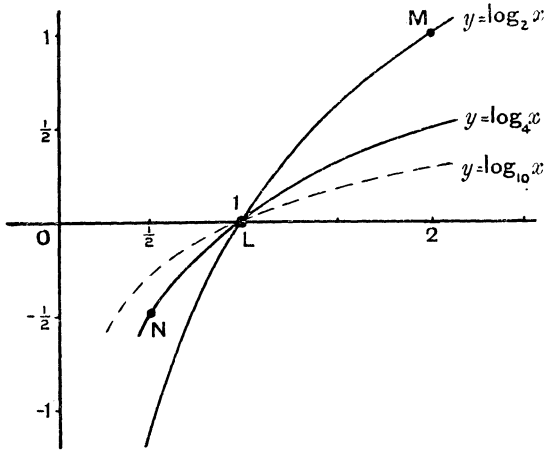
I prefer to differentiate $\log_a x$ rather than a^x, using the following method, for which no originality is claimed. Plot $y=\log_a x$ for $a=10$, 2 and 4. Since $\log_a x=\log_a b \times \log_b x$, the curves $y=\log_a x$ for variable a are such that any one of them is a sideways stretch from the x-axis of any other.



On the curve $y=\log_a x$ consider two points P, Q whose abscissae are x_1 and $x_1(1+h)$. The gradient of the line PQ is

$$\frac{1}{x_1}\ \frac{\log_a (1+h)}{h}.$$

The gradient of the tangent at P is μ_a/x_1, where

$$\mu_a=\lim_{h\to 0}\frac{\log_a (1+h)}{h}.$$

Thus (i) the gradient of $y=\log_a x$ is proportional to $1/x$;

(ii) μ_a is the gradient at the point $L(1, 0)$ on the curve and μ_a decreases steadily as a increases.

Consider now in particular the curves $y=\log_2 x$ on which the point $M(2, 1)$ lies and $y=\log_4 x$ on which the point $N(\frac{1}{2}, -\frac{1}{2})$ lies. Since the gradient of these curves decreases as x increases they are concave downwards and

$$\mu_2 > \text{gradient of } LM = 1;$$

$$\mu_4 < \text{gradient of } LN = 1.$$

Thus there is a number e between 2 and 4 for which $\mu_e=1$, and the gradient of $y=\log_e x$ is $1/x$. Yours, etc.,

R. C. Lyness.

TECHNICAL MATHEMATICS.

To the Editor of the *Mathematical Gazette*.

Sir,—As a mathematics teacher who has just returned to school after four and a half years doing research work in industry and helping with part-time technical classes, I was very interested to read the report in the October

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Gazette of the discussion at the General Meeting on Technical Mathematics. I would like to support the suggestion that the M.A. approach the Engineers' Institutions with a view to drawing up a syllabus. From my experience, unlike Dr. Booth's, there is a growing awareness in industry of the need for mathematics, especially among the younger engineers. During the war it has been very difficult to find suitably-trained people for industrial posts from the lowest to the highest, and both the Ministry and some firms have shown a willingness to consider temporary methods of filling this gap. With peace this need is not going to decrease if all branches of British industry are to reach the top scale of technical efficiency With more time and the changes in education due to the new Act we will be able to tackle the problem more effectively I would suggest that perhaps the discussion could be continued at the next General Meeting of the Association, that joint meetings of the local branches of the Association, the Engineering Institutes, and other interested bodies could be arranged to discuss the common problems, and that some arrangements could be made to report the discussions in the technical engineering press.

It is clear that the discussions taking place in Britain can be paralleled by those in other countries. Several speakers mentioned America, where there was considerable disquiet about the fact that most inter-war mathematical research was on the pure side, and that very little had been done on the applied side. Amongst other things Brown University arranged several Vacation courses on Mechanics, and helped to sponsor the new *Quarterly of Applied Mathematics* with a distinguished editorial board. In this country we lack a suitable journal of applied mathematics. I think the M.A. would be performing a very useful service if it obtained articles relating these and other experiences which would help us here. There are several pointers, too, that the experience of the U.S.S.R. would be helpful; they are known to have placed more emphasis on applied mathematics, but still, unfortunately, co-operation with this country leaves a lot to be desired.

It would be a great mistake if grammar school teachers were to assume that this discussion about technical mathematics did not concern them. Several speakers mentioned the possibility of a common course for grammar and technical sides of secondary schools, whether or not they were part of the same school. It seems obvious that many more grammar-school-trained students will find employment in industry in the future, and we will have to consider what repercussions this will have on our syllabus, even if we are not teaching in the experimental multilateral schools. If I may give a few examples, I think we should consider whether we have gone far enough with our changes in the geometry syllabus, and whether there are other branches of mathematics more worthy of inclusion for both educational and vocational reasons. Every course of logarithms should include some remarks about the slide rule as a very commonly-used means of calculating and, where possible, children should be shown the simple adding machine. In my experience I hardly used any of the geometry I had learnt, and did all the calculation I needed either on a slide rule or a calculating machine and hardly ever opened a book of logarithm tables.

Another grammar-school problem of some importance for all except the largest and most liberally staffed is that of giving adequate time and attention to sixth form work—I was distinctly amused to read of the discussion on sixth form work, where it was assumed that teachers had about twice as many periods for the sixth form as I am allowed. It is obvious that time does not press only in technical part-time courses. Under the new Act, however, all pupils will remain at school until sixteen and then have two years at the County College. What is going to happen to the sixth forms of grammar

schools where they maintain an independent existence and are not part of a comprehensive school? It seems to me that there are educational as well as social reasons for combining sixth form and intermediate pupils as much as possible so as to allow a better staffing ratio. If I may quote my own experience, we have at the moment four grammar schools and a technical college all trying to provide an adequate H.S.C. and Intermediate course. Then, in addition, are the evening courses at the technical college. I strongly support the plea made by several speakers that technical training should be given during the day. Already more private firms are making this possible, and with the nationalisation of other industries the government could make this practice the rule. I do not agree with the suggestion made by one speaker that day courses should be vocational, but think that the cultural side should be treated as well, with further facilities being provided in the evenings for the more enthusiastic. In case any reader supposes that I am against cultural education, I would here like to urge that as a profession we mathematics teachers must do much more to make our subject really cultural, but to deal with this adequately would take a whole article.

Mention was made of the possibility of industrial personnel being able to bring mathematical problems to the staff of technical colleges for assistance. While I support this whole-heartedly, as one to whom such problems might be brought, I would also like to urge the organisation of a consultative service on a wider sphere, because very often the industrial problems raise mathematical difficulties in which only the really expert specialist can help. If there are much wider facilities for post-graduate research, there will be many more engineers with the necessary mathematical background and mathematicians with the engineering knowledge to make co-operation between the two sides much more fruitful.

Several speakers mentioned details of possible syllabuses. I would like to point out that very few mentioned what is normally called mechanics, and this agrees with my experience that what is required is a more fundamental knowledge of pure mathematics and, first of all, of functions of a complex variable. Many of us found that our most-used book was Whittaker and Watson's *Modern Analysis*, although there is obviously room for a book covering approximately the same ground and designed for the engineer. It may interest others to know that we used elliptic functions very extensively (although naturally not as much as Bessel functions), particularly in connection with Schwarz transformations to provide solutions of the Laplace equation. This example also brings home the lack of suitable textbooks for the mathematical engineer, who is hardly at all concerned with the Weierstrass $\wp$ function and only with the theta functions to the extent that they are necessary to deal with the elliptic integral of the third kind, which is only just mentioned in Whittaker and Watson.

After complex variables I would place numerical methods as very necessary and important. A real understanding of these methods is important even when a closed mathematical solution is possible, as the answer required is generally numerical. Usually, however, an exact solution is not possible, and some estimation of the error made is important. Often different methods have to be combined, such as using series expansions about two different points to estimate the value of the function at an intermediate point, or in finding an approximate solution of Laplace's equation by using bits of exact, numerical and graphical (curvilinear squares) solutions of allied problems. In connection with machines, it must be realised that it is insufficient to know how to use a machine to perform ordinary operations, but it is desirable to know and develop methods for use on particular machines, quite apart from the large research problem of developing new machines and adapting existing

machines to solve new problems. Other topics I would like to mention include determinants and matrices, vectors, and Fourier series and integrals.

In conclusion, Sir, may I wish the Mathematical Association all good luck in its efforts to improve the understanding of technical mathematics at all levels. Yours sincerely,

P. J. WALLIS.

AN OLD PROBLEM.

To the Editor of the *Mathematical Gazette*.

SIR,—I have been interested lately by the old unsolved problem of placing n points in m straight lines of p in a line, so that m is a maximum. I should be grateful if any members could inform me of the literature on the subject: I believe both Newton and Sylvester studied it.

It might interest some of your readers to try their hands at the following selection of possibilities, to each of which I have at least one solution. The numbers are the values of p, n, and m, in that order. 3, 11, 16; 3, 12, 19; 4, 18, 18; 4, 24, 28; 4, 25, 30; 4, 36, 55; 5, 22, 15; 5, 26, 21; 5, 30, 26.

The question originally proposed was to find n_0, the minimum value of n for which $m \geqslant n$ for any value of p. When $p=3$, $n_0=9$, and when $p=4$, $n_0=18$, so it is tempting to guess that when $p=5$, $n_0=36$ and, in general, $n_0=9\cdot 2^{p-3}$, but I have not so far found such a solution even for $p=5$.

Yours, R. H. MACMILLAN.

SIXTH FORM SYLLABUSES

To the Editor of the *Mathematical Gazette*.

SIR,—There is temerity in commenting at a distance on the discussion about Sixth Form syllabuses recorded in the *Mathematical Gazette* for October, especially as the clearly valuable report on the training of physicists is not available here, nor are the Cambridge Entrance Scholarship papers which are so highly praised. But our experience for a score of years with Descriptive Mathematics appears to be relevant, and some attempt may be made to state its significance for the changing situation in England.

The discussion concerned two subjects which are well and clearly distinguished, the mathematical preparation best for various future courses of study, and that for winning scholarships with a view to the study of mathematics. Only to the former can I refer, though I note no reference to the consequences of concentration on scholarship work for those who fail to get a scholarship, and though I gather from the constitution of the Joint Advisory Committee the impression that University representation is rather strong. About this I feel that teachers should have clearer views as to aims in teaching which are not too directly linked to the study of advanced mathematics; University teachers can keep in touch with such aims, and help to determine them, but for this they need not have great voting strength on committees.

First, I note too little emphasis, notwithstanding the prominence given to Statistics and the mention of actuarial mathematics, on the uses of mathematics in connection with the social sciences. Not forgetting Marshall's warnings, it may be said that these uses are specially urgent now in regard to Economics. (An example of fumbling may be easily seen at p. 285 of *Employment, Interest and Money*.*) We need a committee or group of

* More positively, there is the apparent fact that *Econometrica* flourishes in the U.S.A., and it is doubtful if it is appreciated as generally in England.