Part 2

Introduction to Dark Matter in Galaxies

Dark Matter in Galaxies
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Alternatives to Dark Matter (?)

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Abstract. It has long been known that Newtonian dynamics applied to the visible matter in galaxies and clusters does not correctly describe the dynamics of those systems. While this is generally taken as evidence for dark matter it is in principle possible that instead Newtonian dynamics (and with it General Relativity) breaks down in these systems. Indeed there have been a number of proposals as to how standard gravitational dynamics might be modified so as to correctly explain galactic dynamics without dark matter. I will review this general idea (but focus on "MOdified Newtonian Dynamics", or "MOND"), and discuss a number of ways alternatives to dark matter can be tested and, in many cases, ruled out.

1. Introduction

The great majority of astronomers now believe that the universe is dominated by cold, collisionless, non-baryonic dark matter. But despite more than 20 years of intense effort, no *non-gravitational* evidence for dark matter has ever been found: no direct detection of dark matter, no annihilation radiation from it, no evidence from reactor experiments supporting the physics (beyond the standard model) upon which dark matter candidates are based. We know nothing about dark matter, except for the properties that we have attributed to it, and also that it is not enough: we need to postulate an even more mysterious "dark energy" to supplement it.

It therefore seems worth keeping in mind, even now, the possibility that what we have interpreted as evidence for dark matter is, in fact, evidence for the breakdown of Einstein's (and Newton's) gravity on scales of galaxies and cosmology. And indeed, over the years, a number of rebellious researchers have proposed modifications of gravitational dynamics as substitutes for dark matter. The idea behind all of these is to increase the strength of gravity on galaxy scales and above so as to explain (for example) the flat rotation curves of galaxies using only the visible baryonic matter in them. To see how this works in detail, and elucidate some of the key points of making such a modification of gravity, let us go through an extended example of an attempt to do away with dark matter.

A first guess would be to add to the normal Newtonian gravitational acceleration a new term that becomes dominant for large radius r:

$$a = M\left(\frac{G}{r^2} + g(r)\right), \tag{1}$$

where $g(r)$ is a free function that does not fall off as fast as $1/r^2$. Now let us require that the rotation curves of galaxies are, as observed, flat at large r. An obvious way to do this is to set the acceleration at large radii equal to the centrifugal acceleration for a constant velocity to obtain:

$$g(r) = A/r. \tag{2}$$

the problem with this proposal is that if A is independent of M, then $v_\infty \equiv v(r \rightarrow \infty) \propto \sqrt{M}$, contradicting the observed Tully-Fisher (TF) relation for spiral galaxies that $M \propto v_\infty^\alpha$ with $\alpha \approx 3.5 - 4$. This important contradiction rules out many alternative gravity theories (see Aguirre et al. 2001); nearly any alternative gravity in which the modification becomes important at a fixed *length* scale leads to the wrong TF relation.

We can, however, repair our candidate theory by setting $A \propto M^{-1/2}$, which yields flat rotation curves as well as $\alpha = 4$. So we now have:

$$a = \frac{GM}{r^2} + \frac{BM^{1/2}}{r}. \tag{3}$$

This theory has a couple of funny features. First, it is nonlinear, so doubling the mass of a galaxy will not double the gravitational acceleration; also Newton's third law is not obeyed if the equation is written in terms of forces (i.e. momentum is not conserved). Second, the modification becomes important at a fixed physical *acceleration* $a_0 = B^2/G$. As we will see this theory is very similar to Milgrom's well known MOdified Newtonian Dynamics (MOND), which is no coincidence: Milgrom used similar arguments to develop MOND, which is, I suspect, essentially the unique modification of Newtonian gravity giving *asymptotically flat* rotation curves as well as the correct TF relation.

A fixed acceleration scale appears to be a part of any explanation of the systematics of galaxy rotation curves. For example, we can look at three modified gravity theories from the literature that claim to provide fits to galaxy rotation curves as well as explain the TF relation. First, Mannheim's (1993; 1997) "conformal gravity" theory has, in the non-relativistic limit of a test particle near a spherical mass distribution of mass M, the acceleration law:

$$a = \frac{GM}{r^2} + BM + a_0, \tag{4}$$

where B and a_0 are fixed constants. It turns out that to fit galaxy rotation curves, B must be fairly small so that the first and third terms dominate; thus the modification becomes important at a fixed acceleration scale a_0. Note, however, that rotation curves in this theory are not asymptotically flat: just as in dark-matter theory, there is only a range in which they are approximately flat. Moffat's "nonsymmetric gravity" theory (see Moffat & Sokolov 1996) has a different value of G at small and large radii, but the transition radius depends on $\sqrt{M}$, as is familiar from our example theory.

Finally, Milgrom's (1983) MOND theory has the force law that $a = a_N$ for $a_N \gg a_0$ and $a = \sqrt{a_N a_0}$ for $a_N \ll a_0$ (with some interpolated behavior between these regimes). It thus explicitly transitions at a fixed acceleration scale and behaves as $\sqrt{M}/r$ at low accelerations.

2. MOND, galaxies, and MIFF

Although all of these theories are fun to think about, I'll focus for the rest of this talk on MOND, making occasional comments regarding other modified gravity theories. For an extensive review of MOND, see McGaugh & Sanders (2002). In brief, MOND as a modification of gravity (See Bekenstein & Milgrom 1984) replaces the usual Poisson equation:

$$\nabla^2\phi = 4\pi G\rho \Rightarrow \nabla \cdot [\mu(|\nabla\phi|/a_0)\nabla\phi] = 4\pi G\rho, \tag{5}$$

where $\mu(x)$ is some interpolating function with the property that $\mu(x) = x$ for $x \ll 1$ and $\mu(x) = 1$ for $x \gg 1$. Because this can be derived from a Lagrangian, it ensures conservation of energy, momentum, and angular momentum. While the mutual acceleration between two similar-mass objects is complicated, the acceleration of a test particle near a spherically symmetric mass distribution is given by the recipe

$$\mu(|\vec{a}|/a_0)\vec{a} = \vec{a}_N, \tag{6}$$

where a_N is again the usual Newtonian acceleration. This formula gives the previously described behavior in the high-and low-acceleration regimes. An important feature of this formulation, and one that is essential to any phenomenologically viable formulation of MOND, is what may be called the "external field effect". It can be shown that if a system is embedded in an external gravitational field of Newtonian acceleration a_{ext}, then if the internal (Newtonian) acceleration of the system a_N satisfies $a_N < a_{\text{ext}} < a_0$, then the modified acceleration, rather than being $\sqrt{a_0 a_N}$, is instead $a_N(a_0/a_{\text{ext}})$ in a coordinate system $(x', y', z') = (x, y, 2z)$, where $\hat{z} \perp \vec{a}_{\text{ext}}$ and the Newtonian acceleration is computed in the unprimed coordinates. That is, the acceleration becomes *Newtonian* (but with a larger effective gravitational constant) and *anisotropic*. This is an important point to which I will repeatedly return.

So what were the predictions of this (1984) theory? There were four principal ones: first, that rotation curves of isolated galaxies should be asymptotically flat. Second, that the TF relation for isolated galaxies would satisfy $M = v_\infty^4/4Ga_0$ exactly. Third, that the effects of modified gravity would manifest at a critical surface density, and that if galaxies existed that always fell below this critical density, they would appear to be dark matter-dominated everywhere. Fourth, the rotation curves of galaxies should be calculable given *only* the distribution of baryonic matter, using the MOND formula.

Twenty years later, it must be said that these predictions are holding up pretty well. In particular, the fourth prediction that galaxy rotation curves should be calculable only given their baryonic mass distributions, has been demonstrated in several studies (see Sanders & McGaugh 2001 for a summary) and appears to work to an amazing degree. This is not to say that other theories cannot fit these rotation curves – for example, Mannheim has shown that he can fit a number of galaxy rotation curves in detail with his model, and reasonable fits using disks and dark matter halo profiles can be found for most of the galaxies – but MOND accomplishes this with a single parameter, the baryonic mass-to-light ratio (which is, in fact, almost a constant when the analysis is done in I-band). This remarkable regularity in the properties of galaxies is a phenomenon that, as many people have pointed out, requires explanation.

Although extremely successful in spiral galaxies, it is somewhat less clear how well MOND does in other galaxies, e.g., ellipticals. MOND is consistent with scaling relations such as the Faber-Jackson or fundamental plane (Sanders 2000), but somewhat more difficult to test in ellipticals due to the ambiguity in the anisotropy of the velocity dispersion. Nonetheless, there are some interesting recent studies bearing upon MOND. For example, the study of Gerhard et al. (2001) finds that the M/L ratio in ellipticals starts to rise (leading to the inference of dark matter) at a higher characteristic acceleration than in spirals, disfavouring a universal critical acceleration a_0. On the other hand, the recent study of Romanowsky et al. (2003) indicates that several large ellipticals are *deficient* in dark matter, a fact which apparently finds easy explanation in MOND (Milgrom & Sanders 2003) because the acceleration regime probed is Newtonian or near-Newtonian. On a similar topic, Prada et al. (2003) have recently measured velocity dispersion curves for dwarf galaxies around massive field galaxies in the SLOAN survey. They find a profile that agrees well with the standard CDM prediction and claim inconsistency with MOND. However, the external field effect may be important here: at distances $\gtrsim 50\,\mathrm{kpc}$ from an L_* spiral galaxy, the internal acceleration can fall below the typical large-scale flow acceleration, and a Keplerian velocity falloff – rather than a flat profile – would be expected.

To summarize the status of alternatives to dark matter in galaxies: first, rather simple considerations lead inexorably to a modification of gravity similar to MOND, and in particular with a characteristic acceleration built into the theory. Second, the formula given by Milgrom provides an excellent description of the systematics of spiral galaxy rotation curves. Milgrom's fitting formula (MIFF) encompasses, but goes well beyond the TF relation, and could prove to be at least as useful. Regardless of the status of MOND as a modification of gravity, MIFF should be tested (especially in other sorts of galaxies) and used, without embarrassment, as an excellent phenomenological relation.

But what about when we leave the realm of galaxies? If MOND genuinely constitutes an alternative to dark matter, then it should be able to reproduce the successfully replace dark matter on cluster, supercluster, and cosmological scales as well.

3. MOND and clusters

It is now well-established that MOND can*not* account for the dynamics of clusters in terms of only the visible (galaxies and X-ray emitting gas) matter in them. There are two good ways of seeing this.

The first is that in the cores of clusters, strong gravitational lensing indicates a much larger mass than can be accounted for by visible stars and gas. Moreover, the surface density in cluster cores is sufficiently high that MOND should not apply; thus we have a system in the Newtonian regime that requires unseen matter (Milgrom 1999; Sanders 1999)

A second probe of gravitation in clusters is the X-ray emitting gas, which can to a sufficiently good approximation be considered to be in hydrostatic equilibrium in cluster's gravitational potential. Then, just as a rotation curve can be predicted using the observed mass and MOND, so can the temperature profile of the cluster be predicted, up to one free parameter which is the central cluster temperature. This was first done in The & White (1988), where they found a poor fit unless the MOND constant was significantly larger than that inferred from galaxies. However, the observational data was not of high quality, and the agreement within a factor of ~ 2 was considered "reasonably good" by MOND optimists (Sanders 1999). More recently, with the advent of higher-quality X-ray data, a more detailed analysis became possible. In Aguirre, Schaye & Quataert (2001) it was shown that MOND fails to fit the temperature profiles of several well-observed clusters. Quantitatively, fitting the observed profiles within MOND requires additional unobserved matter of $\approx 2-5\times$ the observed baryonic mass within 1 Mpc, and $\approx 10\times$ the observed mass within 100 kpc. Similar results were obtained in a subsequent larger but less-detailed study by Sanders (2003).

The necessity of dark matter even given MOND is discouraging at the very least for a theory formulated to remove the need for dark matter; but it is not immediately fatal, as it would have been if clusters in MOND required *less* than the observed amount of matter. Instead, clusters can be "saved" in MOND – just as galaxies were saved in Newtonian mechanics – by postulating extra dark matter. What might this be?

It would be difficult to swallow traditional cold non-interacting dark matter (e.g. WIMPs) alongside MOND, but even if one could, it would not help: the dark matter in MOND must be unique to clusters. One might imagine that there is some baryonic, but cluster-specific form of dark matter such as warm gas or MACHOs, but I think it is fair to say none that of the possibilities for this are particularly attractive, and many can be ruled out.

Probably the most interesting possibility would be neutrinos of mass $\approx 2\,\mathrm{eV}$, an idea adumbrated in Sanders (2003). These would aggregate in cluster-scale potentials but not typical galaxies, and could provide a mass several times that of the baryons. The required neutrino mass is near the upper range allowed by current laboratory experiments, however, and may be tested soon. This possibility has the additional advantage that $\approx 2\,\mathrm{eV}$ neutrinos would flagrantly violate limits in the standard cosmological model based on CMB and large-scale structure. Thus this is an interesting prediction of MOND that would be extremely difficult to explain in the standard cosmological model.

4. MOND as a fundamental theory

MOND, as formulated for example by Bekenstein & Milgrom (1984) is a replacement for Newtonian physics; both fully explain the dynamics of non-relativistic particles under the gravitation of an arbitrary mass distribution, but require the fixing of an unaccelerated reference frame by fiat. There is, however, no satisfactory generalization of MOND analogous to General Relativity, and there are in fact "no-go" theorems ruling out relativistic MOND theories of various types (e.g., Bekenstein & Sanders 1994; Soussa & Woodard 2003). Clever people with a deep knowledge of physics have tried and failed to relativise MOND; the conclusion that it therefore cannot be done is often referred to as "Bekenstein's theorem" (though I think this does some disservice to Milgrom and Sanders). But this theorem should not be taken too seriously, as similar theorems also imply that neither quantum gravity, nor M-theory, nor a realistic model of inflation exist. I shall not say more about relativistic MOND, but refer the reader to the review of Sanders & McGaugh (2002) and references therein.

In the meantime, MOND simply does not make unambiguous predictions for phenomena involving relativistic physics such as the cosmic expansion, the early universe, or gravitational lensing.[1] Nevertheless, MOND is not completely mute on these subjects, as discussed in the next few sections.

5. MOND and Lensing

Although there is no rigorous prediction for the dynamics of relativistic particles such as photons in MOND, a number of arguments have nevertheless been put forward claiming to test MOND using lensing data. In some, a heuristic prescription is applied to generate lensing predictions in MOND, e.g. that the deflection angle is half that predicted in the $m \to 0$ limit of the (MOdified) Newtonian dynamics. Such arguments, I think, can never rigorously test MOND, because the assumptions made are not strictly required.

In a more interesting and dangerous (for MOND) class of arguments one looks for tests that can be applied independent of the details of how lensing occurs in MOND. We encountered one such argument above in the context of cluster cores, where dark matter was detected via lensing even though the characteristic Newtonian acceleration of the system put it outside of the MOND regime.

A second argument has been offered by Hoekstra, Yee & Gladders (2002), who have looked at the statistical galaxy-galaxy lensing signal in the SLOAN survey. They find that the signal is compatible with massive halos, but more importantly that the halos must be elliptical and aligned with the lensing galaxies. This is significant because far from an isolated galaxy the Newtonian potential – and hence the MOND lensing signature – should arguably be highly spherical. This is a problem for MOND, but I can think of three ways that non-spherical lensing could occur around galaxies in MOND. First, the external field effect should be important at such large radii, and would lead to a non-spherical MOND potential. However, as pointed out by Hoekstra et al., it does not empirically appear to be the case that galaxies are aligned with large-scale structure (and hence the external field). Second, baryonic filaments in which the galaxies are embedded could conceivably add an anisotropic lensing signal. Again, the difficulty would be in accounting for the alignment. Finally, if dark matter such as neutrinos is required in MOND, galaxies might be embedded in rather distended neutrino halos that, if elliptical, could add to the lensing signal just as in CDM. However, this is nothing more than speculation. In short, I would judge that the data of Hoekstra et al. provide a strong, though perhaps not ironclad, argument against MOND.

A final and significant worry for MOND is the recent accumulated data on flux ratio anomalies in strong lensing of quasars by galaxies. It appears now to

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[1] It is worth noting that some other modifications of gravity, for example Mannheim's conformal gravity, *do* follow from relativistic theories, and hence make firm predictions for relativistic phenomena that can be tested.

be widely agreed that – at least for radio observations – these flux ratios cannot result from any reasonable lens model based on smoothly distributed matter looking anything like the distribution of visible matter. This has been taken as evidence for dark matter substructure in galaxies (e.g., Dalal & Kochanek 2002). While the issue has not (to my knowledge) been explicitly discussed in the literature, I have an extremely hard time seeing how these data could be accounted for in MOND, given that the lensing occurs in the Newtonian (not MOND) acceleration regime, and that in MOND the required substructure simply should not be there. This argument, if it cannot be somehow circumvented, would similarly appear to doom other modified gravity theories.

6. MOND and the CMB

As in the case of lensing, MOND cannot be used to make a rigorous prediction regarding the CMB anisotropies, because there is no relativistic cosmological framework in which they can be calculated. Nonetheless, as for lensing there are several extant arguments concerning MOND and the CMB.

The first argument proceeds as follows. Plot the WMAP power spectrum of CMB anisotropies, and overlay a predicted power spectrum from a standard ΛCDM model. Even those most skeptical of standard cosmology must admit that the agreement is amazing. It is hard to see how a theory with significantly different physics at the recombination epoch could give a similar agreement while still fitting data concerning the large-scale structure growing from the same perturbations. The counter-argument that this agreement requires the introduction of not one but *two* forms of completely mysterious and unidentified stuff (dark matter and dark energy) is compelling, but substantially weakened by the completely independent astrophysical evidence pointing to – and quantifying both of these mysterious components. While this sort of "wow" argument does not, logically speaking, say anything whatever about MOND, it goes a long way toward convincing many cosmologists that the standard model is on the right track, and hence (unless nature is very cruel) that any other track must be incorrect.

The second argument for testing MOND using the CMB is to assert that whatever the relativistic generalization of MOND, it will have the features that a) the acceleration scale a_0 is fixed in time, b) the relevant acceleration for determining whether dynamics are modified is the peculiar acceleration (and not that of the cosmic expansion). It can then be argued that at very early times, at all relevant length scales the characteristic accelerations of density perturbations are $> a_0$, and hence evolve exactly as in standard gravity (e.g., McGaugh 1999). This means that the physics of the CMB should be substantially unchanged except for the absence of cold dark matter, and a possible change in the angular diameter distance due to MONDifications of dynamics at low-z. This provides a strong test for MOND, because without collisionless, cold dark matter the power spectrum peak heights decrease monotonically as specified by the damping term due (primarily) to the finite thickness of the last scattering surface and photon diffusion. Thus observation of alternating peak heights would be extremely difficult to account for in MOND (or other alternatives to dark matter). The only recourse is probably to assume that gravity *is* modified at the recombination epoch, but then the "wow" argument of the success of the standard scenario returns in force. The present CMB data suggests a high third peak, but the reader will have to judge for themselves whether the data is strong enough to doom MOND on this count.

Even sans a third peak MOND is by no means safe. It was asserted by David Spergel at this conference that the first two peaks alone contain ample data to rule out possibilities very different from the standard parameter values, for example, a $\Omega_b = 0.04$, $\Omega_\Lambda = 0.96$ universe. On the other hand, in MOND one might have the freedom to rescale the angular diameter distance, and to include massive neutrinos (which would become non-relativistic near the recombination epoch). No detailed study has been performed of this question, and it would be interesting to know if there is any good fit to the WMAP data in the allowed MOND parameter space.

7. Summary: Tests of Alternatives to Dark Matter

Table 1. Summary of tests of alternatives to dark matter.

Test/Prediction	**MOND**	**DM**
Correct rotation curve shapes	√√	√×
Correct T-F slope and intercept	√√	×
Visible matter → rotation curves	√√√	×
Elliptical galaxy properties	√?	√
Cluster temperature profiles	×	√
CMB spectrum shapes (inc. alternating peaks)	×?	√√√
Correct lensing	×	√√
Correct large-scale structure, BBN	?	√√√

The wonderful thing about physical theories is that they are testable, and good theories give ample specific predictions that are capable of falsification. I think it can be argued that for a long time CDM did *not* do this, and led to no small discomfort among more skeptical parties. But the situation has clearly changed, and CDM has passed important and stringent tests, especially on cosmological scales. At galactic scales the situation is less clear, and much of this conference concerned the question of to what level current observations conflict with the theory of CDM, given that making those predictions is presently very difficult.

Just as for dark matter, there are a suite of tests for alternatives. I have summarized a number of these in the table, and have focused on a comparison between MOND and CDM. The table is provided largely without detailed justification as an expression of my opinion, with check marks indicating (in my judgment) successes and crosses signifying what I consider to be difficulties.

Here are some key points, however:

1. The success of MOND/MIFF in galaxies strongly suggests any theory (modified gravity or dark matter) that cannot reproduce MONDian behavior in spiral galaxies is in trouble.
2. MOND makes no firm prediction for relativistic phenomena but at least has a possible way to avoid altering the (well tested) physics of the early universe

drastically. This may not be the case for other modified gravity theories that do make, in principle, rigorous predictions for, e.g. nucleosynthesis and CMB anisotropies which ought to disagree with the standard analysis.

3. Nearly independent of details, the shape of the CMB anisotropy spectrum and evidence from lensing for unseen substructure in galaxies constitute grave challenges for MOND or other dark matter alternatives.

8. Unanswered Questions

I would like to conclude with a list of questions to which I would like to know the answers and which suggest to me interesting avenues of future research regarding the issues of galaxy formation, dark matter,and modified gravity.

1. Does a satisfactory modified relativistic gravity (MORG?) exist? The no-go theorems suggest this will not be a simple extension of GR. Milgrom has done interesting work exploring MOND as a modification of inertia; the hope would be to make an explicit link to cosmology via Mach's principle and to explain the coincidence between a_0 and $c\sqrt{\Lambda}$ or cH_0 (which are all numerically similar).

2. Can MOND, by hook or crook, wiggle out of its difficulties with clusters, the CMB, and lensing?

Neutrinos seem a good bet for explaining clusters, but may be ruled out soon. The CMB constraint awaits a careful study, but it does not seem impossible to me that a fit could be found if extra ingredients are added. I can offer no good ideas for explaining the lensing flux anomalies in MOND.

3. How well does MIFF really work in galaxies?

Does the extraordinary success of Milgrom's Fitting Formula (MIFF) in spirals extend to ellipticals of various masses, or irregular and dwarf galaxies? Also, how much of the regularity arises from the selection of very "clean" galaxies to analyze? It would be good to know, independent of MOND, how well MIFF works so that it could be employed like the TF and fundamental plane relations.

4. If MIFF works as well as it presently appears to, why?

Assuming that MIFF's success is not an artifact selecting particular galaxies, it implies that there is an enormous regularity to galaxy formation, and in particular in the correspondence between visible and dark matter. It is unclear to me how this great regularity arises in realistic CDM galaxy formation scenarios which include and require a number of stochastic components such as strong feedback and mergers.

5. If MIFF holds, but not MOND, whence a_0? Does $a_0 \rightarrow$ (Dynamics), or does (Dynamics) $\rightarrow a_0$?

The success of MIFF, and the form of other attempts at modified gravity, strongly suggest a characteristic acceleration scale in galaxy formation. Where does this arise from? There have been a couple of stabs at this question (e.g., Kaplinghat & Turner 2002), but I find them rather unconvincing. In addition, as pointed out by Milgrom (2002), a_0 plays several somewhat independent roles in MOND/MIFF. Why so, if the near-universal acceleration at the onset of dark-matter domination is just coincidental? (Here, testing MIFF in a variety of objects to determine the acuteness of this question is important.) There is a fundamental question of logical priority to address, I believe: is a_0 simply an emergent number that arises through the complexity of galaxy formation and just happens to be nearly-universal in any phenomenologically viable galaxy formation scenario, or is a_0 associated with some specific physical effect (such as a surface density threshold) that *leads* to regularities in galaxy formation – such as the particular slope, and surface-brightness independence of the TF relation – that we observe?

References

Aguirre, A., et al. 2001, Classical and Quantum Gravity, 18, 223

Aguirre, A., Schaye, J., & Quataert, E. 2001, ApJ, 561, 550

Bekenstein, J., & Milgrom, M. 1984, ApJ, 286, 7

Bekenstein, J., & Sanders, R. H. 1994, ApJ, 429, 480

Dalal, N., & Kochanek, C. S. 2002, ApJ, 572, 25

Gerhard, O., Kronawitter, A., Saglia, R. P., & Bender, R. 2001, AJ, 121, 1936

Hoekstra, H., Yee, H. K. C., & Gladders, M. D. 2002, NewAR, 46, 767

Kaplinghat, M., & Turner, M. 2002, ApJ, 569, L19

Mannheim, P. D. 1993, ApJ, 419, 150

Mannheim, P. D. 1997, ApJ, 479, 659

McGaugh, S. S. 1999, ApJ, 523, L99

Milgrom, M. 1983, ApJ, 270, 365

Milgrom, M. 1999, in Dark Matter in Astrophysics and Particle Physics, eds. H. V. Klapdor-Kleingrothaus & L. Baudis (Philadelphia: Institute of Physics Pub.), p.443

Milgrom, M. 2002, ApJ, 571, L81

Milgrom, M., & Sanders, R. H. 2003, astro-ph/0309617

Moffat, J. W., & Sokolov, I. Y. 1996, Physics Letters B, 378, 59

Prada, F., et al. 2003, astro-ph/0301360

Romanowsky, A. J., et al. 2003, Science, 301, 1696

Sanders, R. H. 1999, ApJ, 512, L23

Sanders, R. H. 2000, MNRAS, 313, 767

Sanders, R. H. 2003, MNRAS, 342, 901

Sanders, R. H., & McGaugh, S. S. 2002, ARA&A, 40, 263

Soussa, M. E., & Woodard, R. P. 2003, astro-ph/0307358

The, L. S., & White, S. D. M. 1988, AJ, 95, 1642