uncertainty in the frequency domain rather than the stochastic model that had been used in state-space methods up to that time. Critically, this enabled control engineers to now derive robust rules that could be guaranteed to satisfy certain performance criteria under a prespecified but arbitrary range of deviations from their nominal or assumed model. This led to one robust method known as H_∞, which provided guaranteed performance against the worst-case environment that the designer wished to consider. The first two papers in this special issue, by Peter Whittle and Pierre Bernhard, provide reviews of the developments in this literature. The paper by Peter Whittle maps the development of an important related area known as risk-sensitive decision theory to which he has made fundamental contributions. It was a surprising realization that the stochastic formulation employed in the risk-sensitive literature could be shown to deliver the same decision rule *in extremis* as the deterministic H_∞ approach to robust control. In effect, H_∞ considers the optimal decision in the face of a worst-case distribution that you could face. This implies another interpretation of H_∞ methods as minimax decision rules and they can be derived as solutions to a saddle point problem in a dynamic game between yourself and a malevolent opponent who is seeking to do the worst he can for you. Pierre Bernhard, with Tamer Başar [Başar and Bernhard (1995)], made significant contributions to the game theoretic interpretation of robust decision theory, which may be more easily understood by economists [see also Hansen and Sargent (2002)].

Robust methods have advanced substantially since Zames' original article in 1981 and a large number of introductory texts now exist; see, for instance, Zhou et al. (1996) and an excellent review of "the state of the art" by Petersen et al. (2000). The severity of the worst-case H_∞ solution can be moderated in a number of ways—for instance, by the use of an optimal guaranteed cost control [see Petersen and MacFarlane (1992)] or a mixed H_2/H_∞ formulation [see Hadad et al. (1991)]—and there are a range of different approaches to robust control available, which are reviewed by Petersen et al. (2000). At a fundamental level, robust methods depend on a description of the uncertainty that the decision problem faces, and a range of different descriptions can now be addressed, such bounds on parameters or transfer functions that allow for time-varying uncertainty and nonlinearities; integral quadratic constraints can address unmodeled dynamic misspecifications and provide a richer class of uncertainties than norm-bounded constraints. In addition, an integral quadratic constraint formulation can be extended to describe systems subject to stochastic noise processes, to stochastic relative entropy constraints, which enables us to once again consider disturbances that might be *expected*. This is clearly an advantage since the standard deterministic worst-case design problem effectively treats all uncertainties as equally likely and follow a uniform distribution across all the potential uncertainties. Robust H_∞ methods can then be applied to stochastic systems through stochastic minimax optimization problems and stochastic dynamic game formulations of the decision problem. Clearly, we have only just started to appreciate the power of these techniques in economics, and the papers collected here represent a sample that indicates that potential.

The paper by Hansen, Sargent, and Wang applies a robust decision theory where detection error probabilities are used to determine the desired degree of robustness across models. They consider a permanent-income framework in which the decision maker has imperfect knowledge of the state and needs to estimate the state prior to making allocative decisions. This robust joint filtering and control framework will have many applications elsewhere in economics. They estimate the market price of risk and compare this with the results from Hansen et al. (1999) where the state was perfectly observed and show that there is, in fact, no increase due to a counteracting effect from the increased volatility created by filtering with the increased ease with which a distorted model may be detected.

Onatski and Stock and Giannoni, using different formulations, consider the issue of optimal monetary policy in the face of uncertainty. Giannoni uses a forward-looking model whereas Onatski and Stock use a backward-looking model. They both find that the robust monetary policies are generally more aggressive than those that Bayesian decision theory has suggested. Brainard (1967), using Bayesian methods, found that uncertainty would induce a cautious reaction and it is important to note that robust methods can deliver the opposite conclusion. This is not a generic result, however, but context dependent, as shown by Bernhard in his review.

Kasa extends Whiteman's (1986) approach to the design of stabilization policy with rational expectations to incorporate model uncertainty using robust H_∞ methods. The analysis of time inconsistency and commitment in economic policy occupied the literature for a considerable period in the 1980's and this paper represents an important generalization to an uncertain world. Again, the robust policies are found to be more activist in that their impulse responses are larger and more autocorrelated, and in consequence, the commitment constraint is greater under model uncertainty.

The final paper, by Marcellino and Salmon, discusses the role of the Lucas critique in this robust world. Given that the private sector is able to include all relevant potential government policies within its prior set of models for which it wishes to design a robust policy, they show that the Lucas critique will fail to apply. The specification of the uncertainty set in this case is critical and whether the Lucas critique remains valid depends on whether we regard rational economic agents as employing sufficiently robust decision rules.

While these papers have clearly made significant contributions, the potential range of applications for robust decision theory within economics would seem to be enormous.

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