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## Feedback

On [1]: Paul Belcher writes: The Note talks about a *shear-like* transformation that preserves area. The term *shear-like* is not defined and actually the transformation given is a vertical stretch with a factor of 2. This does not preserve area but multiplies it by 2. However as only half of the circle has been transformed the integral given for  $\pi$  is correct.

To obtain the result given (not using the double angle formula), we do not have to use geometrical transformations. If we take the area enclosed between  $y = +\sqrt{1 - x^2}$  and the  $x$ -axis it is half a circle, so  $\frac{1}{2}\pi = \int_{-1}^1 y \, dx$ . Using parametric coordinates  $x = \cos \theta$ ,  $y = \sin \theta$  and integration by substitution with  $\frac{dx}{d\theta} = -\sin \theta$  then

$$\frac{\pi}{2} = \int_{\pi}^0 \sin \theta (-\sin \theta) d\theta = \int_0^{\pi} \sin^2 \theta \, d\theta$$

The algebra is essentially that shown in the Teaching Note.

### Reference

1. Colin Foster, Teaching Note  $\int_0^{\pi} \sin^2 \theta \, d\theta$  from a shear-like transformation of a circle, *Math. Gaz.* **108** (November 2024) pp. 541-542.

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