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The optical differentiation coronagraph

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Abstract. Direct detection of exoplanets is a topic of increasing interest since the first exoplanet discovery by indirect methods. It represents a formidable task because of the small angular separation and large contrast ratio between planet and parent star. We present a novel family of stellar coronagraphs based on the standard coronagraph design but transformed to perform optical differentiation. The proposed coronagraphic masks are used to perform the first or the second derivative of the incoming field. This concept offers a new method to detect exoplanets providing both, deep starlight extinction and high angular resolution. To perform optical differentiation the coronagraph's occulting disk is replaced by an especially designed mask. A further improvement on the coronagraph performance is made by adding a gaussian profile to the differentiation mask in order to reduce the amount of diffracted light. The theoretical rejection rate of our coronagraph is infinite. Computer simulations carried out for the ideal case show that it achieves deep starlight reduction corresponding to a gain of at least 37 mag (10^{-15} light intensity reduction). To take full advantage of the capabilities of our coronagraph atmospheric distortions must be reduced by the use of extreme adaptive optics systems or by its use on space telescopes.

Keywords. techniques: high angular resolution, astrobiology, instrumentation: adaptive optics.

1. Introduction

The detection of faint sources around bright objects is a topic of increasing interest since the first indirect discovery of an exoplanet orbiting a Sun-like star (Mayor & Queloz (1995)). Since then, direct detection has risen as a probably viable method to uncover exoplanets in the visible. The goal of direct exoplanet detection is to obtain images of Earth-like planets around solar-type stars and to perform spectroscopy of its atmosphere to look for habitability conditions or even to search for signs of life (Woolf & Angel (1998)). However, the detection of such faint objects presents a major difficulty because of the small angular separation and the large contrast ratio between a Sun-like star and the Earth-like planet; 10^{-6} in the mid-IR and $10^{-9} - 10^{-10}$ at visible wavelengths (Beichman *et al.* (1999)).

In the past years, new coronagraphic designs based on the Lyot coronagraph were described. These proposals include the use of specially designed focal-plane phase masks to achieve better starlight background reduction. Roddier & Roddier (1997) proposed a coronagraphic phase mask covering the inner part of the star's image providing better background starlight reduction than the standard coronagraph. Further starlight extinction can be achieved by the Achromatic Interfero-Coronagraph (AIC) (Gay & Rabbia; Baudoz *et al.* (2000)) or by the Four Quadrant Phase Mask coronagraph (FQ-PM) (Rouan *et al.* (2000)), which uses an alternative phase-step mask arranged in a quadrant-scheme. Kuchner & Traub (2002) (see also Kuchner & Spergel (2003)) described other technique that uses band-limited masks, presenting promising results. Theoretically

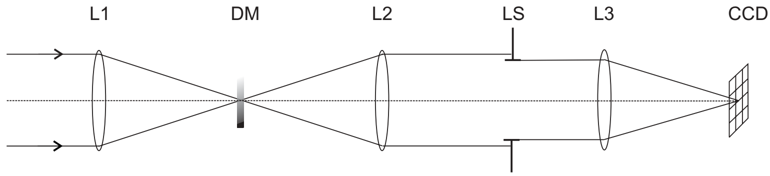

Figure 1. Set-up of the optical differentiation coronagraph. It is composed of three achromatic lenses (L1, L2, and L3), the Lyot stop (LS), the differentiation mask (DM) and a CCD camera.

these techniques, as the optical differentiation coronagraph described here, achieve perfect cancelation of the starlight.

Here we describe a whole new family of coronagraphic masks based on the optical differentiation technique that are implemented on a standard Lyot coronagraph. Theoretically, these differentiation masks allow perfect starlight extinction. Moreover, they provide high angular resolution so they can be used in the detection and discovery of Earth-like exoplanets. As in other coronagraphic designs, the presence of wavefront phase distortions or pointing errors degrades the performance of our coronagraphic technique. Therefore, it is mandatory to reduce atmospheric distortions and jitter in order to take full advantage of the capabilities of our coronagraph.

In § 2 the principle of operation of our coronagraphic design is presented, the mask used to perform optical differentiation is described briefly and the detection procedure is also introduced. Simulations of our coronagraph, in the perfect case with no errors, are presented in § 3. As a simulation example, our coronagraph is used to successfully detect an Earth-like exoplanet. Next, in section § 4, the new designs of coronagraphic masks are introduced and some of their properties are described. Finally, in § 5 some conclusions are given.

2. The optical differentiation coronagraph

The main idea on this new coronagraphic concept is the use of the optical differentiation technique to reduce the light of on-axis stars. In fact, under ideal conditions a centered unresolved star presents a plane wavefront so its derivative is zero. On the other hand, a planet orbiting this star has a tilted wavefront and consequently its derivative is not zero. This coronagraphic concept has been presented in a previous work (Oti *et al.* (2005)). Next, we briefly describe the principle of the optical differentiation technique. From the definition of the Fourier transform (FT), the derivative of a one-dimensional function $f(x)$ can be expressed as:

$$\frac{df(x)}{dx} = \int_{-\infty}^{\infty} 2\pi u_x i \cdot F(u_x) \cdot e^{i2\pi u_x x} du_x \tag{2.1}$$

$F(u_x)$ represents the FT of $f(x)$, u_x is the spatial frequency coordinate in the x direction and i is the imaginary unit. The basic set-up required to carry out the optical differentiation operation consists of a telescopic system made of two achromatic lenses. From (2.1) follows that if a mask of the form $2\pi u_x i$ is placed on the common focal plane of this telescopic system it performs the optical differentiation of the incoming field along the direction of the mask slope (Iizuka (1987); Oti, Cagigal & Canales (2003); Oti, Cagigal & Canales (2005a)). Hence, the idea is to transform a standard Lyot coronagraph into a system able to perform optical differentiation to improve its capacity for exoplanet detection. This is accomplished by substituting the occulting disk by a differentiation mask

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as shown in figure 1. For a incoming electric field $E(x,y) = E_0 e^{i\phi(x,y)}$ at the entrance pupil of the coronagraph, where $\phi(x,y)$ is the wavefront phase and E_0 is the constant amplitude, the detected intensity at coronagraphic pupil, $I(x,y)$, can be expressed as:

$$I(x,y) = |FT\left[2\pi u_x i \cdot FT\left[E(x,y)\right]\right]|^2 = \left|\frac{\partial E(x,y)}{\partial x}\right|^2 = |E_0|^2 \left|\frac{\partial \phi(x,y)}{\partial x}\right|^2 \qquad (2.2)$$

where FT stands for the operation of Fourier transformation. The coronagraph performs a perfect cancelation if the phase $\phi(x,y)$ is constant so its derivative is also equal to zero and no intensity is detected at the output. However, a tilted source presents a phase different than zero that provides a derivative estimate different that zero. Therefore, the light coming from the centered star is removed allowing the search for a potential faint source in the circumstellar environment.

2.1. *Mask description*

The mask used to obtain the first derivative of the incoming field has been described in detail in Oti *et al.* (2005b). It is made of two different complements. First, an amplitude transmittance mask proportional to $|2\pi x|$, and a phase mask consisting of a phase step of $-\pi/2$ for the negative valued coordinates and of $+\pi/2$ for the positive ones. The phase step is required to obtain the complex unit, i, and the correct sign of the differentiation mask. Although this first derivative mask produces a high level of starlight extinction, the diffraction generated by the sharp edges increases the starlight background making hard the detection of faint sources as they are also affected by absorption in the mask. To overcome this limitation we propose to remove the sharp edges of the mask by the use of a transmittance gaussian profile so the differentiation masks ends smoothly. This gaussian profile varies only along the differentiation direction. Furthermore, the gaussian profile concentrates the diffracted light on a narrow area near the pupil border and as a consequence, it can be removed more efficiently by the Lyot stop. Extinction rates may vary largely depending on the width of the gaussian profile and the size of the mask. In general, the larger the differentiation-mask size, the deeper the extinction rate achieved.

2.2. *Off-axis sources detection procedure*

To make a complete survey of all possible off-axis sources locations it is necessary to rotate the coronagraph between 0 and $\pi/2$ and take images during this rotation process. As stated in (2.1), the differentiation mask provides the derivative along the mask slope direction. Consequently, if the exoplanet is located along the perpendicular of the mask-slope direction the measured exoplanet intensity will be zero. In addition, as the differentiation mask rotates, or equivalently the coronagraph rotates, the exoplanet intensity will increase until it reach a maximum corresponding to a position parallel to the mask-slope. This is the optimum position to perform spectroscopy of the exoplanet. Like the rest of the coronagraphic designs our coronagraph is sensitive to pointing errors. A procedure to minimize pointing inaccuracy is to monitor the on-axis starlight and search for a minimum of the detected light intensity. Once this minimum is reached, a rotation of the mask (or coronagraph) should not produce changes in the detected on-axis light intensity.

3. Computer simulations

To check the performance of our coronagraph we carried out computer simulations of an ideal system using the fast Fourier transform routine (Press *et al.* (1995)). To achieve a good sampling and hence, to avoid aliasing effects we use arrays of 1024×1024 data

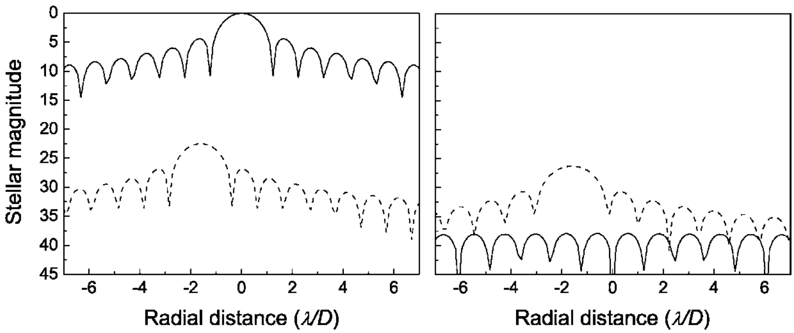

Figure 2. Simulated cross sections of the optical differentiation coronagraph for a successful exoplanet discovery. a) The parent star (solid curve) completely overwhelms the exoplanet (dashed curve) if no coronagraphic system is present. b) With our coronagraph the exoplanet clearly appears in the simulation.

samples. The coronagraph pupil is simulated with a data sampling of 128×128. Figure 2 show the results of a simulation of our system for the perfect wavefront case applied to an Earth-like exoplanet detection example. The mask used is the standard differentiation mask multiplied by the gaussian profile. It has a radius in the x direction of about $50\lambda/D$ and the gaussian profile has a variance of $\sigma^2 = 100(\lambda/D)^2$. The Lyot stop size has been properly adjusted so the extinction rate achieved is the best without a large decrease in exoplanet light throughput. We have found that in this case the optimum Lyot size is 85% of the entrance pupil of the coronagraph. The intensity ratio between the parent star and the exoplanet is 10^9 and the angular separation is $1.5(\lambda/D)$. The starlight intensity reduction achieved is 15 orders of magnitude, corresponding to about 37 mag, hence, the exoplanet clearly appears in the final simulated image.

4. Second derivative differentiation mask

From (2.1) it is easy to show that using a mask of the form $(2\pi u_x)^2$, instead of the linearly increasing transmittance mask, an optical differentiation set-up gives the second derivative of the incoming field. Working out this second derivative for an incoming field $E(x,y) = E_0 e^{i\phi(x,y)}$ follows:

$$I(x,y) = |E_0|^2 \left[\left| \frac{\partial \phi(x,y)}{\partial x} \right|^4 + \left| \frac{\partial^2 \phi(x,y)}{\partial x^2} \right|^2 \right] \tag{4.1}$$

As in the first derivative case presented in § 2, this equation implies that the on-axis light is removed since it presents a plane wavefront. This second derivative mask is pure amplitude mask whose amplitude transmittance increases along the derivative direction quadratically. Consequently it does not require the use of a phase step avoiding problems in the manufacture of an achromatic phase mask. The exoplanet detection procedure is similar to one described in § 2.2. Then, again it is necessary to rotate the mask to make a complete survey of all possible planet locations. To avoid this procedure we propose a further development in the second derivative differentiation mask by making it a symmetric mask of the form $(2\pi)^2(u_x^2 + u_y^2)$, being u_x and u_y the spatial frequency coordinates in the x and y direction correspondingly. With this differentiation mask

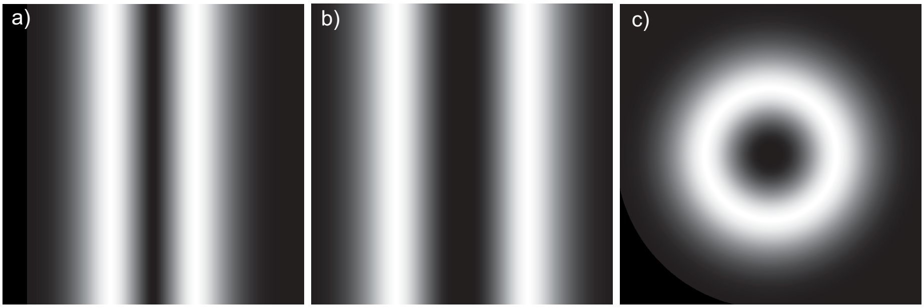

Figure 3. Plot of the proposed differentiation masks. a) Modulus of the first derivative mask of the form $2\pi u_x i$. b) The second derivative mask $(2\pi u_x)^2$. c) And the symmetric second-derivative mask $(2\pi)^2({u_x}^2 + {u_y}^2)$.

placed on the coronagraph the detected intensity can be derived as:

$$\begin{aligned} I(x,y) &= \left| \frac{\partial^2 E(x,y)}{\partial x^2} + \frac{\partial^2 E(x,y)}{\partial y^2} \right|^2 \\ &= |E_0|^2 \left[\frac{\partial^2 \phi(x,y)}{\partial x^2} + \frac{\partial^2 \phi(x,y)}{\partial y^2} \right]^2 + |E_0|^2 \left[\frac{\partial \phi(x,y)}{\partial x} + \frac{\partial \phi(x,y)}{\partial y} \right]^2 \end{aligned} \tag{4.2}$$

Once more, the on-axis light is removed without removing the off-axis exoplanet light. The first term in this expression will vanish in all cases because both the on-axis star and off-axis exoplanet second derivative are zero. A plot of all the differentiation masks described here is shown in figure 3.

These second derivative masks are less affected by pointing errors since the central region of the masks presents a larger darker area than in the first derivative case, as seen in figure 3. To check the last statement we perform computer simulations of the coronagraph for a star with various displacements from axis. These displacements are performed along the mask slope direction of the first derivative differentiation mask. The results of the simulations are displayed in figure 4. For the first derivative mask the pointing error seriously degrades the performance of the coronagraph (solid curve) since small pointing errors produces a large starlight leakage. On the other hand, the second derivative mask presents a slower degradation of its performance (dashed curve).

5. Conclusions

We described a novel family of coronagraphic masks based on the optical differentiation technique. The masks presented here perform the first and second derivative of the incoming field and are implemented on a standard coronagraph transforming it into a system able to carry out optical differentiation. Therefore, the light from an on-axis source is successfully removed since its wavefront is plane. On the other hand, the light from an off-axis faint source is not greatly attenuated as it presents a tilted wavefront. In order to improve the starlight extinction achieved further, we multiply the differentiation mask by a gaussian profile. Then, the starlight diffracted by mask is reduced as the masks ends smoothly. To check the capabilities of our coronagraph we have performed computer simulations for the ideal case with no errors. The achieved starlight reduction is as high as 15 orders of magnitude (37 mag). The phase step is not required for the second-derivative mask making its manufacture easier. Furthermore, a symmetric second derivative mask has been developed to modify the exoplanet search procedure so it is no

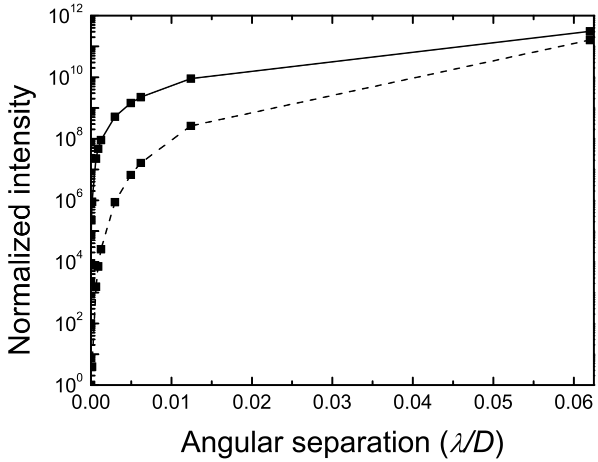

Figure 4. Normalized intensity obtained with our coronagraph for the first (solid curve) and second (dashed curve) derivative differentiation mask as a function of the star displacement from the optical axis. The intensity is normalized by the value of the detected intensity of the on-axis star.

longer necessary to rotate the mask. The coronagraphic designs introduced here can be very useful in the search of Earth-like exoplanet due to its high starlight extinction rate. Nevertheless, more checks of our coronagraphic design should be made under more realist conditions (with phase distortions and jitter errors) in order to completely characterize its performance.

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