COMPARISONS OF ET(SOLAR), ET(LUNAR), UT, AND TDT

Wm. Markowitz
Nova University
2800 E. Sunrise Blvd., Apt. 15 B
Fort Lauderdale, FL 33304, USA

SUMMARY. A fundamental dynamical scale, ET(solar), was formed from absolute, meridian circle observations, from 1835 to 1982. At 1967.0 the ET(solar) and SI seconds were equal to 1 part in 10^9 and, from Morrison's (1979 a) ET(lunar) scale, the ET(ILE) and SI seconds were equal to 1 part in 10^{10}. The long term fluctuation in the Earth's rotation has been parabolic, instead of sinusoidal, for at least 350 y.

1. INTRODUCTION

TDT = TAI + 32.184 s provides a highly accurate, atomic reference scale for time and time interval measurements, but only since 1955.5. Several problems, however, require the use of dynamical scales and their units for before then. Dynamical time, denoted ET(solar), was defined fundamentally by two constants, L_0 and L_1, in Newcomb's (1898) Tables of the Sun (NTS). In practice, however, ephemeris time was obtained from lunar observations and the Improved Lunar Ephemeris (ILE,1954). The SI second was defined to equal the ILE second.

Stumpff and Lieske (1984) noted a discrepancy in the precessional speed which involves the equality of the ET(solar) and SI seconds. The principal objects of this article are to construct an ET(solar) scale, determine the relations between the seconds of ET(solar), ET(ILE), and SI, and study the long term fluctuation in the Earth's rotation. The precession problem will be discussed in a separate article.

The ET(solar) scale is formed from absolute, meridian circle observations made from 1835 to 1982. It is tabulated in two forms, as the difference from UT and from TDT', a composite scale. TDT' = TDT after 1955.5, and a lunar extension of TDT devised by Morrison (1979 a,b) before then.

2. THEORY

2.1 ET(solar)

Let the Sun's altitude be measured when crossing the meridian at a time UT. Then in principle a series of such observations, say for 6 years, will determine the obliquity of the ecliptic and the Sun's

A. K. Babcock and G. A. Wilkins (eds.), The Earth's Rotation and Reference Frames for Geodesy and Geodynamics, 413–418.

absolute declinations, right ascensions, and longitudes. The Sun's mean longitude, L, increases 1" in 24.35 s. Let δL = L(obs) - L(eph), where L(eph) is the ephemeris longitude for the UT of observation. Then ΔT(solar) = ET(solar) - UT = 24.35δL s.

Let ΔT(TDT') = TDT' - UT. Let H = ΔT(TDT') - ΔT(solar) = TDT' - ET(solar). Assume that TDT' correctly extends TDT back to 1835. Then R = dH/dT is the rate of gain of TDT' on ET(solar). One Julian century (cy) = 3 155 760 000 s. If, for example, R = +3.16 s/cy then the ET(solar) second exceeds the SI second in duration by 1 part in 10^9. T is measured in cy from 1900.0.

2.2 Lunar Scales

A non-gravitational, tidal term, CT^2, must be included in a lunar ephemeris to provde a dynamical scale. C = -11.22"/cy^2 in ILE, a value derived by Clemence (1948) from results of de Sitter (1927) for the Moon, based on ancient eclipses, and of Jones (1939) for the Sun, based on observations made since 1677. Morrison and Ward (1975) obtained C = (-13",± 1")/cy^2 from a study of Mercury. Stephenson and Morrison (1985) adopted C = (-13.0" ± 0.5")/cy^2 as the mean of four methods, including the use of lunar laser ranging and artificial satellites.

Let λ be the Moon's mean longitude in ILE. Morrison (1979 a) compared ΔT from 36 000 occultations observed from 1955.5 to 1975.0, using the lunar ephemeris j=2 based on λ, with TDT. He adopted C = -13"/cy^2, solved for two other terms in longitude, and obtained

$$Q = -1.54'' + 2.33''T - 1.78''T^2 \tag{1}$$

as the correction to λ that makes ET(lunar) equal to TDT, as nearly as may be.

λ increases 1" in 1.821 s. Morrison (1979 b) added -1.821Q s to the initial $\Delta T(\lambda)$ to obtain annual values of ΔT(TDT') from 1861.0 to 1943.0. He interpolated values for 1943.0 to 1955.5. McCarthy and Babcock (1986) added the same correction to $\Delta T(\lambda)$ of Martin (1969) for 1627 to 1860.5 to obtain ΔT(TDT'), and smoothed the results. Their Table 2 gives ΔT(TDT') from 1656.0 to 1983.5. $\Delta T(\lambda)$ of both Morrison and Martin are based on the same system of star positions and motions.

2.3 SI second

In 1960 the General Conference of Weights and Measures (CGPM) defined the SI second as the ET(solar) second, i.e., as the fraction 1/31 556 925.794 7 of the tropical year for 1900 Jan.0 at 12 h ET. The fraction is derived from the adopted constant L_1 in NTS. This second was not accessible. In 1967 the CGPM redefined the SI second as the duration of ν_c = 9 192 631 770 periods of a specified radiation of cesium-133. This made the SI second equal to the ET(ILE) second, which had been defined to equal the ET(solar) second, as nearly as may be.

3. THE DATA

Jones (1939) compiled initial δL's, denoted δL_0, based on Greenwich plus combinations of 13 other observatories. For reasons which he gave I discarded those based on right ascensions and those made for before 1835. Column 2 of Table 1 gives his values to 1902.5. Later ones are only those of the meridian circles of the U.S. Naval Observatory at the present Washington site, which form a homogeneous set. Results are given in Publications of the U.S. Naval Observatory, Second Series, Volumes 9 to 23, 1920 to 1982, except for 1980.3, which are provisional. There are about 68 years of old observations and 79 of the more recent.

Newcomb included only one term of period longer than 400 y in NTS, that of about 1 780 y. Stumpff and Lieske (1984) used Bretagnon's (1982) solar theory to obtain a correction to L(NTS) for these terms for 1700-2100. For the shorter interval 1835-1985 I obtained a correction

$$P = -1.260'' - 0.006''T + 0.046''T^2, \qquad (2)$$

TABLE I. Basic data.

Epoch	δL_0	ΔT(TDT')	ΔT(solar)	H
1837.8	-0.03"	+ 6.26 s	- 1.25 s	+7.51 s
1842.9	+0.15	+ 6.32	+ 3.20	+3.12
1848.9	+0.22	+ 6.52	+ 5.00	+1.52
1856.7	+0.12	+ 7.22	+ 2.65	+4.57
1863.3	+0.22	+ 6.81	+ 5.16	+1.65
1868.9	+0.12	+ 2.07	+ 2.77	-0.70
1874.6	-0.40	- 2.89	- 9.84	+6.95
1881.1	-0.42	- 5.36	-10.30	+4.94
1886.2	-0.29	- 5.67	- 7.10	+1.43
1889.5	-0.24	- 5.79	- 5.86	+0.07
1897.0	-0.29	- 5.62	- 7.06	+1.44
1902.5	-0.16	+ 0.62	- 3.90	+4.52
1907.5*	+0.26	+ 7.00	+ 6.33	+0.67
1915.0	+0.80	+17.19	+19.48	-2.29
1922.1*	+0.98	+22.57	+23.83	-1.26
1933.4	+0.91	+23.92	+22.08	+1.84
1940.3*	+1.01	+24.50	+24.46	+0.04
1945.3	+0.90	+26.93	+21.74	+5.19
1952.5	+1.19	+30.19	+28.74	+1.45
1959.5	+1.31	+32.92	+31.58	+1.34
1967.5	+1.57	+37.88	+37.80	+0.08
1980.3	+2.07	+50.76	+49.78	+0.98

* 9-inch meridian circle; other USNO are 6-inch.

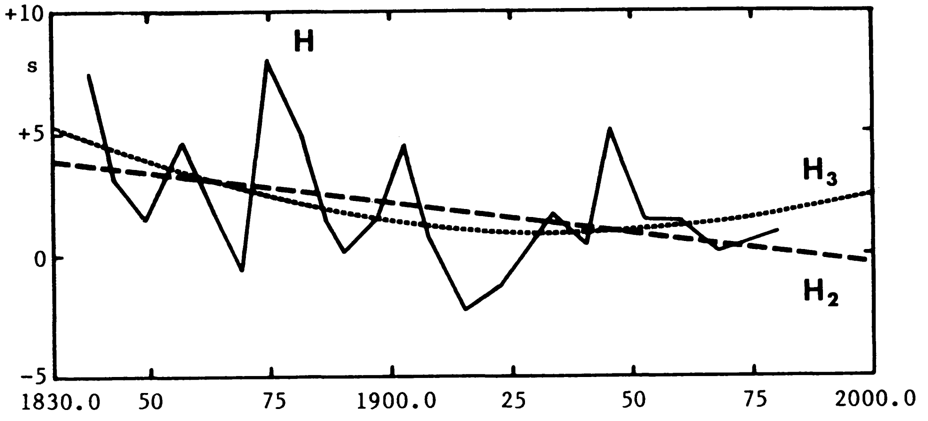

Figure 1. Observed H, and solutions H_2 and H_3.

nearly the same as theirs. Omitting the constant term, I corrected δL_0 and obtained ΔT(solar), given in column 4 of Table I. ΔT(TDT'), given in column 3, was formed from the smoothed values of McCarthy and Babcock (1986). Column 5 lists H, shown in Figure 1.

4. COMPARISONS OF SECONDS

If $C = -13''/cy^2$ is correct and there is no cosmological T^2 term in H then a linear equation should be used in a least squares solution for H. I made solutions, however, for both two and three unknowns. The weighted mean epoch of Morrison's (1979 a) comparison of ET(λ) with TDT is near 1967.0 and we wish to compare seconds near then. Solutions in terms of $t = T - 0.67$ for equal weights of H are:

$$H_2 = +(0.51 \pm 0.95) - (2.41 \pm 1.25)t, \quad (3)$$

$$H_3 = +(1.22 \pm 1.12) + (2.10 \pm 4.03)t + (3.74 \pm 3.19)t^2, \quad (4)$$

where the unit is s. R = -2.40 s/cy for H_2 and +2.22 s/cy for H_3 at 1967.0. There are no strong reasons for preferring either solution.

The term $+3.74t^2$ is only slightly larger than its mean error. If $C = -13''/cy^2$ is correct then the indicated cosmological change in the duration of the ET(solar) second relative to the SI second is an increase of (2.4 ± 2.0) parts in 10^{11} per year.

C is uncertain by $\pm 0.5''/cy^2$. If Morrison (1979) had used $C = -13.5''/cy^2$ then changes in the solutions would be $\delta H_2 = -0.18 - 1.10t$, and $\delta H_3 = -0.01 - 0.02t + 0.90t^2$. The coefficients would change sign if $-12.5''/cy^2$ had been used.

I conclude from the above solutions that the ET(solar) second has been equal to the SI second to within 1 part in 10^9.

ν_c was determined initially from a 3-y comparison of cesium and the Moon for mean epoch 1957.0, with an observational probable error of

https://doi.org/10.1017/S0074180900119825 Published online by Cambridge University Press

±10 periods (Markowitz et al., 1958). From eqn. (1) I find that the ILE second equalled ν_c + 0.3 period at 1967.0 and ν_c − 1.6 periods at 1957.0.

4.1 Errors

The mean error of a residual of H is ±2.42 s. Assume that the error in H is due solely to δL_0. The m.e. for a USNO residual is ±2.3 s, and ±2.6 s for an older value. A USNO δL_0 is based on one instrument and an older one on an average of four. Hence, a considerable increase in the accuracy of the 20th century instruments and techniques is shown. The average internal m.e. of a USNO δL_0 is ± 0.093", which corresponds to ±2.3 s. This agrees with the external error derived from H.

5. THE FLUCTUATION

Analysis of ΔT(obs) for about 25 cy provides a computed ΔT, of form $a + bT + cT^2$. Essentially, this gives the tidal variation in UT, although other effects may be involved, and is denoted δT(tidal). Let

$$\Delta T(\text{obs}) = \delta T(\text{tidal}) + F, \qquad (5)$$

where F is a residual called the fluctuation. Fairly accurate values of F are available from about 1660. Eqn. (5) separates the total variation of UT into an external, tidal part and an internal, geophysical part.

F was considered to be an unexplained, periodic fluctuation in the Moon's motion up to about 1925. P.A. Hansen included an empirical, long period Venus term in his lunar theory. Newcomb (1878, 1912) derived an empirical, 275-y sine term, which he called the "great fluctuation". On this he found superimposed "minor fluctuations", now called decade

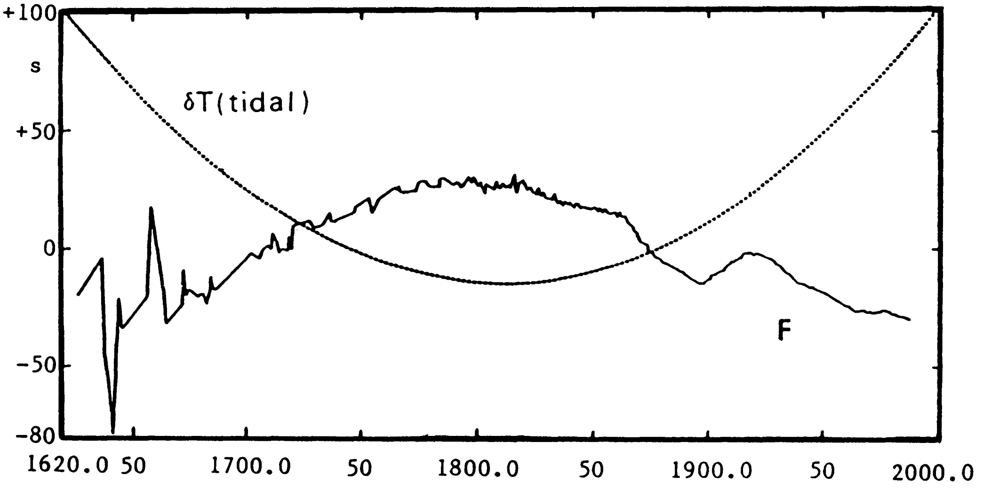

Figure 2. Tidal and non-tidal components of ΔT(TDT') = TDT' − UT.

https://doi.org/10.1017/S0074180900119825 Published online by Cambridge University Press

variations. Brown (1915), influenced probably by an upturn in F about 1896, adopted 257 y for the period. Brouwer (1952) analyzed F in a study of changes in the Earth's speed of rotation, ω.

F was obtained for 1627 to 1986.0 by letting the unsmoothed $\Delta T(TDT') = \Delta T(obs)$, and letting Stephenson and Morrison's (1983) eqn., $\Delta T = (11 + 58.4T + 32.4T^2)$ s, equal $\delta T(tidal)$. Figure 2 shows $\delta T(tidal)$ and F. The principal, long term component of F is not sinusoidal, like the decade variations. F has been parabolic for the past 350 y, at least, causing a secular acceleration in ω which almost canceled the tidal deceleration. This component and the decade variations may have different geophysical causes.

REFERENCES

Bretagnon, P., 1982, Astron. Astrophys. 114, 278-288. Dr. Bretagnon kindly furnished a listing of the long period terms.

Brouwer, D., 1952, Astron. J. 57, 125-146.

Brown, E.W., 1915, Mon. Not. R. astron. Soc. 75, 508-516.

Clemence, G.M., 1948, Astron. J. 53, 169-179.

deSitter, W., 1927, BAN. 4, 21-38, and corrections, p. 70.

ILE, 1954; Improved Lunar Ephemeris, a joint supplement to the American Ephemeris and the (British) Nautical Almanac, U.S. Govern. Printing Office, Washington.

Jones, H.S., 1939, Mon. Not. R. astron. Soc. 99, 541-558.

Markowitz, W., Hall, R.G., Essen, L., and Parry, J.V.L., 1958, Phys. Rev. Letters, 1, 105-106.

Martin, C.F., 1969, thesis, Yale University.

McCarthy, D.D. and Babcock, A.K., in Physics of the Earth and Planetary Interiors, in press.

Morrison, L.V., 1979 a, Mon. Not. R. astr. Soc. 187, 41-82.

Morrison, L.V., 1979 b, Geophys. J.R. astr. Soc. 58, 349-360.

Morrison, L.V. and Ward, C.G., 1975, Mon. Not. R. Astr. Soc. 173, 183-206.

Newcomb, S., 1878, Astr. Met. Obs., Washington, 1875, Appendix II.

Newcomb, S., 1898, Astr. Papers Amer. Ephem., 6, Part I.

Newcomb, S., 1912, Astr. Papers Amer. Ephem., 9, Part I.

Stephenson, F.R. and Morrison, L.V., 1983, in P. Brosche and J. Sündermann (eds) 'Tidal Friction and the Earth's Rotation, II', Springer, Berlin, Heidelberg, New York, pp. 29-50.

Stephenson, F.R. and Morrison, L.V., 1985, Geophys. Surveys, 7, 201-210.

Stumpff, P. and Lieske, J.H., 1984, Astron. Astrophys. 130, 211-266.

ACKNOWLEDGEMENT

I thank James A. Hughes, Dennis D. McCarthy, P. Kenneth Seidelmann, and Gernot M.R. Winkler for many helpful discussions held and for data received.