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The thermal conductivity of seasonal snow

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ABSTRACT. Twenty-seven studies on the thermal conductivity of snow (k_{eff}) have been published since 1886. Combined, they comprise 354 values of k_{eff}, and have been used to derive over 13 regression equations predicting k_{eff} vs density. Due to large (and largely undocumented) differences in measurement methods and accuracy, sample temperature and snow type, it is not possible to know what part of the variability in this data set is the result of snow microstructure. We present a new data set containing 488 measurements for which the temperature, type and measurement accuracy are known. A quadratic equation,

$$k_{\mathrm{eff}} = 0.138 - 1.01\rho + 3.233\rho^2 \qquad \{0.156 \leq \rho \leq 0.6\}$$
$$k_{\mathrm{eff}} = 0.023 + 0.234\rho \qquad \{\rho < 0.156\},$$

where ρ is in g cm^{-3}, and k_{eff} is in W m^{-1}K^{-1}, can be fit to the new data (R^2 =0.79). A logarithmic expression,

$$k_{\mathrm{eff}} = 10^{(2.650\rho - 1.652)} \qquad \{\rho \leq 0.6\},$$

can also be used. The first regression is better when estimating values beyond the limits of the data; the second when estimating values for low-density snow. Within the data set, snow types resulting from kinetic growth show density-independent behavior. Rounded-grain and wind-blown snow show strong density dependence. The new data set has a higher mean value of density but a lower mean value of thermal conductivity than the old set. This shift is attributed to differences in snow types and sample temperatures in the sets. Using both data sets, we show that there are well-defined limits to the geometric configurations that natural seasonal snow can take.

INTRODUCTION

Snow is an excellent thermal insulator. When covered by snow, heat flow from the ground, from ice on an ocean, lake or river, or from a man-made structure, is greatly reduced. Because a significant fraction of the Earth's land and sea-ice surface is often snow-covered, this reduction plays an important role in the Earth's heat balance and has a profound effect on human lives. Consequently, measured or estimated values of the insulating properties of snow are used in a wide range of studies including planetary geophysics, climate modeling, and glaciology. In studies of permafrost or ground freezing, accurate values of the thermal conductivity of the snow cover are essential. They are also critical in assessing the potential impact of changes in snowfall resulting from climate change.

It is customary to quantify the insulating property of snow by its thermal conductivity (k). Compared to many other naturally occurring substances (i.e. rocks, liquid water, soil, ice), snow has an exceedingly low thermal conductivity. This attribute has been recognized for hundreds or perhaps even thousands of years, and Man has tried to take advantage of it. Snow has been banked against cabins and barns to help keep out the winter cold, and is known to provide valuable additional insulation if left on the roof of a dwelling. Conversely, winter travelers have long been aware that a thick cover of snow can retard the rate of freezing of river and lake ice, creating potentially dangerous conditions for crossing.

The first formal measurements of the thermal conductivity of snow date back to at least 1886 (Andrews, 1886). In the intervening 110 years, there have been an additional 26 studies in which thermal conductivity has been measured (Table 1). These measurements have been compiled in several summary papers (Kondrat'eva, 1954; Mellor, 1964, 1977; Yen, 1969; Anderson, 1976; Langham, 1981; Fukusako, 1990) intended to bring information together in one place and make it concise and easy to use. The summaries include little or no critical discussion or assessment of the original data, and present only curves fit to the data, not the data themselves. Over time, this has had the unintended effect of making the collection of measurements seem more orderly and less scattered than it actually is. It has also made it impossible for the reader to assess data quality. Curves based on robust experiments with many data are presented with the same weight as curves based on few data. Curves developed using indirect methods of measurement (i.e. deriving the conductivity from variations in snow temperature) and low accuracy are weighted the same as curves derived from data measured using more precise laboratory techniques.

Another weakness of the existing summaries is that they present thermal conductivity as a function of snow density alone. While for engineering purposes this is generally the variable of choice, other snow attributes like grain-size, bonding and temperature are the primary variables affecting the thermal conductivity.

In light of these weaknesses, and because it has been our

Table 1. Summary of existing thermal conductivity studies comprising the "Others" data. No., number of measurements; DS, in data set?; T, temperature

#	Date	Author	Location	No.	Method	Problems	DS
1	1886	Andrews	Gr. Britain (Sheffield)	1	T change of snow volume with time	Snow compressed; no density taken; no explicit value calc.	No
2	1889	Hjelström	Sweden (Stockholm)	1	T amplitude ratios @ 3 depths plus surface	Highly conductive thermometers, snow settlement and change in k_{eff}	Yes
3	1893	Abel's	Russia (St Petersburg)	13	T amplitude ratios @ 2 depths plus surface	Solar radiation, conductive thermometers, density at therms. not measured	Yes
4	1901	Jansson	Sweden (Uppsala)	33	Unguarded hot plate; heat flow divided by T gradient	Possible heat flow around, not through, snow sample; compressed snow	Yes
5	1905	Okada	Japan (Sapporo)	2	T amplitude ratios @ 3 depths plus surface	Same as Hjelström and Abel's	Yes
6	1908	Okada and others	Japan (Sapporo, Asahigawa)	8	T amplitude ratios @ 3 depths plus surface	Same as Hjelström and Abel's	Yes
7	1924	Ingersoll and Koepp	U.S.A. (Wisconsin)	1	Response at depth to a step change in temperature	Compacted snow, not initially thermally equilibrated	Yes
8	1933	Devaux	French Alps	12	Amplitude ratio of natural T variation	Possible solar radiation; snow not in thermal equilibrium	Yes
		Devaux	French Alps	23	Analysis of time-dependent heating of a cylinder	Possible solar radiation; snow not in thermal equilibrium	Yes
		Devaux	French Alps	21	Analysis of time-dependent heating of a sphere	Possible solar radiation; snow not in thermal equilibrium	Yes
9	1933	Niederdorfer	Germany (Potsdam)	3	Method given as Albrecht's; no other details	Values given for only 3 snow types, no density	No
10	1939	Kuz'min	Russia (Kuchino)	10	T amplitude ratios @ 3 depths plus surface	Solar radiation penetration	Yes
11	1949	Bracht	Germany (Leipzig)	17	2 parallel needles, one heated, one records T with time	Snow compaction between needles; new snow not possible	Yes
		"	"	1	Amplitude ratio and phase shift of diurnal T variations	Concludes phase shift gives twice that of amplitude ratios	Yes
12	1954	de Quervain	Switzerland (Davos)	2	Unguarded hot plate with heat flow determined across Bakelite	Possible heat flow around snow sample; compressed snow	Yes
13	1954	Kondrat'eva	Russia	16	Fourier soln. for time change of T @ 4 depths (lab. samples)	Used compacted snow; has to smooth data heavily for finite difference approximation.	Yes
14	1954	Yosida and Iwai	Japan (Sapporo)	16	Time response of snow cylinder to step change in T	T measured by air pressure; inaccurate; non-isotropic heat flow	Yes
15	1962	Yen	U.S.A. (New Hampshire)	1	Solves steady-state heat eqn. from vertical T profiles	Used aged boxed snow samples; extrapolated to 0 air flow rate	Yes
15	1965	Yen	U.S.A. (New Hampshire)	4	Solves steady-state heat eqn. from vertical T profiles	Used aged boxed snow samples; extrapolated to 0 air flow rate	Yes
16	1967	Pitman and Zuckerman	U.S.A. (Massachusetts)	8	Guarded cut-bar apparatus	Only compressed freezer-hoar frost tested	Yes
17	1970	Jaafar and Picot	Canada (New Brunswick)	5	Transient change in T with needle-probe apparatus	Lab. samples possibly suffered some compaction	Yes
18	1971	Weller and Schwerdtfeger	Antarctic Plateau	2	Fourier-type analysis of mean annual T at depth	Possible non-conductive heat-transfer mechanisms; snow T: −60°C	Yes
19	1973	Shesterikov	Kara, Laptev, Siberian Seas	7	Based on rate of sea-ice growth; needs ice thickness, conductivity	Calc. values twice as high as those measured by needle probes	No
20	1975	Izumi and Huzioka	Japan	24	No translation available, method not known	??	Yes
21	1975	Kuvaeva and others	Russia (Caucasus)	15	Insertion of cold plate in snow; determine T at distance	Compaction of snow around plate; errors in determining x	Yes
22	1975	Voitkovsky and others	Siberia (Yakutsk)	20	No method stated	No method given; 20 values averages of 174 measurements	Yes
23	1980	Reimer	Canada (Manitoba)	4	Amplitude ratio and phase shift for monthly ave. T at 6 depths	Windpumping likely; values average over layers	Yes
24	1985	Lange	Antarctica (Filchner–Ronne Ice Shelf)	31	Transient change in T with needle-probe apparatus	No data to indicate if air convection possible	Yes
25	1989	Murakami and Maeno	Japan	7	2 parallel needles, one heated, one records T with time	Steel vessel affected results; samples compacted, only init. density used	Yes
26	1991	Östin and Andersson	Sweden	58	Transient change in temperature of heated strip	Used samples consisting of freezer frost	Yes
27	1992	Sturm and Johnson	Alaska	101	Transient change in T with needle-probe apparatus		No
		Total (less Sturm and Johnson)		366			
		Measurements not used		12			
		Total used in analysis		354			

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experience that most end-users of thermal conductivity values extract them from summary papers, we felt that it was time to review, evaluate and consolidate existing measurements. Most of the original data are published in foreign journals and are relatively inaccessible, so one of our goals was to give the user the advantage of knowing the quality of the data without the need to search it out and translate it. A second goal was to emphasize the significant scatter in the data. The scatter is real and arises from differences in the microstructure of snow. Differences in grain-size, grain type and bonding can lead to an order-of-magnitude range in thermal conductivity at a given density. This scatter renders regressions of thermal conductivity vs density inherently inaccurate, and we wanted readers to be aware of this.

We had one other motive for undertaking a review at this time. Only a limited number of the nearly 500 measurements of snow thermal conductivity that we have made are published (Sturm, 1991; Sturm and Johnson, 1992). This set, which encompasses a wide range of densities and nearly all types of snow, comprises more data than all the other studies combined. In addition, it includes sufficient snow-sample temperatures and descriptions for the effect of attributes other than density on the thermal conductivity to be examined. Using our data, we develop improved regression equations for thermal conductivity as a function of density. With the regressions, we stress that at any given density there is an order-of-magnitude range of thermal conductivity. This scatter, real and not the result of measurement error, implies that density is not the fundamental controlling variable for thermal conductivity. We show that reasonable estimates of thermal conductivity can be obtained using a description of the snow type in lieu of the density, a result that highlights the fact that bonding and the shape and arrangement of snow grains is the underlying controlling parameter.

BACKGROUND

Discussions of heat transport in snow have been published by Mellor (1964, 1977), Yen (1969), Combarnous and Bories (1975), Anderson (1976), Arons (1994) and Arons and Colbeck (1995). Here, we restrict our attention to issues related to the making and interpretation of thermal conductivity measurements. We define the thermal conductivity using a continuum view: thermal properties can vary across a layer of snow, but are averaged across many snow grains and grain clusters. Grains, grain clusters and bonds do not explicitly appear in the formulation, even though they actually control the thermal conductivity. The thermal conductivity is defined as the proportionality constant between the heat transport and the temperature gradient. For unidirectional steady-state heat flow in a solid, the defining equation (the Fourier equation) is:

$$q = -k\frac{\mathrm{d}T}{\mathrm{d}z} \tag{1}$$

where q is the heat flow, T is the temperature, Z is length and $\mathrm{d}T/\mathrm{d}Z$ is the temperature gradient. For reference, the thermal diffusivity (α) is related to the conductivity by:

$$k = \rho c \alpha \tag{2}$$

where ρ is the snow density, and c is the specific heat of ice.

Because snow consists of three phases, air, ice and water vapor (we limit our attention to dry snow), the heat transport is more complicated than for a solid. It has three components: (1) conduction through the ice lattice, (2) conduction through the air in the pore spaces, and (3) latent-heat transport across pore spaces due to vapor sublimation and condensation. Two other heat transport mechanisms, radiation and convection, can also operate but are not important in the context of thermal conductivity measurements. For common snow temperatures and pore sizes, radiation transfers several orders of magnitude less heat than other mechanisms. Air convection in pore spaces can dominate heat transport when it occurs, as shown by Sturm (1991) and Sturm and Johnson (1991), but it is unusual in most natural snow covers, and unlikely to occur under the conditions common in thermal conductivity tests. The three main mechanisms of heat transport are generally combined in a single value, the effective thermal conductivity (k_{eff}). Equation (1) becomes:

$$q = -k_{\mathrm{eff}}\frac{\mathrm{d}T}{\mathrm{d}z}\,. \tag{3}$$

Apportioning the total heat transport into transport mechanisms is difficult since the thermal and water-vapor pressure gradients in snow are coupled and depend on the microstructure, which is difficult to quantify. The ice skeleton (grains and necks) is about 100 times more conductive than the air in the pore spaces (Fig. 1). Since the ice skeleton provides a much better thermal pathway than the pore spaces, temperature gradients across the pore spaces are enhanced compared to those in the ice skeleton (de Quervain, 1973). Pore-space temperature-gradient enhancement also increases the vapor-pressure gradients and hence the transport of water vapor. The associated transport of latent heat also increases. At the same time, the ice skeleton has a blocking effect that limits vapor movement to the pore

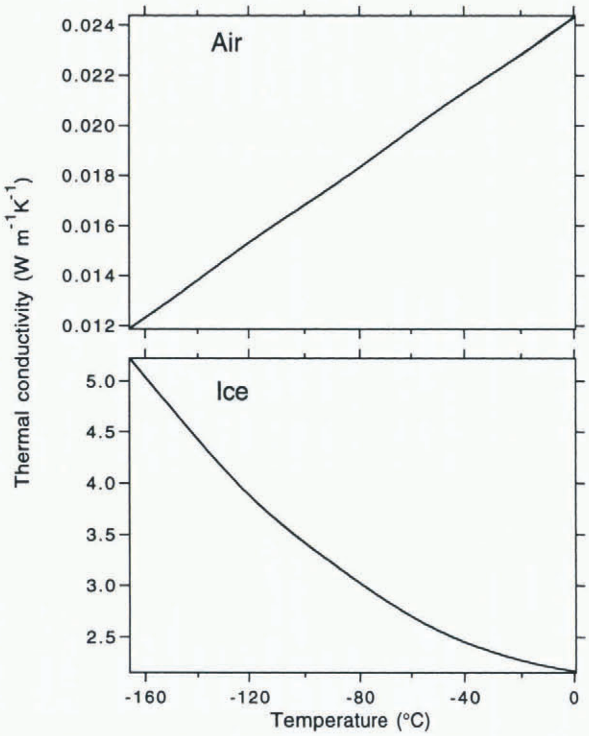

Fig. 1. Thermal conductivity of ice and air as a function of temperature (sources: air: List, 1951; ice: Hobbs, 1974).

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spaces. The balance between these contradictory processes (high thermal conductivity but low temperature gradients vs low thermal conductivity but high gradients; blocking vs enhancement) is a function of grain and pore configuration and size, as well as the temperature and temperature gradients imposed on the snow.

De Quervain (1956), based on an elegant experiment in which he replaced the air in snow with gases of different thermal conductivity, estimated that for his snow samples the ice skeleton carried 55–60% of the heat, the rest moving across the pores as sensible or latent heat. Colbeck (1993), reviewing the literature on vapor diffusion in snow, concluded that about 30–40% of the heat moved by vapor transport because vapor-gradient enhancement predominated over the blocking effect of the ice skeleton. He suggested that the amount of heat carried by water vapor increased as the pore size decreased, though no data exist to test this idea. Estimates by other authors (Yosida and others, 1955; Yen 1965; Arons, 1994) for the heat transported by water vapor range from 10% to 40% depending on the type of snow and its temperature (Fig. 2).

A general assumption is that the temperature dependence of the thermal conductivity of snow is due to variations in the amount of heat moved by water vapor. Therefore variations in k_{eff} with temperature can be used to determine the relative importance of heat-transport mechanisms. Unfortunately, relevant experimental data are sparse and do not completely support theoretical results (cf. Figs 2 and 3). Of the four studies in which measurements have been made over a wide range of temperature and at temperatures sufficiently low to effectively eliminate all vapor heat transport, two may not be reliable. The most widely cited results, those of Pitman and Zuckerman (1967), were obtained on artificial snow (freezer frost). Because of the way they did their tests, data from different samples have to be amalgamated into a single set in order to be used, potentially creating large errors. This may account for their data indicating a 20–40% increase in the contribution of vapor to the total thermal conductivity between −88° and −27°C, a temperature range over which little change in vapor transport would be expected. Similarly, the results of Voitkovsky and others (1975) show an extremely strong temperature dependence for k_{eff} at higher temperatures, but no published details are available to evaluate how they obtained their results.

For the remaining two studies (Arons, 1994, and previously unpublished data of ours), differences in the degree of bonding seem to explain differences in the results. Both samples had approximately the same density. One sample (Arons, 1994) was lightly sintered, sieved snow of density 0.410; the other was extremely well-bonded natural drift snow of density 0.49. The well-bonded snow showed increasing thermal conductivity with decreasing temperature (plus a slight increase in thermal conductivity as the temperature approached 0°C). The less well-bonded snow showed the opposite response. We attribute the differences to the fact that the vapor-transport term was negligible in the well-bonded sample, while it was significant in the less well-bonded one. The well-bonded drift sample had sufficiently good ice connections that the temperature dependence of ice (Fig. 1) became a controlling factor.

The preceding discussion highlights the fact that measurements of thermal conductivity depend on two key attributes of the snow: its microstructure and temperature. For temperature, more data are needed before reliable conclusions can be drawn. For microstructure, Arons and Colbeck (1995) conclude that the tools necessary to parameterize the relevant microstructural geometry are neither available now nor likely to be available in the near future. Several attempts to predict thermal conductivity based on the micro-geometry have been made (Pitman and Zuckerman, 1967; Reimer, 1980; Christon and others, 1987; Arons, 1994), but these have had limited success due to the lack of these crucial geometric data. Also none have included fully coupled heat and vapor flow.

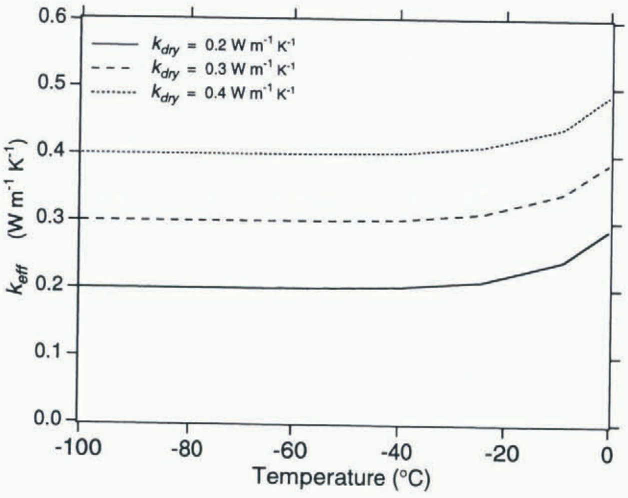

Fig. 2. Temperature dependence of the thermal conductivity of snow, based on kinetic theory and a diffusion equation for vapor transport (after Arons, 1994). Note that most of the increase in k_{eff} occurs between −20° and 0°C. k_{dry} indicates the thermal conductivity of snow with no vapor transport.

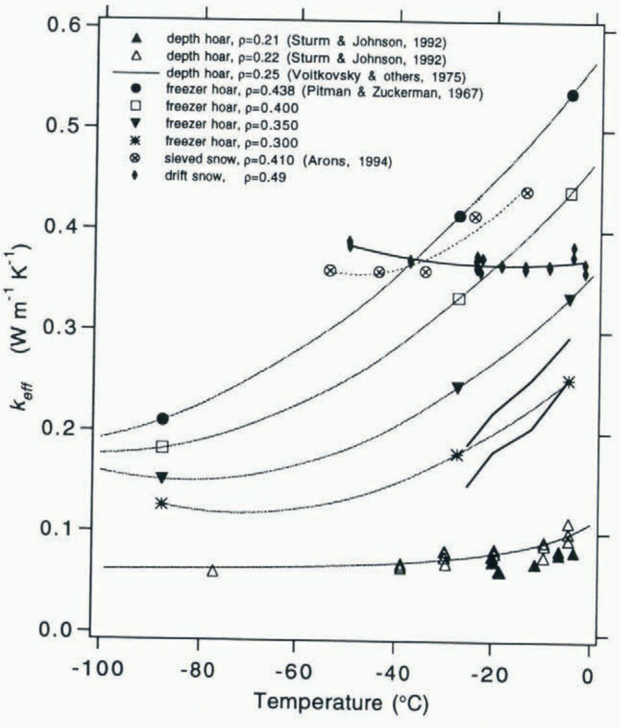

Fig. 3. Experimental results showing the thermal conductivity of snow as a function of temperature. See text for discussion.

With respect to thermal conductivity measurements of snow, several points warrant repeating:

(1) Measurements are temperature-dependent. Values determined at temperatures near 0°C may be as much as 300% higher than had they been obtained at lower temperatures.

(2) Measurements made in natural environments potentially include heat transport by mechanisms like solar radiation or convection, and therefore may be too high.

(3) Grain characteristics and bonding are the fundamental attributes that determine the thermal conductivity. Relationships between thermal conductivity and density exist only because there are relationships (complicated) between microstructure and density.

(4) Vapor transport is the main mechanism from which the temperature dependence of the thermal conductivity of snow arises.

REVIEW OF EXISTING MEASUREMENTS

Table 1 lists 27 published studies in which the thermal conductivity of snow was measured. We refer to this set, excluding the work of Sturm and Johnson (1992), as the "Others" data. Data from Sturm and Johnson (1992) are included with the new data we present below.

Many methods have been used to determine the values listed in Table 1, but they can be roughly divided into three categories: (1) Fourier-type analysis of natural temperature cycles in a snowpack, (2) steady-state heat flow across snow blocks, and (3) transient heat-flow methods, either in situ in a snowpack or on samples. A discussion of the relative merits of different methods of measuring the thermal conductivity of low-conductivity materials has been published by Pratt (1969).

Fourier-type analysis consists of monitoring the temperature at a number of depths in the snowpack and calculating the thermal diffusivity from the attenuation and/or phase shift of the temperature records. Thermal conductivity is computed from the diffusivity using Equation (2). Most studies carried out before 1940 used some variant of this method. Steady-state heat-flow methods consist of placing a tabular block of snow on a heating plate, fixing the energy input to the plate, letting the arrangement come into thermal equilibrium (hence the name) and then measuring the temperature drop across the snow block. The thermal conductivity is calculated by dividing the steady-state heat flow by the steady-state temperature gradient. Three studies (Jansson, 1901; de Quervain, 1956; Pitman and Zuckerman, 1967) have used this method, though only in the most recent study was a guarded type of heated plate used, a method that is now recognized as essential if accurate results are to be achieved. Transient methods consist of heating a point, line or plate source in a block of snow that is initially isothermal and in steady state. The thermal conductivity is calculated from the time response of the temperature of the snow at or near the source. Various configurations of heat source and temperature probes have been used. Devaux (1933) used cylindrical and spherical heat sources. Bracht (1949) used two parallel needles, one a heater and one containing a temperature-measuring device. Jaafar and Picot (1970) used a single needle containing both a heater and a temperature-measuring device. Kuvaeva and others (1975) used a heated plate inserted into a block of snow in which thermocouples had been previously embedded. Östin and Andersson (1991) used a heated strip embedded in frost samples. We have employed a heated needle with a thermocouple in it.

Each method has potential problems. Details of those likely encountered for each study are listed in Table 1. In general, Fourier-type analysis of snow temperatures is prone to the greatest number of problems. Solar radiation, convection and windpumping in natural snowpacks may enhance heat flow, and result in thermal conductivity values that are too high. The instruments used to measure the snow at depth may either conduct heat or absorb solar radiation more readily than the snow, thereby registering temperatures that are unrepresentative. Compaction of the snow during the installation of the temperature probes can bias measurements toward higher values. Lastly, complicated fluctuations in air temperature can produce a series of temperature waves propagating downward into the snow which are incompatible with the simple boundary conditions required for the type of Fourier analysis used in most of the studies listed in Table 1. The best success seemed to be achieved in those studies in which there was a strong diurnal temperature signal and a simple amplitude ratio method could be employed.

Steady-state heat-flow methods have been found to be generally accurate when measuring the thermal conductivity of many materials, but for snow the material begins to change and metamorphose under the imposed temperature gradient. If the test is short enough, reasonably accurate results may be obtained, but if it is too long the thermal conductivity will vary as the test runs. More seriously, because snow is such a good insulator, heat flowing from the warmer end of a sample tends to try to flow around rather than through the snow. To avoid this, a guarded heating plate and good insulation along the sides of the sample are critical. Only one of the tests cited in Table 1 (Pitman and Zuckerman, 1967) employed these precautions. In at least one of the two tests in which an unguarded heater plate was used (de Quervain, 1956), results twice as high as any others at comparable density can probably be explained by heat flowing around rather than through the snow sample.

Transient methods, in our opinion, have the least potential for problems that cause bias or error. The short time of the tests precludes significant snow metamorphism, and the small size of the needle or point heat source minimizes disturbance of the snow. The tests are also short in comparison to the ambient rate of change of temperature in a natural snow cover, so transient temperature effects are minimal and in situ tests can be performed. Because of these advantages, we employed transient methods in our measurements. Errors arise when a sample is changing temperature rapidly or when large contact resistance between the heat source and the material exists.

A total of 354 measurements comprising 23 of the data sets in Table 1 has been plotted in Figure 4. The data set of Sturm and Johnson (1992) has been omitted since it is included with the new data presented below. Niederdorfer's (1933) data, for which he did not measure density, Bracht's (1949) amplitude ratio value which he indicated should be discarded, Shesterikov's (1973) values, which were calculated from the rate of sea-ice formation, an inherently inac-

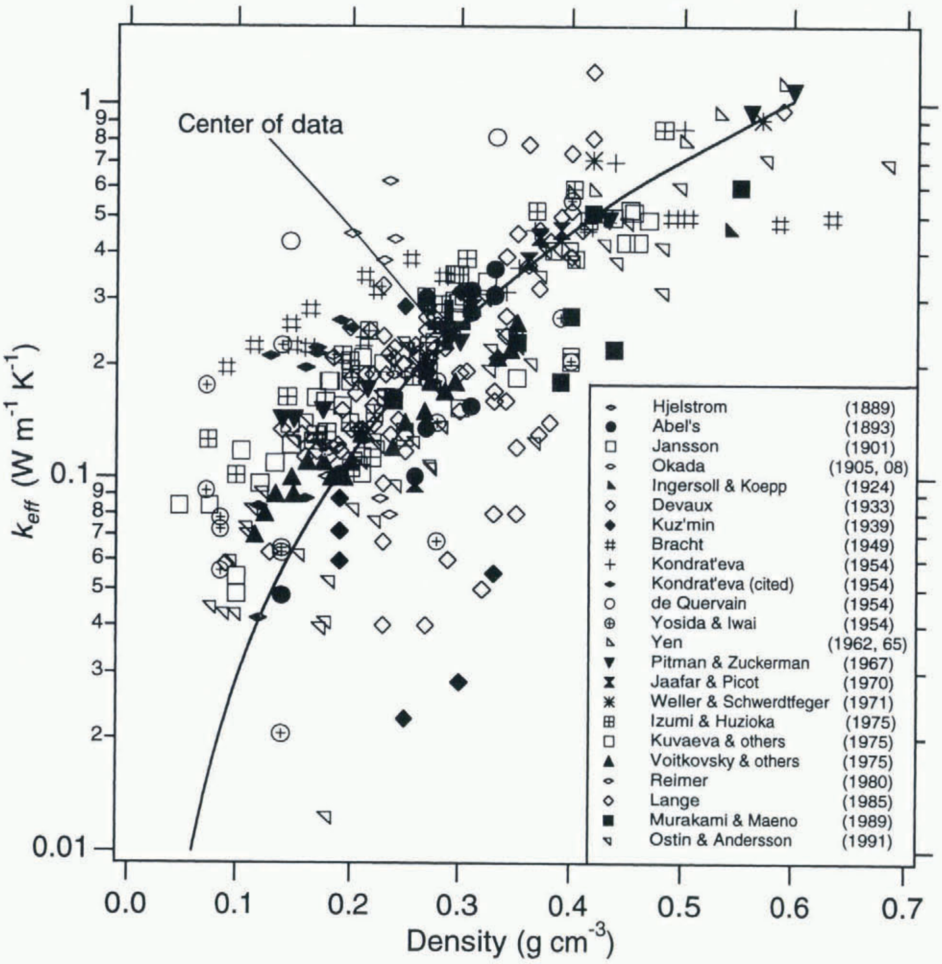

Fig. 4. Thermal conductivity measurements of the "Others". The regression equation of Abel's (1893) (see Table 3), one of the most widely used, is superimposed on the data for reference.

curate method, and Andrews' (1886) single value, which is not explicitly determined, are also omitted. Table 2 lists summary statistics for the remaining data.

The scatter of the data is large. No data have been discarded from the 23 sets used, though it is likely that at least some are incorrect (see Table 1) and contribute to the scatter. However, we estimate that the erroneous data account for only a few per cent of the total number of points. Most of the scatter is the result of natural variation in the conductivity of the snow at a given density, a direct consequence of variations in bonding and microstructure. The scatter is also the result of the fact that the tests were done over a large range of temperatures (Table 3). Measurements were made at temperatures ranging from near 0° to as low as −60°C. For example, Voitkovsky and others (1975) made measurements at an average temperature of −25°C, while most of the authors who used Fourier-type analysis of temperature records made measurements at temperatures higher than −6°C. Unfortunately, the available temperature data (as well as snow-type descriptions) are so vague as to preclude correction to a common or reference temperature, even if theoretical (Fig. 2) or experimental (Fig. 3) data allowed this.

The regression equation of Abel's (1893), the equation most frequently cited and used to predict thermal conductivity, has been superimposed on the data in Figure 4 for comparison. About as many points lie above as below Abel's regression curve, and at any given density there is more than an order-of-magnitude range in thermal conductivity. For later reference, the center of mass of the data cloud ($k_{eff} = 0.251$, $\rho_{snow} = 0.273$) has been marked with a bold cross. For comparison with Figure 4, we have compiled the regression equations (thermal conductivity vs density) developed by a number of authors in Table 3 and plotted them in Figure 5. This style of presentation is similar to figures published by Mellor (1964, 1977), Yen (1969), Anderson (1976), Fukusako (1990), Sturm and Johnson (1992), Arons (1994), Arons and Colbeck (1995) and others in review papers. As with the actual data, quite a spread is encompassed by the ensemble of regression lines, though the regression lines suggest more order in the data than do the data themselves. A line-to-data comparison between Figures 5 and 4 indicates that in many cases the fit of the regression line to the data is poor, and a considerable amount of smoothing has been done through the fitting process.

A NEW SET OF MEASUREMENTS

During the past 12 years we have made measurements of the thermal conductivity of snow using a needle probe and a transient heating/cooling method. Our goal was to collect a large set of data for which we had both accurate conductivity measurements and complete sample descriptions. The set now consists of 488 measurements, plus 72 measure-

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Table 2. Summary statistics for the "Others" data

Date	Author	No. of obs.	Thermal conductivity ($W m^{-1} K^{-1}$) Mean	S.D.	Max.	Min.	Density ($g cm^{-3}$) Mean	S.D.	Max.	Min.
	All	354	0.251	0.197	1.220	0.012	0.273	0.115	0.684	0.047
1889	Hjelström	1	0.212	0.000	0.212	0.212	0.183	0.000	0.183	0.183
1893	Abel's	13	0.219	0.102	0.361	0.048	0.278	0.057	0.330	0.140
1901	Jansson	33	0.268	0.142	0.523	0.084	0.269	0.128	0.470	0.047
1905	Okada 1 and 2	10	0.131	0.039	0.188	0.079	0.222	0.032	0.271	0.179
1924	Ingersoll and Koepp	1	0.460	0.000	0.460	0.460	0.540	0.000	0.540	0.540
1933	Devaux	56	0.202	0.016	0.962	0.059	0.240	0.070	0.590	0.090
1939	Kuz'min	10	0.129	0.122	0.314	0.022	0.269	0.058	0.330	0.190
1949	Bracht	17	0.351	0.117	0.498	0.196	0.299	0.175	0.635	0.090
1954	de Quervain	2	0.622	0.275	0.816	0.427	0.238	0.131	0.331	0.146
1954	Kondrat'eva 1	10	0.182	0.080	0.264	0.042	0.162	0.031	0.200	0.120
1954	Kondrat'eva 2	6	0.501	0.227	0.858	0.310	0.388	0.068	0.500	0.330
1954	Yosida and Iwai	16	0.158	0.132	0.552	0.020	0.211	0.131	0.400	0.072
1962	Yen 1 and 2	5	0.808	0.235	1.130	0.580	0.488	0.079	0.590	0.400
1967	Pitman and Zuckerman	8	0.419	0.386	1.080	0.142	0.322	0.186	0.600	0.140
1970	Jaafar and Picot	5	0.333	0.149	0.460	0.096	0.338	0.053	0.390	0.260
1971	Weller and Schwerdtfeger	2	0.810	0.141	0.910	0.710	0.495	0.106	0.570	0.420
1975	Izumi and Huzioka	24	0.261	0.179	0.856	0.101	0.240	0.092	0.483	0.073
1975	Kuvaeva and others	15	0.241	0.098	0.400	0.100	0.125	0.051	0.211	0.149
1975	Voitkovsky and others	20	0.136	0.052	0.260	0.070	0.222	0.074	0.350	0.120
1980	Reimer	4	0.473	0.104	0.623	0.382	0.226	0.018	0.240	0.200
1985	Lange	31	0.333	0.273	1.220	0.040	0.352	0.045	0.420	0.230
1989	Murakami and Maeno	7	0.310	0.173	0.600	0.160	0.399	0.094	0.550	0.240
1991	Östin and Andersson	58	0.207	0.159	0.715	0.012	0.274	0.127	0.684	0.077

Table 3. Regression equations and the temperatures at which the "Others" data were taken

Date	Author	Temp. range °C	Regression formula
1889	Hjelström	−3 to −11	
1892	Abel's	−10 to −30	$k = 2.846\rho^2$
1901	Jansson	−2 to −13	$k = 0.02093 + 0.7953\rho + 2.512\rho^4$
1905	Okada	−1 to −5	
1908	Okada and others	−2 to −12	
1929	VanDusen	?	$k = 0.021 + 0.42\rho + 2.16\rho^3$
1933	Devaux	−5 to −20	$k = 0.0293 + 2.93\rho^2$
1949	Bracht	−3 to −13.5	$k = 2.051\rho^2$
1954	de Quervain	−1 to −8	
1954	Kondrat'eva	−2 to −13	$k = 3.558\rho^2$
1954	Yosida and Iwai	−1 to −6	
1955	Sulakvelidze	−2 to −13	$k = 0.5107\rho$
1955	Yosida and others	−1 to −6	$\log_{10}(k) = -1.378 + 2\rho$
1962	Yen	−20	
1965	Yen	−6 to −11	$k = 3.223\rho^2$
1967	Pitman and Zuckerman	−5 to −88	
1970	Jaafar and Picot	−4	
1971	Weller and Schwerdtfeger	−17 to −60	
1975	Izumi and Huzioka	?	$\log_{10}(k) = -1.11 + 2.16\rho$; $\log_{10}(k) = -1.20 + 1.70\rho$
1975	Kuvaeva and others	−2 to −13	
1975	Voitkovsky and others	−25	
1980	Reimer	0 to −20	$k = k_a(1-\rho) + \rho k_{ice} \sin(a_m\rho^d)$
1985	Lange	−4 to −20	$\log_{10}(k) = 6.9\rho - 3.0$
1989	Murakami and Maeno	−11	
1991	Östin and Andersson	−6.5 to −19.9	$k = -0.00871 + 0.439\rho + 1.05\rho^2$
1992	Sturm and Johnson	−1 to −77	

$k = k_{eff}$ of snow ($W m^{-1} K^{-1}$); ρ = density ($g cm^{-3}$); k_{ice} = thermal conductivity of ice.

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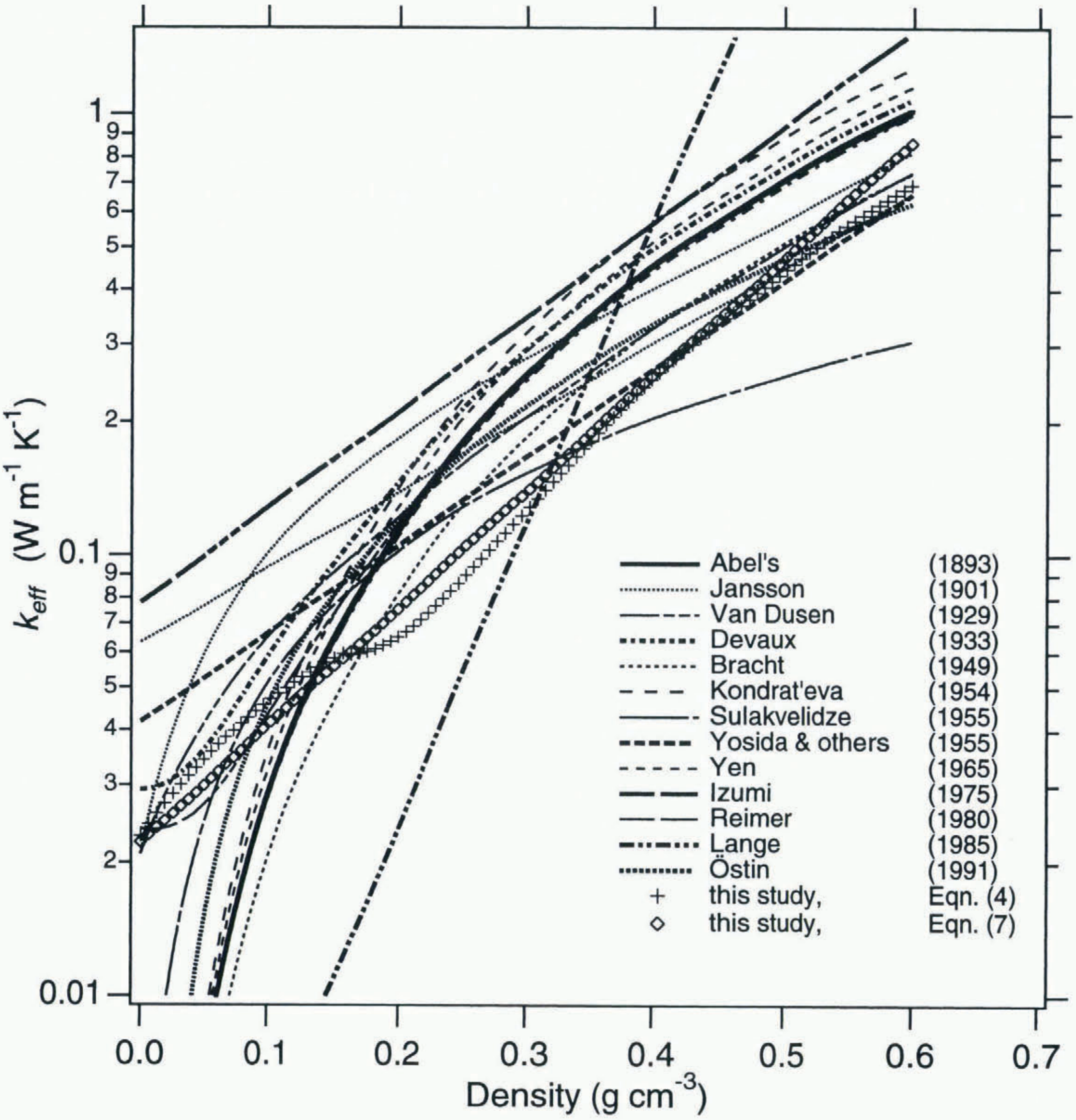

Fig. 5. Published regression equations of thermal conductivity vs density (see Table 3). Regressions developed in this study are shown for comparison.

ments on standards. It encompasses most types of seasonal snow.

Methods

Thermal conductivity was measured using a thin needle probe inserted into the snow. The probe was heated, then allowed to cool while its temperature was monitored. This procedure gave two independent values of thermal conductivity for each test, one from the heating cycle and one from the cooling cycle, which were averaged together. The theory behind the method, and details about its use in snow, can be found in many articles, including those of Blackwell (1954), Buettner (1955), Lachenbruch (1957), de Vries and Peck (1958), Jaeger (1958), Von Herzen and Maxwell (1959), Jaafar and Picot (1970), Moench and Evans (1970), McGaw (1984), Lange (1985) and Sturm and Johnson (1992).

Originally, our tests were done on cylindrical samples placed in an environmental chamber in order to control the sample temperature, as described in Sturm (1991) and Sturm and Johnson (1992). A Custom Scientific Inc. (CSI) needle controlled by a Hewlett Packard data logger and computer was used. The system required 120 VAC power and an ice-water bath as a thermocouple reference junction. In 1992 we rebuilt the apparatus in order to make it field portable and easier to use. A custom needle built by Soiltronics, 1.2 mm in diameter and 200 mm long, containing a helical nicrome heater wire and a Type T thermocouple, replaced the CSI needle. Only the distal 120 mm of the new needle are heated in order to minimize non-axial heat losses during a test. The ferrule of the needle is of only slightly greater diameter than the needle itself, allowing both to be fully embedded in a snow sample, further limiting heat loss from the needle end. Tests are done in situ in the snow-pit wall when there is little or no wind, or, alternately, large (30 cm × 40 cm × 20 cm) boxed snow samples are removed to a cold room and thermally stabilized, then the needle is inserted through the wall of the box to do the test.

The new needle is connected to a Campbell CR-10 data logger controlled by a Zenith 286 laptop computer. A resistance temperature device (RTD) measures the temperature of the thermocouple 0°C reference junction at the data logger and automatically adjusts the thermocouple reading. Software controls the test including activating and shutting off the heater current. A 2 min heating cycle is followed by a 10 min cooling cycle. During both cycles, the needle temperature and heater voltage are recorded every second. Heat input to the needle is adjusted using a variable potentiometer to achieve a 2–5°C rise in needle temperature during the heating cycle. Results (needle temperature vs time) are analyzed using automated statistical software (IGOR Pro on a Macintosh computer). Heating and cooling temperature data are fit independently using least-squares regression, and the slopes from this procedure are used in calculations of the thermal conductivity using standard needle-probe theory (i.e., the thermal conductivity is proportional to the slope of temperature vs log(time)).

Four checks are made to ascertain test quality. First, tests in which the sample temperature is not stable are identified by cross-checking the initial and final needle temperatures. If these differ by more than 0.5°C, the test is discarded. Secondly, if the trace of the needle temperature vs time (for both heating and cooling cycles) is not monotonic and regular, the test is also discarded. Thirdly, the thermal conductivity of the heating cycle is compared to the value computed from the cooling cycle. These must be within 10% for the test to be accepted. Finally, periodically throughout each test set, "standards" (polyurethane foam and glycerol; see Sturm and Johnson, 1992) are tested and compared to known values to ascertain that there is no instrument drift. The standard deviation of all measured values for foam is less than 6% of the mean (0.019 W m^{-1} K^{-1}), and the mean is within 5% of the value determined using a guarded hot plate. For glycerol, the standard deviation is less than 9% of the mean (0.310 W m^{-1} K^{-1}), which is within the range of published values (Touloukian and others, 1970).

Results

Each snow sample was classified by type using the International Classification for Seasonal Snow on the Ground (ICSSG; Colbeck and others, 1990). Type is indicated by a numerical code related to the symbols used in this widely recognized system (Table 4). Since grain character, not bonding, is the primary basis of the ICSSG system, a close relationship between snow type and thermal conductivity was not expected. Based on our experience however, we felt that it might be possible to assign relative degrees of bonding based on the snow type. For example, depth hoar (types 5.1, 5.2) tends to be weakly bonded, and slab and drift snow (9.1–9.4) tends to be well bonded. Refrozen melt-grain clusters (6.2) can have highly variable bonding, even in a small sample.

Our data ($n = 488$) are plotted as a function of density in Figure 6 and summarized in Table 5. The data form a widely scattered field similar to the "Others" data in Figure 4. Separate symbols have been used for each type of snow, and a limited organization by snow type is evident, with wind slabs and drift snow, predictably, having higher densities and higher thermal conductivities than depth hoar, new snow or recent snow. The variance of k_{eff} increases with density. The center of mass of the data field has a lower thermal conductivity (0.178 vs 0.259 W m^{-1} K^{-1}) and a higher density (0.317 vs 0.272 g cm^{-3}) than the "Others" data (Fig. 4).

Analysis

The new data can be fit with a quadratic equation:

$$k_{\mathrm{eff}} = 0.138 - 1.01\rho + 3.233\rho^2 \qquad \{0.156 \le \rho \le 0.6\}$$
$$k_{\mathrm{eff}} = 0.023 + 0.234\rho \qquad \{\rho < 0.156\} \qquad (4)$$

where ρ is in g cm^{-3}, and k_{eff} is in W m^{-1} K^{-1}. The fit has an R^2 equal to 0.79 and is shown in Figure 6 with 95% confidence limits. The limits imply that (a) predicted values of k_{eff} will have an uncertainty of 0.1 W m^{-1} K^{-1}, a large amount, and (b) about 5% of all measured values will lie outside this predicted range.

When Equation (4) is extrapolated to the density of ice (0.917 g cm^{-3}), it has the attractive property of closely predicting the correct value (≈2.2 W m^{-1} K^{-1}), though this was not a constraint of the least-squares fitting routine. This suggests that Equation (4) might reasonably be extrapolated beyond the limits of our data to denser snow and firn without incurring a large error. At the same time, the equation predicts a minimum value of k_{eff} at a density of 0.156 g cm^{-3}, with higher values at densities lower than 0.156 (dashed line in Figure 6). Many highly insulative granular materials show exactly this type of increase in conductivity at densities less than 0.05 g cm^{-3} due to convection and radiation (Pratt, 1969). However, the increase in the regression is solely the result of the least-squares fitting procedure, not thermal physics. We have no experimental data that show

Table 4. Snow-type codes and descriptions

Symbol	Code No.	Description	ICSSG Code*
	9.4	very hard wind slab	9d and 3a
	9.3	hard drift snow	9d and 3a
	9.2	hard wind slab	9d and 3a
	9.1	soft to moderate wind slab	9d and 3a
	6.2	refrozen wet poly-grains, wet clusters	6b
	6.1	clustered rounded grains	6a
	5.2	indurated depth hoar	5a, 5b, 5c
	5.1	weak chains-of-grains depth hoar	5b
	3.3	mixed forms	3c, 4c
	3.2	large rounded grains	3b
	3.1	small rounded grains	3a
	2	recent snow	2a, 2b
++++	1	new snow	1a to 1e

* The International Classification for Seasonal Snow on the Ground (Colbeck and others, 1990)

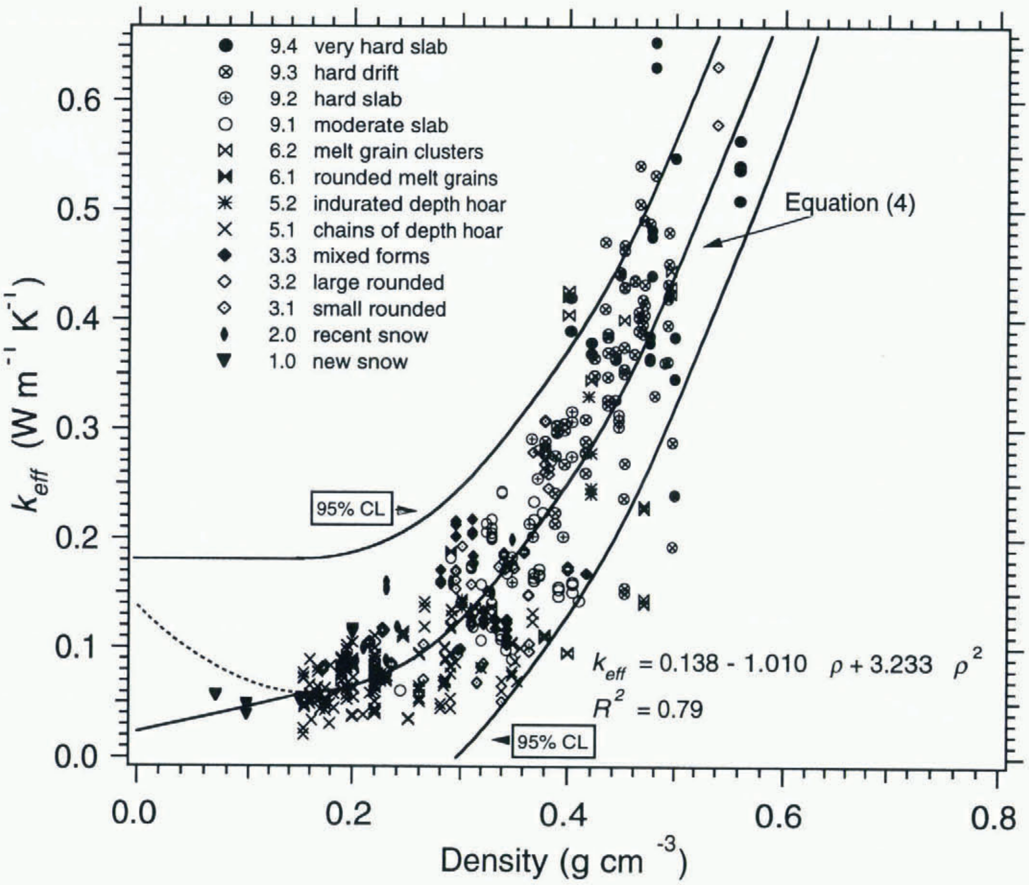

Fig. 6. All of our thermal conductivity data as a function of density and snow type (see Table 4). Equation (4) has been superimposed on the data. Below a density of 0.156 g cm^{-3}, a linear regression with y intercept of 0.023 W m^{-1} K^{-1} (k_{air}) is used (solid line); the dashed line shows the continuation of the quadratic equation. 95% confidence intervals are also shown.

increased thermal conductivities at low density in snow, so for densities less than 0.156 we use a linear extrapolation to the thermal conductivity of dry air (k_{air} = 0.023 at −15°C) at zero density. If this piecewise fitting approach is not used, the error in estimated value at zero density would be 0.115 W m^{-1} K^{-1}, still within the 95% confidence limits. For comparison, Equation (4) is shown on Figure 5. It lies close to the regressions of Bracht (1949) and of Yosida and others (1955), differing at the lower densities where these authors did not force their curves to converge to k_{air}.

An alternate curve fit is suggested by the distribution function for both our conductivity data and the data of the "Others". Both functions are negatively skewed, with more data points at lower values of k_{eff}. This fact, combined with the fact that for both data sets the variance in k_{eff} increases with density, suggests using the log transform of k_{eff}

Table 5. Summary statistics; data from this study

		Thermal conductivity (W m^{-1} K^{-1})				*Density (g cm^{-3})*				*Temperature (°C)*			
Snow type	*No. of obs.*	*Mean*	*S.D.*	*Max.*	*Min.*	*Mean*	*S.D.*	*Max.*	*Min.*	*Mean*	*S.D.*	*Max.*	*Min.*
All	488	0.178	0.134	0.654	0.021	0.317	0.106	0.560	0.070	−14.6	7.9	−1.0	−77.1
9.4	22	0.452	0.104	0.654	0.240	0.488	0.050	0.560	0.402	−21.2	9.8	−6.5	−34.4
9.3	77	0.359	0.084	0.541	0.150	0.444	0.034	0.498	0.380	−18.4	8.9	−7.2	−34.4
9.2	16	0.237	0.066	0.316	0.141	0.379	0.045	0.445	0.300	−17.8	5.7	−10.2	−24.9
9.1	50	0.167	0.051	0.281	0.061	0.348	0.033	0.410	0.243	−12.9	4.7	−1.7	−25.1
6.2	20	0.250	0.141	0.445	0.095	0.422	0.058	0.496	0.314	−10.8	4.4	−3.1	−18.5
6.1	1	0.188	0.000	0.188	0.188	0.290	0.000	0.290	0.290	−25.1	0.0	−25.1	−25.1
5.2	9	0.183	0.095	0.330	0.062	0.345	0.073	0.420	0.260	−12.1	4.3	−5.3	−18.5
5.1	171	0.072	0.025	0.142	0.021	0.225	0.055	0.369	0.154	−14.4	8.9	−1.0	−77.1
3.3	26	0.153	0.040	0.218	0.099	0.321	0.029	0.416	0.280	−12.1	2.4	−4.1	−15.2
3.2	9	0.163	0.084	0.278	0.083	0.345	0.036	0.380	0.297	−9.1	5.2	−2.2	−13.7
3.1	51	0.169	0.111	0.632	0.051	0.320	0.078	0.540	0.173	−12.9	6.5	−2.3	−25.3
2	21	0.128	0.050	0.200	0.056	0.254	0.068	0.350	0.169	−13.5	2.8	−10.6	−19.3
1	11	0.070	0.030	0.117	0.039	0.135	0.051	0.200	0.070	−11.3	4.4	−5.5	−19.6
Missing	4	0.112	0.012	0.124	0.096	0.306	0.021	0.337	0.295	−12.4	0.2	−12.2	−12.6

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($\log_{10}(k_{\mathrm{eff}})$). Doing so allows for some statistically desirable improvements in the curve fitting. Under log transformation, (1) the data become homoscedastic (the variance is constant over the range of density), (2) the residuals become normally distributed about the fit, (3) a linear and continuous (vs piecewise) fit can be employed, and (4) extrapolation of the fit to zero density yields a value of $k_{\mathrm{eff}} = 0.02\ \mathrm{W\,m^{-1}\,K^{-1}}$, close to the thermal conductivity of dry still air (Fig. 7). For our data the regression equation is:

$$\log_{10}(k_{\mathrm{eff}}) = 2.650\rho - 1.704 \qquad \{\rho \leq 0.6\} \qquad (5)$$

with $R^2 = 0.76$. For the "Others" data, the log regression equation is $\log_{10}(k_{\mathrm{eff}}) = 2.037\rho - 1.272$ ($R^2 = 0.52$). This regression predicts higher values of k_{eff} than Equation (5) for a given density, and has a slope that is significantly lower. The extrapolation of the log fit to the "Others" data to zero

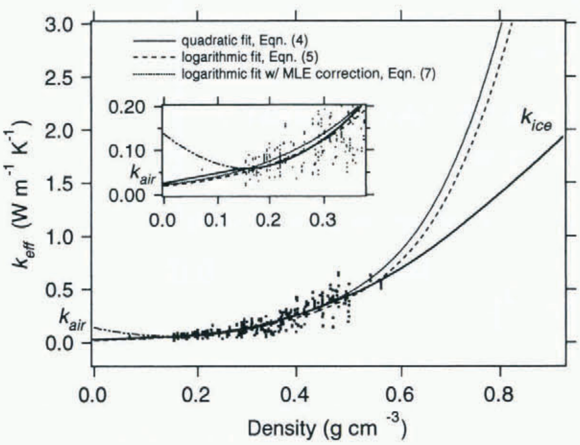

Fig. 7. A comparison of regression Equations (4), (5) and (7). Equation (4) is best if extrapolation beyond the maximum density of the data is necessary, as it predicts the correct value of k_{ice} at 0.917 g cm^{-3}, but it is not as good as Equation (5) or (7) for low-density snow. Equations (5) and (7) give values very close to k_{air} at zero density, but would predict estimates with large errors for densities above 0.6 g cm^{-3}. All three regressions give reasonable estimates for the range of densities encountered in most seasonal snow.

density gives a value of $k_{\mathrm{air}} = 0.053\ \mathrm{W\,m^{-1}\,K^{-1}}$, higher than the true value (Fig. 8).

Most applications require values of k_{eff}, not $\log(k_{\mathrm{eff}})$, necessitating a back transformation of the results of Equation (5). This back transformation is biased (see Thompson, 1992, p. 208–214). The bias is apparent if the mean density for our data set (0.317 g cm^{-3}; see Table 5) is used in Equation (5). The resulting value of $\log_{10}(k_{\mathrm{eff}})$ is −0.864, which when back-transformed equals 0.137 W m^{-1} K^{-1}, not 0.178 W m^{-1} K^{-1}, the mean for the data set. Statistically, the regression given by Equation (5) should pass through the mean value of k_{eff}. Several methods can be used to correct for the bias. We use the maximum likelihood estimator (MLE) as detailed by Helsel and Hirsch (1992). Using this method we get:

$$k_{\mathrm{MLE}} = 10^{[(2.650\rho - 1.704) + (V_{\log}/2)]} \qquad \{\rho \leq 0.6\} \qquad (6)$$

where k_{MLE} is the value of k_{eff} corrected by the MLE method, ρ is the snow density, and $V_{\log}$ is the variance of the log

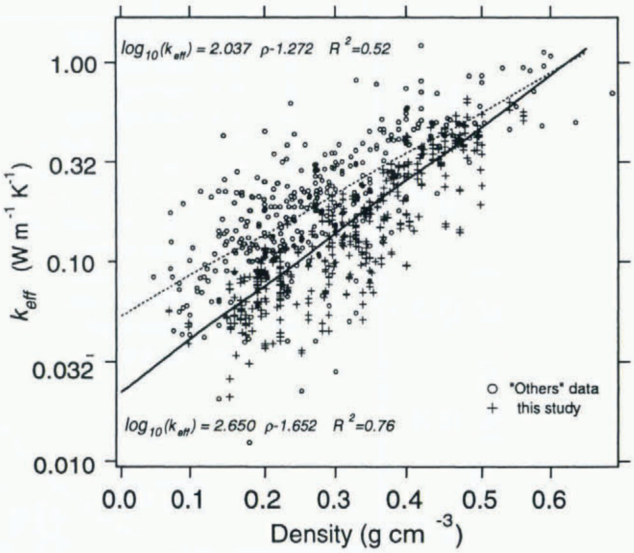

Fig. 8. A comparison of all our data (crosses) with the "Others" data (circles). The log regressions to each data set are shown. The "Others" regression predicts higher values of k_{eff} at a given density and does not predict k_{air} correctly at zero density, while our regression does.

values of k_{eff} in the data set. The small correction factor, $V_{\log}$, equals 0.1025, so Equation (6) simplifies to:

$$k_{\mathrm{MLE}} = 10^{(2.650\rho - 1.652)} \qquad \{\rho \leq 0.6\}. \qquad (7)$$

As can be seen in Figure 7, there is little practical difference between Equations (5) and (7), but, statistically, Equation (7) is a better approximation.

Unfortunately, both Equations (5) and (7) have the undesirable property of diverging rapidly beyond the upper limit of our data ($\rho > 0.6$ g cm^{-3}) (Fig. 7). Unlike the quadratic curve (Equation (4)), the values predicted by Equations (5) and (7) at the density of ice are more than twice the measured value. For the normal range of density encountered in seasonal snow, Equation (4), (5) or (7) is adequate. Equation (4) is best if estimates are needed for snow and firn with densities beyond the limits of our data. Equation (7) is best if estimates are needed for low-density snow.

We have examined the thermal conductivity as a function of snow type for those types for which we have 20 or more measured values (Table 5). Two different characteristic patterns appear. For depth hoar (type 5.1), indurated depth hoar (type 5.2) and mixed forms (partially faceted rounded grains; type 3.3), there is little relationship between the density and the thermal conductivity. This can be seen in Figure 9, where a line has been fit to each data subset. For depth hoar and mixed forms, the lines intersect the quadratic fit (Equation (4)) at sharp angles, showing that k_{eff} is essentially independent of density. Depth hoar and mixed forms are the result of kinetic growth processes (Colbeck, 1983, 1987) and imply a high rate of water-vapor transport. Bonds tend to be weak. For these types of snow, choosing the mean value of k_{eff} for a snow type results in as good an estimate of the thermal conductivity as using a regression equation based on density.

For recent snow (type 2.0), small rounded-grain snow (type 3.1) and wind-slabs and drifts (types 9.1, 9.2, 9.3, 9.4), lines fit to data subsets lie parallel to the quadratic fit (Fig. 9), showing density dependence. For types 3.1, 9.1, 9.2, 9.3 and

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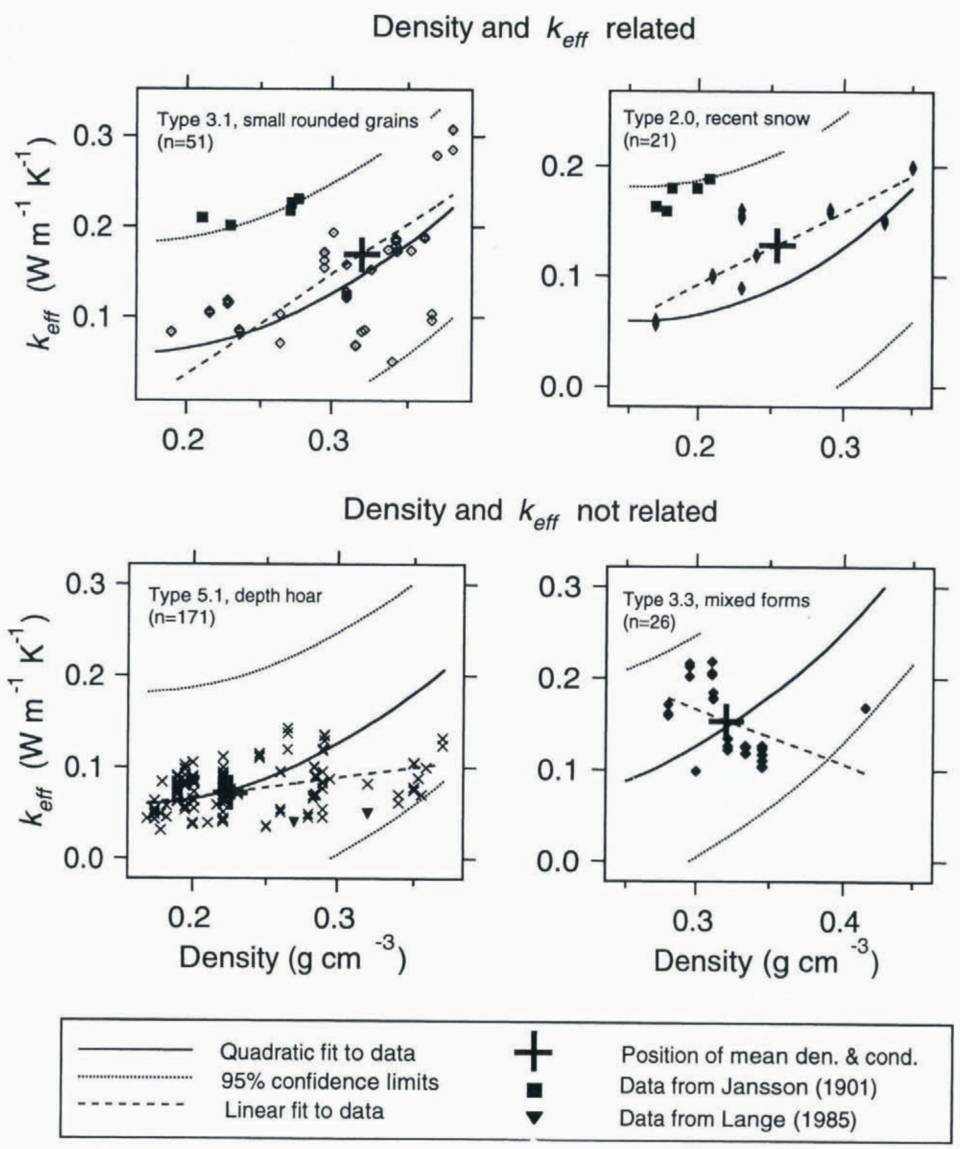

Fig. 9. Data clusters by snow type showing (a) types in which density and conductivity are related, and (b) types in which they are not. See Figure 6 for symbols.

9.4, snow grains tend to be equant and rounded, with limited variation in shape but a high degree of bonding. For these snow types, estimated values of thermal conductivity are best obtained using regression Equation (4), (5) or (7).

For a limited number of measurements in the "Others" data set we can identify the snow type. The results of Jansson (1901) and Lange (1985) contain sufficient descriptions or stratigraphic diagrams to allow us to assign snow types to each value of k_{eff}. These data have been plotted along with our data in Figure 9. In some cases (snow types 3.1 and 2.0) there seems to be an offset between the different sets, perhaps the result of tests done at different temperatures. Despite the offset, the trends indicated by the "Others" data are consistent with those indicated by our data and discussed above.

The relationship between density, k_{eff}, and the degree of bonding is further illustrated in Figure 10. Here, only data for wind-blown snow have been used. The four types (9.1, 9.2, 9.3 and 9.4) were differentiated by differences in their hardness using the standard technique of resistance to penetration by fist, finger, pencil or knife. This test, while qualitative, is unambiguous. For each type there is a cluster of

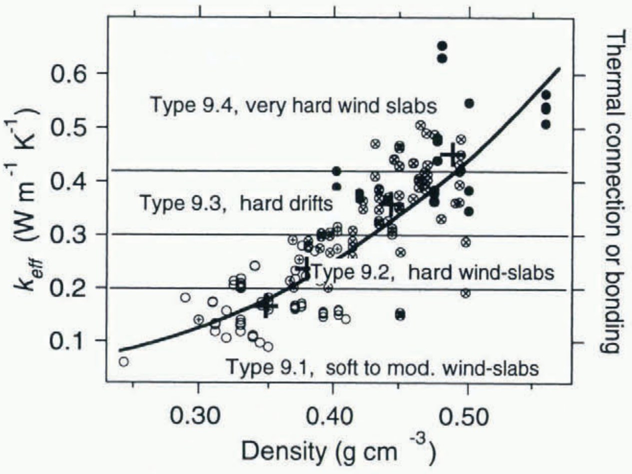

Fig. 10. Data clusters for wind-blown snow. Heavy crosses show the center of each cluster (mean density and mean thermal conductivity). Increasing thermal conductivity and increasing hardness suggest that the vertical axis could be replaced by a relative scale of bonding or thermal "connectedness".

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data in Figure 10. The clusters are distinct and separate. To highlight this, the mean value of density and conductivity for each cluster is indicated by a bold cross, and these crosses define nearly a straight line of increasing density and thermal conductivity. Linear regressions fit to each data cluster are nearly parallel and reasonably consistent with the quadratic fit to all the data. An alternate vertical axis to thermal conductivity is hardness, a measure of the bonding. In that way the vertical axis in Figure 10 can be thought of as the degree of bonding or the degree of thermal "connectedness". The linear trend indicated by the mean values suggests that for wind-blown snow, density, thermal conductivity and bonding (or its proxy) all are linearly related.

DISCUSSION

There is a distinct offset between our data and the data of the "Others" (Fig. 8), an offset that appears to be the result of differences in sampling and the average temperature at which the tests were done. The center of our data cloud is at a higher density but a lower conductivity (Tables 2 and 5).

Our set samples the full range of natural seasonal snow, but over-represents two snow types frequently encountered in high latitudes: depth hoar and wind-slab. This is a result of the fact that most of our research is done in the Arctic or Antarctic. Our set contains 180 measurements on depth hoar (types 5.1 and 5.2) and 165 measurements on wind-blown snow (types 9.1, 9.2, 9.3 and 9.4). Based on the locations where the "Others" took their data (Table 1), we infer that they sampled less wind-blown snow. With the exception of Jansson (1901), who collected wind-blown snow from near Uppsala University in Sweden on one occasion, and a couple of measurements by Lange (1985) from the Antarctic, this type of snow appears to be unrepresented in the "Others" set. On the other hand, depth hoar seems to be well represented, having been sampled by Abel's (1893), Izumi and Huzioka (1975), Lange (1985) and possibly one or two others. As wind-blown snow tends to be denser than most other types of snow (Fig. 10), we conclude that our sample set is biased toward this dense material when compared to the "Others". We think that our set is close to parity with the "Others" for depth hoar. Combined, these facts suggest why the mean density of our data set is higher than the "Others".

The difference in mean thermal conductivity between the two sets can be accounted for largely by differences in the temperature at which the tests were done. Table 5 lists the temperature at which our tests were done. The mean value is −14.6°C. Comparable data is unavailable for the "Others" tests, though the general range for some of the investigators is given in Table 3. Calculating a mean value of temperature from these ranges is difficult, particularly for those investigators who used Fourier-type analysis. In those cases, the range given is the ambient range of air temperature, not the snow temperature itself. Our best estimate is that the "Others" data were taken at an average of about −5°C. The difference (−5° vs −14.6°C) between the two sets, almost 10°C, could account for a drop of anywhere between 0 and −0.06 $\text{W m}^{-1}\,\text{K}^{-1}$ in thermal conductivity, based on the results given in Figure 3. The observed shift between the mean of our data and the "Others" (0.251 to 0.178 $\text{W m}^{-1}\text{K}^{-1}$) is −0.073 $\text{W m}^{-1}\,\text{K}^{-1}$, slightly greater than, but of similar magnitude and sign to, the amount that could be accounted for by differences in temperature. We conclude that this is the major cause of the shift in k_{eff} values.

Which data set to use? In general, both sets contain good values of k_{eff} for seasonal snow at temperatures and densities routinely encountered in practical problems. Our set, and the regressions based on it, offers six advantages over individual sets or composites of the older data:

(1) The new set was made using a needle probe and a dynamic test method that is known to be more accurate for low-conductivity materials than other methods. The test is short, a particular advantage for a material that can metamorphose.

(2) Ours is a single data set, with all data collected in a consistent manner.

(3) It is one of the few sets for which measurements on standards were made, allowing assessment of systematic errors and accuracy.

(4) Measurements were made at known temperatures, which did not vary greatly. They were also relatively low, so that they fell in the flatter part of the temperature curves shown in Figures 2 and 3, minimizing sensitivity to temperature dependence.

(5) Ours is practically the only set for which the type of snow is described.

(6) It contains more data than the combined set of the "Others".

Based on these reasons, we recommend using the regression equations for our data.

It is unfortunate that the two data sets cannot be combined. Initially we had hoped we could temperature-correct the "Others" data to −14.6°C in order to create a combined set, but the potentially large errors that would have been incurred in making the correction forced us to drop the idea. In addition, differences in test methods, lack of information about the accuracy of many of the older sets and lack of description of the snow for most tests made combining the two sets statistically inadvisable, and would have resulted in more confusing and less reliable regressions.

The gross differences between the two sets of data highlight the pressing need for a better method of accounting for the temperature dependence of k_{eff}. For example, our regressions for estimating k_{eff} are strictly valid only for temperatures near −14.6°C. If estimates are to be applied to snow at different temperatures, particularly higher temperatures, corrections must be applied. The best data available to do this are shown in Figure 3, but, as stated before, they are confusing and contradictory. The data suggest that the correction will vary depending on the temperature and the microstructure of the snow. Pending more comprehensive studies, we make the following recommendations based upon our interpretation of the data:

(1) Use the curves of Pitman and Zuckerman (1967) with care since they were measured on artificial snow and had to be composited from multiple samples. The pronounced temperature dependence indicated by these curves below −27°C does not seem realistic to us and indicates that the form of the curves may result, to a large extent, from the way the data were compiled.

(2) Similarly, the data from Voitkovsky and others (1975) show a very strong temperature dependence that cannot be verified.

(3) Use different curves for different types of snow. For depth hoar (and perhaps other low-density, loosely bonded types of snow like recent (type 2.0) and new (type 1.0)), use the curves from Sturm and Johnson (1992). These indicate that k_{eff} nearly doubles between −40° and 0°C for low-conductivity depth hoar. An empirical fit to the data gives:

$$k_{\mathrm{eff}} = k_{\mathrm{dry}} + \frac{51.8}{\left[(T - 27.8)^2 + 211.2\right]} \qquad \{-40^\circ \leq T \leq 0^\circ\mathrm{C}\} \qquad (8)$$

where k_{dry}, the thermal conductivity with no vapor transport, equals about 0.06 W m^{-1} K^{-1}, and T is the temperature in °C. For dense, well-bonded snow, results in Figure 3 suggest there will be little or no temperature dependence above −40°C. Below −40°C, the conductivity will increase due to the increasing conductivity of ice. However, if the dense snow is poorly bonded, data from Arons (1994) suggest a temperature correction of the form

$$k_{\mathrm{eff}} = k_{\mathrm{dry}} \qquad \{T \leq -30^\circ\mathrm{C}\}$$
$$k_{\mathrm{eff}} = k_{\mathrm{dry}} + 0.004(T + 30) \qquad \{-30 \leq T \leq -1^\circ\mathrm{C}\}$$

might be used.

We stress that all of these corrections are provisional. They highlight the need for more controlled tests on the effect of temperature on k_{eff}.

What can the data tell us about the microstructure of snow? A number of models of snow, with varying degrees of realistic geometry, have been proposed (see the review by Arons and Colbeck, 1995). Two of the simplest models consist of plates of ice arranged either parallel or perpendicular to the heat-flow direction (de Quervain, 1973; Combarnous and Bories, 1975; Auracher, 1978; Fig. 11, insets). Conceptually, the models represent limiting cases for both microstructure and thermal conduction in snow. In the parallel model, the ice is arranged in optimal pathways for heat conduction. In the series model, the opposite is true, and no continuous ice pathways exist; all heat must be conducted across the air-filled pores as well as the ice plates. For both models, the conductivity (latent-heat transport is ignored) can be calculated directly from the geometry. The calculated values provide limits on what is possible for more realistic geometry and snow.

Curves for both the parallel and series models, along with our data and the "Others" data for k_{eff}, are plotted in Figure 11. The combined data lie between the parallel and series limits, as they must, but significantly closer to the series limit. The samples range in density from about 0.05 to 0.60 g cm^{-3}, generally acknowledged to be the range found in seasonal snow that has not been saturated and refrozen (see Anderson and Benson, 1963).

We have drawn a trapezium about the main body of data. In drawing the upper limit of this trapezium, we have discounted about 16 data from the "Others" set as being too high and likely to be questionable. Otherwise, all data (ours

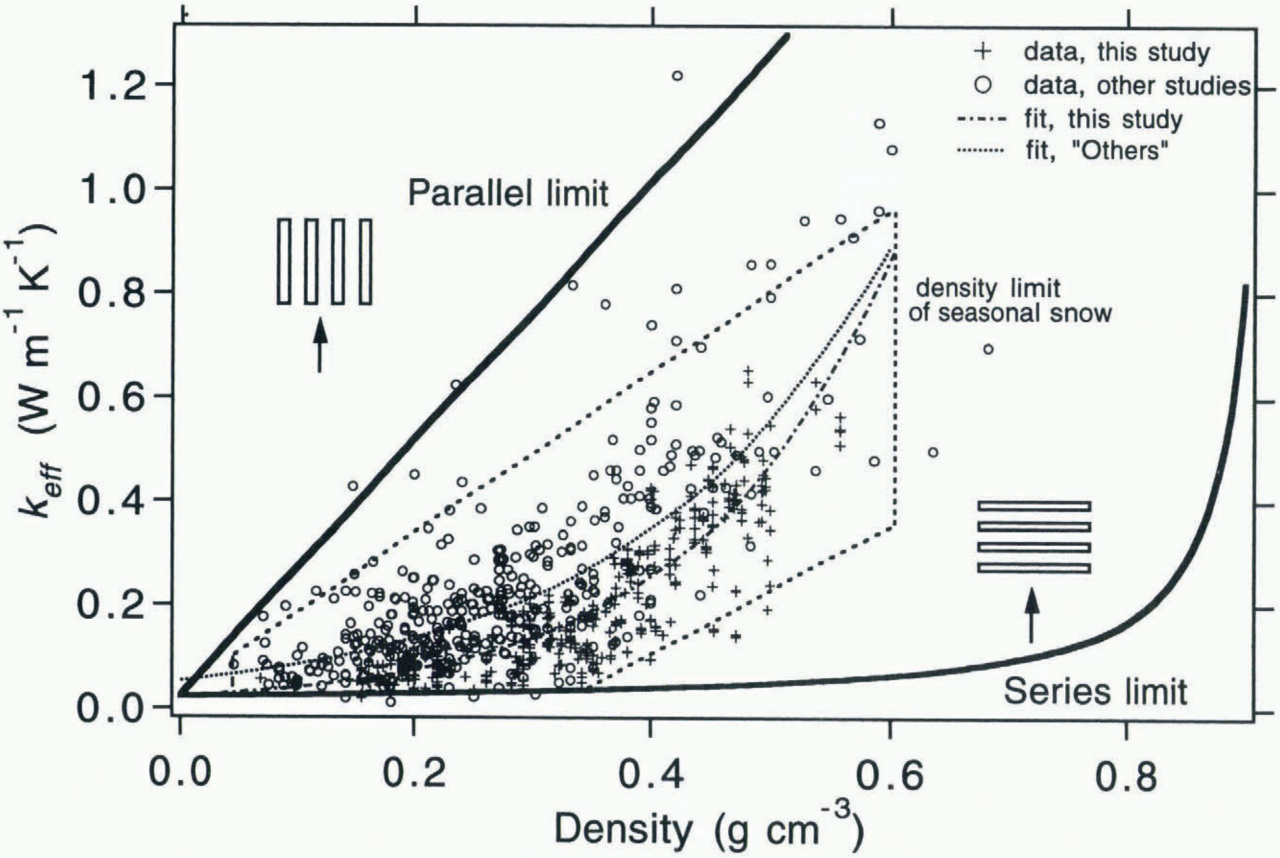

Fig. 11. All data (ours (crosses) and the "Others" (circles)), along with curves showing the theoretical thermal conductivity of parallel or series ice plates. The approximate limit in the density–thermal-conductivity space occupied by natural seasonal snow is shown to be a small subset of the possible domain.

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and the "Others") are included. It is surprising that the resulting enclosed area occupies only about half the area of density–conductivity space that it could. Large gaps exist along both limits of the data, with the data nowhere approaching the parallel limit. In contrast, between 0 and approximately 0.35 g cm^{-3}, many data (from both sets) actually lie along the series limit. At about 0.35 g cm^{-3}, the lower limit of the data cloud rises away from the series limit in a fairly regular fashion, on a line that is nearly parallel to the upper limit of the trapezium. Akitaya (1974) found that the upper limit of density for depth hoar was approximately the same: 0.35 g cm^{-3}.

The results displayed in Figure 11 suggest that there are constraints on the bonding–density configurations that can exist in natural seasonal snow. As suggested by Figure 10, there exists a strong relationship between thermal conductivity and the bonding or ice "connections" in a snow sample. If we replace the vertical axis in Figure 11 with the degree of bonding, it becomes clear that natural limits exist. Certain ranges of density exist only with certain degrees of bonding. We see the information as useful in conjunction with (or for the testing of) microstructural models. Combined with a model, Figure 11 could be used to determine what grain, bond and density relationships might exist in seasonal snow.

Lastly, we want to mention trajectories in density–conductivity space. These are related to metamorphic trajectories for snow (see Colbeck, 1987). Under equilibrium growth conditions, particularly with overburden pressure, snow tends to densify, and, as it does, its thermal conductivity increases. Under those conditions, Equations (4) and (7) can be used to describe the evolution or trajectory of k_{eff} if the density evolution is known. However, if kinetic growth processes occur, then density and conductivity are not coupled, as shown by Figure 9. In this case, the trajectory of thermal conductivity may be quite complicated. Sturm and Johnson (1992) show several trajectories for depth hoar (types 5.1 and 5.2). Similar information for new (type 1.0) and recent snow (type 2.0), for mixed-form snow (type 3.3) and perhaps other types, is not available. These trajectories, however, are unlikely to follow Equation (4) or (7).

CONCLUSIONS

The 27 studies (comprising 354 values of k_{eff}) published since 1886 have been used to derive over 13 different regression equations predicting k_{eff} vs density. Large (and largely undocumented) differences in accuracy, temperature and snow type exist in this data set. Consequently, it is difficult to use the combined set to investigate how thermal conductivity varies with density or snow type. We introduce a new set that consists of 488 measurements (plus 72 measurements on standards). The accuracy of this new set is thought to be better than $\pm 10\%$, and the type of snow was known for each measurement. The new set can be fit with a quadratic or a logarithmic equation. The quadratic is better when estimating values beyond the limits of the data set; the logarithmic is better when estimating values for low density.

The data suggest that two distinctly different relationships between k_{eff} and density can be found. Snow composed of depth hoar or faceted grains (i.e. the result of kinetic growth) shows little connection between density and thermal conductivity. In contrast, snow composed of rounded grains or wind-blown fragments shows a strong density dependence in the thermal conductivity, and the evolution of this type of snow with time (assuming densification) will follow the regression equations we have developed. For density-independent snow types, an accurate estimate of k_{eff} can be made by simply using a mean value for that type of snow. For density-dependent snow types, better estimates result from using the regressions.

Comparison of the new data set with the combined older data set shows that the new set has a mean value of density that is higher than the older set, but a lower mean value of thermal conductivity. This shift can be attributed to differences in the type of snow sampled in each set, plus differences in the temperatures at which the measurements were made.

The combined data sets can be used to delineate the field of density–thermal conductivity that is taken by natural seasonal snow covers (Fig. 11). Since thermal conductivity can be shown to be closely related to the degree of bonding, this is the same as the density–bonding field. The results indicate that natural seasonal snow occurs within much narrower limits of density and bonding than are theoretically possible. The limits delineated can be used to guide and test models of snow microstructure.

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