NUTATIONS AND INELASTICITY OF THE EARTH

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ABSTRACT. The adopted nutation series correspond to an elliptical uniformly rotating Earth with an elastic inner core, a liquid core and an elastic mantle. There exist nowadays a difference between the theoretical results and this theory. In this paper, we introduce the mantle inelasticity in the equations in order to give an idea of its contribution to the nutations.

INTRODUCTION

Due to the lunisolar attraction, there is a forced motion of the mean Earth rotation axis with respect to the inertial space and with respect to the Earth. In the first reference frame, they are called precession and nutations and in a reference frame uniformly rotating, they appear in the form of diurnal wobbles. Nowadays, the most complete, published nutation series are computed by Wahr (1981b) for an elliptical, rotating Earth with an elastic inner core, a liquid outer core and an elastic mantle.

The analysis of Very Long Baseline Interferometry observations (Herring et al., 1986) has yielded estimates of important corrections to the seven largest nutation series coefficients in the Wahr's tables. The nutation series computation need assumptions about the Earth model. A part of these corrections may be explained by taking a more complete Earth model into account; for example, Herring et al. suggest to take a more complete core-mantle coupling in the model. In this paper, we will examine the effect of mantle inelasticity on the nutations. Recently, Wahr and Bergen (1987) have analysed this effect in terms of a first order perturbation theory and by constructing partial derivatives with respect to the Earth rheological parameters. They conclude that mantle inelasticity cannot resolve the discrepancy between the observations and the theory but that the effects are potentially observable. This is a good reason to develop the equations

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using complex arithmetic and integrating directly the new equations (which are briefly presented in the first paragraph).

Resonance effects are taken into account in Wahr's computations by using normal mode expansion (Wahr, 1981a). The problem with this theory is that the normal mode expansion is no more valid in the complex variable space (Wahr & Bergen, 1987). We will analyse this situation in paragraph 2. Finally, we will give new numerical values for some of the nutations (paragraph 3).

1. INTEGRATION OF THE DEFORMATION EQUATIONS FOR AN ELLIPTICAL, ROTATING EARTH WITH AN INELASTIC MANTLE.

By using the correspondence principle of Biot (1954), the stress-strain relationship may be expressed in the frequency domain, in the same form as in the elastic case (i.e. Hooke law), but with complex and frequency dependent shear and bulk moduli. The partial differential equations become then complex as in the spherical case (Zschau, 1979). In order to find scalar equations, we separate real and imaginary parts and as in the elastic case (Smith, 1974), we expand the scalar, the vectorial and tensorial components using the Generalized Spherical Harmonic functions (GSH) introduced by Phinney and Burridge, in 1973. We obtain then twice the number of equations comparing to Smith (1974) and Wahr (1979) theory. The equations are presented in a previous paper (Dehant, 1987a) and in our thesis (Dehant, 1986).

As explained by Wahr (1979 and 1982), the displacement field for an elliptical uniformly rotating Earth contains the effects of the nutations and of the variation of the rotation rate. The nutation effect is in that frame, a toroidal displacement of the order ℓ=1, m=1; the variation of the rotation rate effect is a toroidal displacement of the order ℓ=1, m=0. They are obtained when one computes the Earth response respectively to an external potential of the order (ℓ=2, m=1) and (ℓ=2, m=0). In the case of an inelastic mantle Earth, these toroidal displacements are complex and one can deduce "complex nutations".

In order to integrate numerically those equations, one needs complex profiles of the rheological properties. There are different models in the literature from which one can compute a shear modulus profile. Two models are based on strain retardation time distribution. They give either a quasi constant quality factor Q (Liu et al., 1976) or a frequency to the power α Q model (Anderson & Minster, 1979). Those models assume an Earth rheology comparable to standard linear solids in series. Zschau (Zschau & Wang, 1985) considers a stress relaxation time distribution in order to construct his model and assumes that the Earth is comparable to an infinite set of Maxwell bodies in parallel. This distribution is a cut gaussian distribution (Zschau, 1985). The parameters of this distribution (the mean, the standard deviation and the cut-off period) are computed by using a very large set of data. Details on this model are given in Dehant (1986 and 1987a). As explained in this last paper, we believe that Zschau's model is the most appropriate for the Earth's mantle rheology.

Concerning the bulk modulus profile, there is no need for a model because the effects are to small (see Dehant 1987a). The values published by Dziewonski (PREM, 1981) can than be directly used.

Introducing these rheological profiles and integrating the new equations, one gets the Earth's surface displacements, the response to a specific external tidal potential component. From these results, one can compute inelastic tidal parameters (Dehant 1987b and Dehant & Ducarme, 1987) and new nutations.

2. NORMAL MODE EXPANSION

2.1. Elastic Earth's case - Review

The free oscillation equation of motion is deduced from the forced deformation equation by suppressing the external tidal force. Resonance effects in the forced deformations make it necessary to compute the Earth normal modes. Wahr (1979, 1981a and 1982) has developped a theory capable of accounting for these effects by using a normal mode expansion. The vectorial motion equations describing a deviation from the hydrostatic equilibrium, are first written in an unifomly rotating frame and in the frequency domain. Wahr (1981a) writes them using operators so that he can get the following forms, respectively in the forced and free cases :

$$w\bar{z} = A\bar{z} + \bar{F} \qquad (1)$$

$$w_n\bar{z}_n = A\bar{z}_n \qquad (2)$$

He defines then a function space H in which he introduces an inner product. He uses an equivalence relation and verifies that A is self-adjoint, so that all the hypotheses of the spectral decomposition theorem are satisfied. He expresses then $\bar{z}$ as an expansion of the normal mode eigenfunctions $\bar{z}_n$. He shows also that it is possible to express the response of the Earth to an external potential as a sum of a direct response of the Earth in the same frequency band at a frequency far from the eigenfrequencies, plus a correction due to resonance effects, thus inversely proportional to $(w-w_0)$ and (w_0-w_n).

2.2. Inelastic Earth's case

In the inelastic case, the operator A defined in (1) is complex. The normal mode frequencies are complex and this is the reason why the hypothesis of the spectral decomposition theorem is not satisfied. Wahr and Bergen (1987) say that this is evident from the fact that self-adjoint operators have real eigenvalues, whereas the eigenvalues of A are complex. But there is a way to get out from this problem, considering that we are working at the first order in the ellipticity. If we introduce $w = w^R + i\, w^I$ in the complex form of the equation of motion, at the first order in the ellipticity and if one computes the normal mode expansion in the diurnal band, the terms containing w^I can be neglected. Separating the real part and the imaginary part of the equation, one can work in the same way as in the elastic case with twice the number of vector components. This

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situation is analysed in full details in a paper in preparation (Dehant & Antoine, 1987).

3. NUMERICAL RESULTS AND DISCUSSION

From the surface displacements, one can compute the tides, the nutations and the variation of the length of day. From the tides, one deduces the gravimetric factor and the Love numbers. These results were presented in Dehant (1987a and b) and in Dehant and Ducarme (1987). The inelasticity of the Earth gives a small increase of the gravimetric factor of about 0.1% in the semi-diurnal and diurnal band (far from the NDFW) and about 0.2% for M_f. The most important difference lies in the definition that one uses to compare the theory with the observations. The difference was about 1.5% in Melchior and De Becker's paper (1983). With our definition, the remaining discrepancy is about 0.6% which is probably due to a calibration effect. The increase due to inelasticity for the Love numbers is much more significative. It is about 1.5% in the semi-diurnal and diurnal band and about 2.6% for h and 3% for k for M_f. This is not negligible for VLBI observations.

For evaluating the nutations, we must account for the resonance effects in the diurnal band and it is necessary to compute the normal mode frequencies. From the results, we conclude that for an inelastic Earth, these frequencies shift a little bit to the lower frequency part of the spectrum. Among the most important modes, the Nearly Diurnal Free Wobble (NDFW) also called Free Core Nutation (FCN) is due to an angle between the rotation axis of the core and the rotation axis of the mantle and produces a torque at the core-mantle boundary. Due to the deformations at this boundary, there exist a pressure torque acting against this effect and increasing the period in the inertial space. If one considers the Mantle inelasticity, the Earth deforms more (increase of the Love number h for example). This will produce an increase of the period of the FCN in the inertial space. This is in the opposite sense of the observations because the observed inertial space period is about 435 days (Neuberg et al., 1987). The elastic inertial space period computed by Wahr is about 460 days and the inelastic period is 463 to 467 days depending on the inelastic models presented in paragraph 1. The Tilt-Over-Mode (TOM) also called the Free Diurnal Nutation (FDN), is due to an angle between the mean rotation axis and the instantaneous rotation axis and is associated with the rotation of the Earth as a whole. It will not be so affected by the mantle inelasticity. The Chandler Wobble (CW) is due to an angle between the rotation axis and the figure axis. Its period increases from about 6 days to 15 days, depending on the inelastic models. The nutations are computed from the surface displacement field, response to an external potential of the order $\ell=2$, m=1, thus, in the diurnal band. This is the reason why the most important resonance effect is due to the presence of the NDFW in this band. Due to the difficulties to find the eigenfunctions, we decide to consider only the effects of inelasticity on the NDFW and on the direct computation of the tides. So that the surface displacement field,

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response to an external potential of frequency w, is :

$$\bar{u}(w) = \bar{u}(w_0) + \sum_{n=1}^{9} a_n(w, w_0, w_n)\, \bar{u}_n(w_n) \qquad (3)$$

where $\bar{u}(w_0)$ and $\bar{u}_3(w_3)$ are complex; n=1 corresponds to the CW, n=2, to the TOM; n=3, to the NDFW;n=4 to 9 correspond to the most important Earth classical eigenfunctions of ±54min, ±28min and ±24min.

The numerical results of the direct integration were presented in a previous paper (Dehant, 1987b). The annual, semi-annual and 18.6 years nutation results are presented in table I where they are compared with Wahr's theoretical results (1979) and to the observations (Herring et al., 1986). One can see that the discrepancy between the theory and the observation is not resolved, it is even worse than before.

TABLE I

Periods		Present paper	Wahr	Kinoshita	Observations
18.6 yrs	Δψ	17."1936	17."1922	17."2743	
	Δε	9."2029	9."2025	9."2278	
annual	Δψ	0."1424	0."1426	0."1254	0."1470
	Δε	0."0053	0."0054	-0."0001	0."0072
semi-	Δψ	-1."3169	-1."3180	-1."2770	-1."3195
annual	Δε	0."5731	0."5736	0."5534	0."5739

We believe that a part of the discrepancy may be due to the fact that other phenomena must be considered in the computation of the FCN. Gwinn et al. (1986), for example show that a non-hydrostatic ellipticity (variation of about 500 metres) at the core-mantle boundary may decrease the theoretical inertial space FCN period. Pressure torques due to displacements in the fluid core may also be involved. In the future, we will examine these problems in more details.

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DISCUSSION

Yoder: I presume that the change in *k*, *h*, etc. is due to the use of Maxwell-like dissipation units. Have you considered models with Kelvin units (i.e. dashpot & spring in parallel)?

Reply by Dehant: Yes. I applied also models involving an infinite number of standard linear solids in series. First, the Liu et al. (1976) model (which corresponds to the Anderson and Minster model (1979) with $\alpha = 0$) and, second, the Anderson and Minster model (1979) with $\alpha = 0.15$ which is a realistic value for the Earth. The results obtained by using Zschau's model were in between the results of those two models. I get:

Models	Increases due to inelasticity for M_j	
	h	*k*
Liu et al. 1976	about 2%	
Zschau 1985	2.6%	3%
Anderson and Minster 1979	about 4%	