

Dynamics and instabilities in exoplanetary systems

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Abstract. Extrasolar planet surveys have discovered over two dozen multiple planet systems. As radial velocity searches push towards higher precisions and longer survey durations, they can be expected to discover an even higher fraction of multiple planet systems. Combined with radial velocity data, dynamical studies of these systems can constrain planet masses and inclinations, measure the significance of resonant and secular interactions, and provide insights into the formation and evolution of these systems. Here, we review the dynamical properties of known extrasolar multiple planet systems and their implications for planet formation theory. We conclude by outlining pressing questions to be addressed by a combination of future observations and theoretical research.

Keywords. planetary systems: formation, celestial mechanics, gravitation, instabilities, scattering, techniques: radial velocities

1. Categories of Multiple-Planet Extrasolar Planetary Systems

As is customary for a young science, we begin by observing the properties of individual specimens (i.e., multiple planet systems) and using their properties to identify categories that could be the result of some underlying physical processes that we seek to understand. From the perspective of orbital dynamics, the extrasolar planetary systems can be roughly divided into three categories, based on the strength and timescales of planet-planet interactions. We discuss category each below.

Resonant Systems: Some exoplanets are in (or near) mean motion resonances that result in strong planet-planet interactions. For example, GJ 876 hosts two giant planets with a ratio of orbital periods very nearly 2:1. Indeed, n-body integrations reveal that all three resonant angles associated with the 2:1 resonance to librate about zero (Laughlin et al. 2005). The resonant interactions cause the orbits of both planets to precess by 360° once every $\simeq 9$ years. In this case the precession is particularly rapid due to the short orbital period, low stellar mass, and relatively high planet masses (Laughlin & Chambers 2001). In most known multiple planet systems, the interactions are more subtle and/or have somewhat longer timescales than in GJ 876, making it very difficult to observe the interactions directly.

At the time of writing, five pairs of exoplanets also have a ratio of orbital periods near 2:1, one pair has a period ratio near 3:1, and a few pairs of planet have period ratios of nearly $n:1$ with $n \geq 4$ (Butler et al. 2006; www.exoplanets.org, www.exoplanet.eu). While there is strong evidence that some of these systems are indeed in resonance, some of these systems may merely be near a mean motion resonance. Unfortunately, observational uncertainties often make it difficult to differentiate the two cases. This distinction can have significant implications for planet formation models (see §2).

Secular Systems: Even in planetary systems without any mean motion resonances, angular momentum can be transferred between planetary orbits, resulting in significant eccentricity/inclination evolution on secular time scales. Among the presently known multiple planet systems, 10-15 appear to be undergoing significant secular eccentricity evolution (e.g., Barnes & Greenberg 2006 and references therein).

It is useful to subdivide the secular systems according to the mode evolution of the periastron angles. In the classical Laplace-Lagrange perturbation theory for a pair of planets, the angle between the two periastron directions ($\Delta\varpi$) can either librate about 0° (aligned), librate about 180° (anti-aligned), or circulate over the range $0-2\pi$ (circulate) (e.g., Zhou & Sun 2003; Adams & Laughlin 2006). In the full 3-body problem, short-period terms of the Hamiltonian can cause $\Delta\varpi$ to librate and sometimes circulate for systems near the boundary (e.g., Ford, Lystad & Rasio 2005; hereafter FLR). We label such systems as “borderline” and will make use of them in §2, when we describe how they could be useful for testing planet formation models. In practice, the observational uncertainties can make it difficult to determine the mode of secular evolution even for systems that do not lie particularly close to the boundary between librating and circulating initial conditions.

Hierarchical Systems: Some planetary systems do not contain any known planets near a mean motion resonance and do not appear to undergo significant secular eccentricity evolution. We label these systems hierarchical, since each planet basically orbits the barycenter of the star and interior planets. The planetary perturbations cause each orbit to regress, but there is no apsidal lock or large amplitude eccentricity oscillations. Among the presently known multiple planet systems, $\simeq 6$ are currently categorized as hierarchical systems. Hierarchical systems with large gaps between the known planets may be fertile hunting grounds for additional planets (e.g., Barnes & Raymond 2004).

Frequency of Resonant, Secular, and Hierarchical Systems: As exoplanet searches discover more multiple planet systems, it will become possible to ask questions about the relative frequency of different types of systems. At least two factors complicate the interpretation of the apparent frequency of resonant, secular, and hierarchical systems. First, in some cases, the uncertainties in the current orbital elements preclude determining whether the system is in a mean motion resonance and/or has significant secular evolution. Second, the potential discovery of additional planets in or near a mean-motion resonance could cause a system to be reclassified as resonant. Similarly, the discovery of non-resonant planet(s) in a system believed to be hierarchical could result in a tighter dynamical couple between previously known planets and reveal that the system is undergoing more significant secular evolution. Third, in some cases the discovery of additional planet(s) can result in quantitative or even qualitative changes in the orbital elements of the previously known planets. Historically, the masses and/or eccentricities of the known planets has often been revised downward, reducing the strength of dynamical interactions. Thus, the discovery of additional planets can either increase or decrease the significance of secular evolution.

2. Implications for Planet Formation Theory

Resonant and Near-Resonant Systems: Given the wide range of planet semi-major axes, eccentricities, orientations, and orbital phases, it seems unlikely that two planets would form in the relatively narrow range of parameter space that would result in a low-order resonance. However, if the planets formed in orbits with a larger ratio of orbital periods before a slow and smooth migration caused the planets to approach each other, then such systems could naturally become captured into a resonance. Therefore, planets

in mean motion resonances have been taken as evidence supporting models that result in the convergent migration of giant planets (Lee & Peale 2002). Even more compelling support for migration leading to resonant capture is provided by the current eccentricities of the two giant planets orbiting GJ 876. Assuming that two planets are captured into the 2:1 resonance while on nearly circular orbits, further migration will result in eccentricity excitation for both planets. The observation that the two eccentricities fall along the one parameter family of solutions predicted by this model provides further support for this model (Lee & Peale 2002).

It is important to note that merely having a ratio of orbital periods close to $n:1$ does not guarantee that the system is actually participating in a mean motion resonance. To be in a resonance, there must be a resonant angle that liberates about some value. Thus, determining whether or not a planetary system contains a resonance requires dynamical modeling to determine how the system will evolve in time. The results of such integrations will depend on the initial conditions which are chosen based on the available observations. In many cases, the existing observations leave a considerable uncertainty in the choice of initial conditions (Ford 2005, 2006), preventing a definitive determination of whether a system is in resonance (e.g., Goździewski *et al.* 2007). Identifying a viable orbital solution that results in resonance would demonstrate that a resonance is plausible. However, in order to make the statement that the system is in resonance, one would need to demonstrate that there are no orbital solutions that are: 1) consistent with existing observations, 2) dynamically stable for the age of the system, and 3) not in resonance. Typically, many more observations will be required to satisfy this more demanding criteria than are required to identify a near commensurability of orbital periods. Nevertheless, such observations will be essential for the successful theoretical interpretation of multiple planet systems. For example, a system deep in a resonance could provide evidence for a smooth migration in a dissipative disk (Lee & Peale 2002), while a similar system merely near the same resonance would be more likely to have arisen by chance. As another example, some theoretical models suggest that resonant capture may be followed by additional dynamical interactions that result in the planets evolving out of resonance (Thommes *et al.* 2007; Narayan *et al.* 2002). Thus, resonant systems should be targeted for frequent follow-up observations so as to distinguish models that predict planets near mean motion commensurabilities from models that predict planets actually participating in mean motion resonances.

Secular Systems: Shortly after the discovery of the three planets around v And, theorists recognized that the system was undergoing significant secular eccentricity variations and proposed two models that might explain the significant eccentricities of the two outer giant planets (c & d). Chiang & Murray (2002) proposed that a protoplanetary disk beyond planet d could *adiabatically* torque planet d. If the system was initially configured so that the longitudes of periastron were circulating, then this torque would drive the system towards solutions where the longitudes of periastron librate about an aligned configuration. Once the system was in the librating regime, the torque would damp the libration amplitude. Thus, this model would predict that the pericenters of the outer two planets would currently be librating with small amplitude about an aligned configuration and that the secular evolution would cause only small variations in the eccentricities.

In an alternative scenario, Malhotra (2002) proposed that the outer planet could have been perturbed *impulsively*. In this model, the periapses of the two planets could be either librating or circulating about an aligned (or anti-aligned) configuration, depending on the state of the system at the time of the impulsive perturbation. If the system were librating, then this model would generally predict that the libration amplitude would be large and that there would be significant eccentricity oscillations. What could cause such

an impulsive perturbation? The most natural candidate is a close encounter with another planet (Malhotra 2002). The extra planet could have been ejected from the system or might still remain bound, but in a wide orbit and hence undetected. Other possibilities include the rapid halting of inward migration (Sandor & Kley 2006), perhaps due to an edge or the rapid dispersal of the disk (but see online supplement of FLR).

The viability of these two very different models for eccentricity excitation in the ν And system demonstrated that the mode of evolution could be used to constrain planet formation theory and highlighted the importance of measuring the masses and current orbital elements with enough precision to predict the secular eccentricity evolution. Thanks to an intensive radial velocity campaign, the current angle between the two periaapses was measured to be $\Delta\omega \simeq 38^\circ \pm 5^\circ$ (FLR). This implies a large libration amplitude and therefore supports models with an impulsive eccentricity perturbation over models with an adiabatic torque. While the eccentricity of the outer planet undergoes small oscillations, the eccentricity of the middle planet undergoes very large oscillations with e ranging from from 0.34 to very nearly zero. Such behavior is characteristic of “borderline” systems, where the system lies near the boundary separating librating and circulating regimes, and is the natural outcome of a strong impulsive perturbation. FLR used a Bayesian statistical analysis of the radial velocity observations to determine that the eccentricity of the middle planet periodically returns to nearly zero for *all allowed* orbital solutions (see Fig. 2 of FLR). This provides a strong constraint on the timescale for eccentricity excitation in ν And and supports the model of the outer planet being perturbed impulsively, most likely by strong planet-planet scattering.

Variations on a Theme: Other multiple planet systems likely offer additional insights into their orbital histories and planet formation. For example, HD 128311 contains a pair of planets near a 2:1 mean-motion resonance, again suggesting convergent migration leading to resonant capture. However, the outer planet appears to be undergoing large eccentricity oscillations, quite unlike those of GJ 876, suggesting an eccentricity excitation mechanism subsequent to resonant capture. Sandor & Kley (2006) proposed a hybrid scenario that invokes convergent migration, resonant capture, and strong planet-planet scattering to explain the current orbital dynamics of the best-fit orbital solution. Tinney *et al.* (2006) and Sandor *et al.* 2007 have suggested similar scenarios for explaining HD 73526. While the published radial velocity data for HD 128311 and HD 73526 are consistent with an impulsive scenario (Sandor & Kley 2006; Sandor *et al.* 2007), the observations are still consistent with a range of orbital solutions that is too broad to allow a unique interpretation (e.g., Gozdziewski & Konacki 2006).

These and several other multiple planet systems might be examples of borderline secular evolution, as observed or suggested by several authors. For example, Zhou & Sun (2003) found that each of HD 12661 bc, HDH 82943 bc, and 47 UMa bc could be near the borderline between circularization and libration, and the recently announced two planet solution for HD 155358 bc also exhibits the large eccentricity oscillations characteristic of borderline secular evolution (Cochran *et al.* 2007). Barnes & Greenberg (2006, 2007) claimed that a large fraction of the known multiple-planet solutions appear to exhibit borderline secular evolution (which they refer to as “near separatrix motion”). They claim that a high abundance of such systems would suggest that the impulsive perturbation is unlikely to have been delivered by a planet on a nearly circular orbit at the time of the first close encounter. While a single close encounter between comparable mass planets on circular orbits does not result in ejection (Katz 1997), numerical simulations show that a rapid succession of close encounters can produce eccentric orbits. In order for the perturbation to remain impulsive, the duration of strong interactions must be less than the secular timescale. FLR presented an example requiring only one additional planet,

and also suggested that a second additional planet could have helped to make the strong perturbation impulsive by rapidly raising the periastron distance of the planet that interacted strongly with ν And d. Barnes & Greenberg (2007) have proposed an alternative scenario in which the impulsive perturbation comes from a planet that is already on an eccentric orbit at the time of the first close encounter.

Words of Caution: We eagerly look forward to studies of the relative frequency of various types of dynamical behavior in multiple planet systems. However, before jumping to conclusions, it is important to recognize the limitations of existing observations. Unfortunately, for some secularly evolving planetary systems, the observations are not yet able to measure important orbital parameters (e.g., eccentricity and argument of periastron) with sufficient precision to determine whether the systems must be undergoing borderline secular eccentricity evolution. In particular, one should distinguish between the statements: “the observations of a system are consistent with borderline secular evolution” and “the observations imply that the system is near the borderline dividing circulation and libration”. The latter statement is much more powerful, but typically requires many more observations accompanied by a detailed dynamical and statistical analysis. We have begun a program to perform such analyses of several additional multiple planet systems. In the mean time, we caution against drawing conclusions based on the dynamical behavior of the best-fit model (as opposed to the dynamical behavior of all models consistent with observations and dynamical stability).

3. Future Directions

In the coming years, it will be particularly important to follow-up discoveries of multiple planet systems with intensive radial velocity campaigns to nail down the secular evolution. In some cases (e.g., systems where the orbital period of the outer planet is comparable or greater than the time span of high-precision observations), even the orbital period and radial velocity amplitude can be highly uncertain (Ford 2005), allowing for qualitatively different orbital solutions and limiting the power of dynamical modeling. Therefore, multiple planet systems containing giant planets with short or modest orbital periods appear the most promising for precision dynamical studies and providing constraints on planet formation theory (Ford *et al.* 2007 and references therein).

We are particularly interested in determining the frequency of multiple planet systems that are in or near low-order mean motion resonances. Accurate determination will require discovering more multiple planet systems, measuring the current orbital elements of known multiple planet systems more precisely, and careful consideration of observational selection effects (Ford 2006). Ongoing radial velocity surveys will soon be complimented by searches based on the transit timing variation method, which is particularly sensitive to planets in or near mean motion resonances (Agol *et al.* 2005; Holman & Murraru 2005; Ford & Holman 2007). As more transiting planets are discovered and subjected to follow-up observations, we expect that transit timing will become a powerful tool for detecting multiple planet systems and constraining planet formation models (e.g., Nelson & Papaloizou 2002; Papaloizou & Szuszkiewicz 2005; Cresswell & Nelson 2006; Fogg & Nelson 2007).

As radial velocity and transit searches continue to increase the sample of exoplanets, it will become increasingly practical to compare the distribution of observed masses and orbital elements to those predicted by various models. Theorists have begun making testable predictions for based on various models planet-planet scattering and planet-disk interactions (e.g., Moorhead & Adams 2005; Veras & Armitage 2006; Chatterjee *et al.* 2007; Ford & Rasio 2007; Juric & Tremaine 2007). For example, radial velocity

observations can be used to test the prediction that eccentric planets will be more common when the ratio of escape velocity from the planet's surface exceeds the escape velocity from the host star at the planet (Ford & Rasio 2007). As another example, planet scattering predicts a correlation between eccentricity and inclination (Chatterjee *et al.* 2007), enabling tests with both astrometric measurements of the relative inclination between orbits (McArthur *et al.* 2007) and Rossiter measurements of the inclination of orbital angular momentum to the stellar spin axis (Narita *et al.* 2007; Nagasawa in this volume). We encourage theorists to continue developing models to the point where they make alternative predictions that can be tested by upcoming observations.

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