PROPERTIES AND EVOLUTION OF WHITE DWARF STARS

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ABSTRACT

This paper reviews the properties and evolutionary status of white dwarf stars, focusing most closely on those aspects which are likely to be of significance to understanding the ultimate fate of planetary nebulae and their central stars. White dwarf stars show a broad variety of chemical compositions. Broadly speaking, they are divided into the DA stars (with H-rich photospheres) and the non-DA stars (with He-rich photospheres), though there are a fairly large number of subtypes. The mass distribution of white dwarf stars is quite narrow, with a mean value near 0.6 and with extremes at 0.43 and 1.05. Different varieties of trace elements (such as C, N, O, Si, Ca, and Mg) are quite common. I will review several recent proposals for explaining these abundance patterns. A particularly significant question is whether processes operating while the star cools as a white dwarf can account for their variety, or whether at least part of the white dwarf phenomenology is related to events which took place when the object was a planetary nebula or even earlier.

I. INTRODUCTION: A COOK'S TOUR OF THE BOTTOM OF THE HR DIAGRAM

In the last several years, it has become clear that the white dwarf stars are a phenomenologically very rich class of stellar objects. Broadly speaking, they are divided into a group of stars with H-rich photospheres, called the DA stars ("D"=degenerate and "A" refers to the main sequence analog) and the non-DA stars, which (in all cases but one) almost certainly have He-rich photospheres. However, a number of very peculiar subclasses have been recently identified in which substantial quantities of trace elements are introduced into the photospheres of these objects. The roster of white dwarf stars now includes a number of different types of objects illustrated in Figure 1 on the next page. Complex as Figure 1 seems, it considerably oversimplifies our understanding of white dwarf stars; a figure illustrating white dwarf evolution in its full glory is Figure 1 of Sion's (1986) review article.

S. Torres-Peimbert (ed.), Planetary Nebulae, 555–566.

T(eff)/10³-->	150	75	50	30	15	10	8	6
Type:								
H	//////////	DA		V471*?		ZZ Ceti*		DC
Hybrid H/He	DAO				DBA		DZA	
He	PG1159*	DO		///DB*		DQ		DZ
?	H1504							
DA/non-DA ratio:	0(!)	7:1	oo(!)		4:1			1:1

* denotes variable stars.

Figure 1: A simplified description of the classes of white dwarf stars. Effective temperature decreases from left to right, following the cooling sequence. Each of the major classes of white dwarf star has its place in this table; diagonal slashes denote those places along the four major cooling sequences where no stars are found.

The broadest, most obvious division of the white dwarf stars is into the DA and non-DA categories, but each of these categories includes a number of spectroscopically, chemically, and evolutionarily distinct subclasses. The spectral classification system is fully defined in Sion, Greenstein, Landstreet, Liebert, Shipman, and Wegner 1983; only a resume will be provided here. All white dwarf spectral classes have the prefix "D" for degenerate, which in the present context generally means log g > 7. Thermal pressure plays a role in the structure of hot white dwarfs, so that an object with T(eff) > about 50,000 K may be fully degenerate and not lie on the Hamada-Salpeter (1961) zero temperature mass radius relation. The first letter following the "A" indicates the element with the strongest lines in the star, with "A" being hydrogen, "B" being neutral helium, "O" being ionized helium, "Q" being carbon, and "Z" referring to other elements, sometimes referred to by astrophysicists as "metals." (In the case of white dwarf stars, these usually are genuine metals like Ca, Mg, or Fe rather than substances like oxygen which no respectable chemist would call metallic, despite the astrophysical nomenclature.) A white dwarf with no clear spectral features is designated as "C" (for continuous); improvements in spectral resolution, sensitivity, and coverage of the electromagnetic spectrum have resulted in a considerable decrease in the fraction of white dwarf stars classified as "DC." If a second element is present in the spectrum, it is indicated by a second letter; thus a DBA star (discussed in some detail below) is one showing both He I and H I, with H I being weaker than He I. The numerical digit found in catalogs of white dwarf stars (e.g., McCook and Sion 1987) refers to 50400/T(eff). This catalog, incidentally, is a good reference to the rather confusing nomenclature of white dwarf stars; many investigators continue to prefer to refer to a star by its original catalog designation rather than by a coordinate-based system. This classification system is an elaboration of earlier ones by Greenstein (1960) and Luyten (1952). The "A","B", and "O" designations came from the main sequence analogs to white dwarf spectra. "P" is used to designate polarized white dwarfs, which have strong magnetic fields.

Most broadly speaking, about two thirds of the white dwarfs are DA stars (either just plain DA, or DA with a suffix like DAO,DAZ,DAB), which with only one known exception, the DAB star Gr 488, really have photospheres which are dominated by H. The highest He abundances in the DA stars are found in the very hot DAO stars, where $N(He)/N(H) = 10^{-2}$ (Wesemael, Green, and Liebert 1985). The remainder have He-dominated photospheres; if T(eff) > 11,000 K, they appear as DB or DO stars since the He I or He II lines in accessible spectral regions (which all arise from excited states) are only visible at sufficiently high temperatures.

2. TEMPERATURES, MAGNETIC FIELDS, AND ROTATION

Some of the most extreme stellar properties are found in the white dwarf region of the HR diagram. High magnetic fields, high temperatures, low rotational velocities, low luminosities, and extreme chemical compositions are all aspects in which particular white dwarfs can represent the extremes of directly observable stellar properties. Since the second half of this review deals extensively with chemical peculiarities, the remainder of this introductory section will deal with the other aspects in which white dwarfs are peculiar.

A few percent of them have very strong magnetic fields, ranging from a few to a few hundred megagauss. The highest field found to date is the >500 MG value recently reported for PG 1031+234 (Schmidt, West, Liebert, Green, and Stockman 1986. These fields are recognized by the distortion of the usual pattern of hydrogen Balmer lines as well as by the circular polarization of the light from the white dwarf.

The hottest white dwarf is the very peculiar object H1504+65, with T(eff) = 160,000 K(Nousek, Shipman, Liebert, Holberg, Pravdo, Giommi, and White, 1986). This object, along with several other hot white dwarfs of the PG 1159 class, with T(eff) near 130,000 K, is hotter than all but a very few planetary nebula nuclei, but there are no visible nebulae around H1504 and the PG 1159 objects. (The nucleus of the planetary nebula K 1-16, which is pulsationally and spectroscopically similar to PG 1159, may not be a degenerate object.)

All other things being equal, one would expect white dwarf stars to rotate quite rapidly, as neutron stars do when they are born. A star of one solar mass rotating with a period of 1 month would have a rotational period of about 6 min (or an equatorial velocity of 140 km/sec) if it conserved angular momentum per unit mass and collapsed to a white dwarf of 0.6 solar masses and 0.012 solar radii. The most complete set of white dwarf rotational velocities has been determined by Pilachowski and Milkey (1987) from high resolution observations of the narrow cores of Balmer lines in DA white dwarfs. Of fifteen white dwarfs, they find that ten have negligible velocities (the lowest 2 sigma limits are 20 km/sec), while the rest have velocities detected at the 2 sigma level or better, the highest being 60 ± 10 km/s for the DA star GD 140. These values are consistent with the limit of $v \sin \underline{i} < 30$ km/s set from the C and Si lines in the hot white dwarf Feige 24 (Wesemael, Henry, and Shipman 1984).

Rotational velocities can be determined for the magnetic white dwarfs because the observed magnetic field can change as the star rotates. Changing fields can show up as changing polarization patterns, changing spectra (as the Zeeman shifts change), or both. The five magnetic white dwarfs with detected (or possibly detected) rotation periods all have periods which are appreciably longer than expected: 1.6 hr (PG1015+014), 3 hr (L795-7), 31 hr (G 195-19), 67 hr (tentative detection; BPM 25114), and 3.4 hr (PG1031+234; data from Angel, Borra, and Landstreet 1981 and references therein and Schmidt, West, Liebert, Green, and Stockman 1986). Some extreme lower limits can be set by the absence of polarization changes in three objects; the position angle of linear polarization has changed by less than 3° in several years for three magnetics (GD 229, G 240-72, and Grw+70°8247) , suggesting rotation periods exceeding 100 years! While it is unlikely that all three of these stars are rotating pole-on, it would be useful to have similar data for a larger sample of magnetic white dwarf stars.

The conventional explanation for these low rotation periods is that the white dwarf progenitors rotate as solid bodies when they are red giants. The angular momentum in a slowly rotating red giant is concentrated in the outer layers, and mass loss will then carry away a disproportionate amount of angular momentum. I am not aware of any investigations which have tried to explain whether this scenario can account for century long rotation periods. It would be quite interesting if someone could observe consequences of angular momentum losses during the final red giant stages (for example, effects on planetary nebula morphology or determinations of nebular rotation), though I suspect that such a task is quite difficult. It might also be interesting to explore why some white dwarf stars do appear to retain a fraction of their angular momentum and why others do not.

The coolest white dwarfs are the lowest luminosity stars known. Their temperatures are difficult to determine, largely because their photospheres are partially degenerate in the sense that the perfect gas equation of state cannot be used in model atmosphere calculations. Kapranidis and Liebert (1985) determined T(eff) = 4500 K for the cool degenerate LP 701-29, which at the moment is probably the best analyzed of a small collection of very cool objects. ER 8, discovered as a by-product of a supernova search program at the Universidad de Chile, may be cooler still, since it is as red as LP 701-29, though it is probably not as cool as claimed in the discovery paper (Ruiz, Maza, Wischnjewsky, and Gonzalez 1986; Ruiz, private communication).

About ten years ago, Liebert and co-workers (Liebert, Dahn, Gresham, and Strittmatter 1979; see also Liebert 1980, Liebert, Dahn, and Sion 1983) showed that despite exhaustive searches, very cool white dwarfs like those mentioned in the previous paragraph represent the extreme cool end of the white dwarf distribution. This conclusion is reinforced by the failure to discover any very cool white dwarfs as astrometric companions to nearby stars (Shipman 1983). The interpretation of this cutoff in the white dwarf luminosity function is complicated. If you believe that we can correctly calculate the cooling rate of white dwarfs, then the existence of this cutoff can set limits on the age of the galactic disk. The most recent such determination by Winget et al. (1987) indicates an age for the disk of 9.3 ± 2 Gyr. However, the physics of white dwarf cooling, particularly at the very cool end of the white dwarf sequence which is crucial for using white dwarfs to determine the age of the disk, is complex. The equation of state and opacity in the partially degenerate layers, which overly the core and are the throttle that determines the rate of cooling, must be known accurately in order to calculate correct cooling times. Winget et al. take the audacious step of extrapolating from the age of the galactic disk to the age of the Universe. Whether one can reliably state that the Universe is only 1 Gyr older than the galactic disk is a matter of taste and judgment; consumers should be wary of uninformed, hasty use of cosmic ages based on white dwarf cooling times, in my view.

3. MASSES OF WHITE DWARF STARS AND WHITE DWARF PROGENITORS

Nearly a decade ago, two comprehensive investigations of the masses and radii of DA white dwarf stars were completed, one by the Kiel group (Koester, Schulz, and Weidemann 1979) and the other by Shipman (1979) and Shipman and Sass (1980). In general, the results of the two discussions were quite similar. Both found that the observed sample of DA white dwarfs had a mean mass of 0.6 solar masses. Both investigations suggested that the mass distribution of white dwarfs is quite narrow, far narrower than the range from 0.45 to 1.4 solar masses which is allowed in principle by the physics of white dwarf stars. (The lower mass limit is set by core masses of main sequence stars which can evolve in less than a Hubble time; the upper limit is the maximum mass of a C white dwarf according to the mass radius relation of Hamada and Salpeter 1961). The principal difference between the two investigations is that Shipman (1979) found a significantly higher mass spread, leading to a selection effect which would skew any observed sample of white dwarf stars towards those with larger radii (since they can be observed to greater distances). This selection effect will only apply if the cosmic scatter in the white dwarf distribution is sufficiently large.

Subsequent investigations have tended to confirm these results on the mean masses of the DA white dwarfs, and have suggested that the lower value of the mass spread is more likely to be correct. Weidemann and Koester (1984) and Greenstein (1985) find that the mass distribution of white dwarfs is sufficiently sharply peaked that the selection effect will only skew the mean mass of white dwarfs by 0.05 solar masses or less. While Guseinov, Novruzova, and Rustamov (1983, 1984) find a larger spread of white dwarf masses, they used UBV colors to define the white dwarf temperatures and thus obtained a mass distribution which may well be much broader than the real one, since UBV colors from heterogeneous sources can be subject to uncertainties which are both large and difficult to determine (Koester 1984). At the moment, my best estimate is that the mean mass of the DA white dwarf stars is between 0.58 and 0.63 solar masses.

The observed range of white dwarf masses extends, at its extreme ends, to the reasonably precise masses for white dwarfs in binary systems: 0.43 solar masses (40 Eri B) and 1.05 solar masses (Sirius B). Recent work by the Kiel group on the masses of white dwarf stars in galactic clusters confirms the existence of a high-mass component to the white dwarf mass distribution (see references below). At the moment, neither the observations nor their theoretical calibration in terms of various methods of determining white dwarf masses are sufficiently precise that the characteristics of the high-mass and low-mass components of the white dwarf mass distribution can be given in detail.

Shipman (1979) also determined the mean mass of a sample of non-DA white dwarfs, using model atmospheres and parallaxes, and found no appreciable difference between the masses of the non-DAs and the masses of the DAs. The difficulty with the non-DAs is that the model atmospheres are less certain (because of convection) and that the nice separation between white dwarfs of different gravity in the two-color (*U-B* vs. *B-V* or its analog in other color systems) diagram does not occur because non-DAs don't have a large Balmer jump. Oke, Weidemann, and Koester (1984) did use the two-color diagram to try to find the masses of a sample of DB stars, and agree that there seems to be no appreciable difference between the masses of the DA's and the DB's.

For a number of years, investigators have sought to determine whether there is any relation between the mass of a white dwarf and the mass of its progenitor. If a white dwarf is found in a star cluster, the cooling age of a particular white dwarf can be subtracted from the age of the cluster to determine the nuclear burning age (and hence the mass) of that particular white dwarf's progenitor. Koester and Weidemann (1984; see also Weidemann 1984, 1987) provide a recent summary of these efforts. Investigations of a number of young clusters (see particularly Reimers and Koester 1982, Koester and Reimers 1985) show that even comparatively massive progenitors, with masses of roughly 8 solar masses, still produce white dwarfs with relatively low masses (around 0.8 solar masses).

A second, important result from the investigations of white dwarfs in star clusters is the determination of the initial mass M_w of a star which will die as a white dwarf rather than something else. Reimers and Koester (1982) estimated M_w as being 8 (+3,-2) solar masses. There seems to be no compelling reason to challenge this estimate, though a value of M_w which was as low as solar masses would be difficult to reconcile with the data on the cluster NGC 2451.

4. CHEMICAL COMPOSITIONS: THE ORIGIN OF DA AND NON-DA WHITE DWARFS

"The uniformity of composition of stellar atmospheres appears to be an established fact," wrote one of the pioneers, Cecilia Payne, in her celebrated thesis in 1925. (Payne 1925, quoted in Bidelman 1986). In retrospect, I find it indeed remarkable that Payne could recognize the uniformity of composition of main sequence stars, despite their very dissimilar appearance. In 1925 the Saha equation, the key to understanding the many spectroscopic faces of the "Russell mixture," was scarcely five years old. It took great insight to understand that the great change of hydrogen line strength from spectral types A through O was nothing more than the results of the Saha-Boltzmann equation and radiative transfer, not of compositional differences.

However, the exceptions to Cecilia Payne-Gaposhkin's dictum are indeed quite interesting, and white dwarf stars constitute one of the most numerous exceptions to the uniformity of stellar compositions which tends to prevail elsewhere in the HR diagram. In 1925, only two or three white dwarf stars were known to exist, but there are now over 10^3 of them, and next to main sequence stars they are the most common type of star. They are anything but uniform in composition. It was in the the late 1950s and early 1960s, when spectra of a reasonable variety of white dwarf stars became available and when model atmosphere techniques were reasonably well developed, that the existence of two compositionally distinct classes of white dwarf stars had become clear (Greenstein 1960). Certainly ever since the early 1970s a number of us have struggled with the question of why this compositional dichotomy should exist.

The existence of the non-DA stars is particularly puzzling because a naive view of white dwarf evolution would suggest that all white dwarfs should be DA stars. The photospheres of DA white dwarf stars are very thin indeed. A white dwarf which makes a single passage through a reasonably dense region of the interstellar medium will accrete enough hydrogen to appear as a DA, if there is no fractionation during the accretion process. Furthermore, the high gravity of white dwarf stars means that heavier elements settle to the bottom quite rapidly, in a process which is generally referred to as diffusion. One would thus expect that whatever hydrogen is left or accreted in the outer layers of a white dwarf would float to the top and make the star look like a DA. Why do non-DA white dwarf stars even exist? There is no obvious difference in mass or kinematics between DA and non-DA white dwarf stars, so it is no longer possible to appeal to different accretion regimes in order to explain the existence of the two types of stars (as was popular several years ago, where a mass difference was reputed to be the cause).

As an aside, before considering various explanations for the existence of both DA and non-DA white dwarfs, a definition of what constitutes an acceptable "explanation" is in order. For a number of years many of us in the white dwarf business have sought to delineate the channels of white dwarf evolution, explaining whether DB's become DZ's and how the DQ's fit in, and so forth. Fig. 1, presented some pages back, represents one such attempt, but the true picture is considerably more complex (see Fig. 1 of Sion 1986). Earlier I cited a classic volume of Payne-Gaposhkin; here I go considerably further back in time (Ockham 1488). "Entia non sunt multiplicanda praeter necessitatem." (For bibliographical sources, see Sarton 1947.) In common engineer's parlance, "Keep it simple, stupid." I have elsewhere (Shipman 1987) quoted E.W. Kolb's way of expressing Occam's Razor: "A theory which is too complex to fit on a T-shirt is too complex to be correct." Complicated pictures are not the whole story; while we seek to delineate the channels of stellar demise, we also seek to understand why? What are the underlying causes?

Let me set forth two extreme scenarios for explaining the distinction between DA white dwarfs and non-DA white dwarfs, illustrated schematically below. The first scenario, which I will call "primordial," postulates the distinction between DA's and non-DA's lies in the planetary nebula stage if not before, certainly predating the white dwarf stage itself. In this scenario, DA stars remain DA stars for most if not all of their cooling lifetimes, possibly dredging up subsurface He to become He-dominated DC stars at very cool temperatures. A specific mechanism for producing approximately the right number of non-DA's (~ 25

%) was suggested by Iben and Renzini (1983; see also Iben 1984), where the phase of the final thermal pulse determined whether a star would become a DA or a non-DA. However, the basic outline of the primordial scenario as discussed here does not depend on one particular mechanism for the origin of the two types of white dwarfs; binaries might indeed play a numerically significant role, as suggested by Tutukov (1987).

Another scenario is a "mixing" hypothesis, in which the establishment of convection zones and other events such as accretion from the interstellar medium or nuclear burning transform DA's into non-DA's and vice versa. The contrast between the two scenarios is illustrated in Figure 2 below. Liebert (1987) and Liebert, Fontaine, and Wesemael (1987) have discussed this scenario in the context of contemporary theory. Its origins go back to earlier discussions by, e.g., Strittmatter and Wickramasinghe (1971) and Shipman (1972).

Convective mixing certainly seems to make sense as an explanation for the origin of carbon in the DB stars. A widely cited mechanism for changing the surface composition of white dwarf stars is the establishment of a deep convection zone which will mix interior layers with the surface. In one case, a combined theoretical and observational effort has demonstrated quite convincingly that mixing does occur. The DQ white dwarfs are non-DA stars which contain a trace abundance of carbon; model atmosphere analyses indicate that the carbon abundance peaks at about T(eff) ~ 10,000 K (Wegner and Yackovich 1984, Koester, Weidemann, and Zeidler 1984). These authors suggested the mixing explanation, which was confirmed by more detailed "ab initio" models (Fontaine, Villeneuve, Wesemael, and Wegner 1984; Pelletier, Fontaine, Wesemael, Michaud, and Wegner 1986).

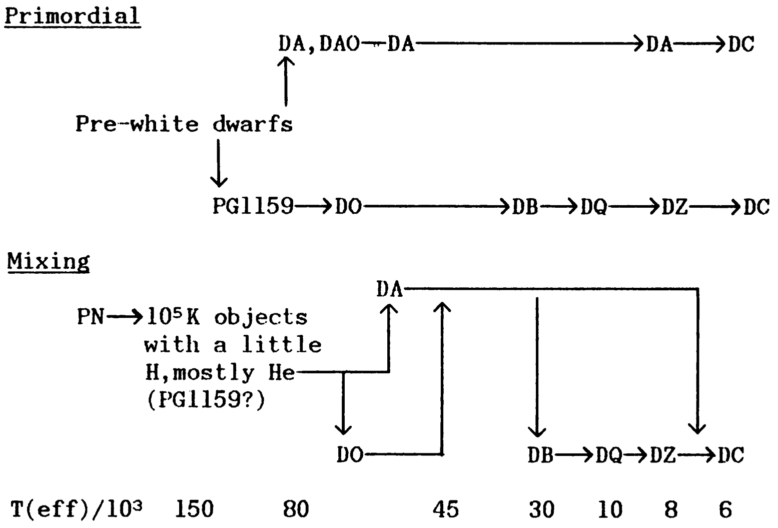

Figure 2. Two extreme explanations for the chemical evolution of white dwarf stars.

There are two important pieces of evidence which tend to favor the primordial scenario. First, as emphasized by Renzini in the discussion at this conference, about 20-30 % of central stars of planetary nebulae are extremely H-deficient (Mendez, Miguel, Heber, and Kudritzki 1987; Kudritzki, this conference). The existence of this abundant class of planetary nebula central stars strongly suggests an evolutionary connection between these central stars and the non-DA sequence, which represents (very!) approximately 20 % of the white dwarf stars. A second, less direct reason for believing in the primordial hypothesis is that white dwarf stars which do originate as DA's should have fairly thick hydrogen shells, with masses of roughly 10^{-4} solar masses, overlying a He envelope of 10^{-2} solar masses, overlying a C/O core. Such hydrogen envelopes are too thick to be mixed with the underlying He. While it is possible that diffusion-induced H burning could thin the H layer down (Michaud, Fontaine, and Charland 1984), Iben and MacDonald (1985) have questioned whether this process can work with the required effeciency to reduce the mass of the H layer by of order 10^4 or more.

The complex phenomenology of the white dwarf stars, outlined in Figure 1, has been used by several of us to argue in favor of mixing in at least some circumstances. The varying ratio of DA's to non-DA's as a function of T(eff), illustrated in Figure 1 at the beginning of this paper, has been used by many to suggest that there is considerable crossing over between the two sequences (Sion 1984, Greenstein 1986, Liebert 1986, and references therein). There are zero DA stars above about 90,000 K, where the ratio turns around and we find 7 DA stars for every non-DA star. Between 30,000 K and 45,000 K, the Palomar-Green survey of the north galactic pole contains no non-DA stars (which, in this temperature range, would be hot DB's); five would be expected from normal statistics (Liebert, Wesemael, Hansen, Fontaine, Shipman, Sion, Winget, and Green 1986) Between 10,000 K and 30,000 K, there are four DA stars for every non-DA star, a ratio which a few years ago was thought to prevail at all temperatures. Cooler than something like 10,000 K, the ratio becomes 1:1. Cooler than about 5,000 K, the ratio is unknown, because Balmer lines are no longer visible. In principle, infrared absorption by the pressure induced dipole of hydrogen could be used as a composition diagnostic, but clear conclusions have not yet appeared.

However, there are other ways to explain these data. One simple one suggested by MacDonald (1987, private communication; see also Iben and MacDonald 1985, 1986) is that the cooling rates, which underlie the conclusions about the DB "gap," might be incorrect, making the significance of the gap considerably less. MacDonald, Iben, and Jason Tillett are currently undertaking calculations to see whether this idea will work. Another suggestion appeals to changes in the star formation or evolution processes in the last few billion years. White dwarfs of differing temperatures are the end products of stars which formed at different times. If star formation in the Galaxy is patchy, then the changing numbers of DB and DA stars could simply reflect a different population of ancestors. It is especially interesting to consider this suggestion in light of the fact that all white dwarfs that we know of, and which are the basis of these statistics, lie within roughly 100 pc of the sun. It is not too unreasonable to suppose that star formation on such a short length scale is inhomogeneous in time. Still another possibility, relevant to the shortage of high-temperature DA's is that H shell burning in the DA precursors will affect their evolutionary tracks in such a way that DA degenerates will join the white dwarf sequence at T(eff) near 80,000 K, rather than a hotter value.

The behavior of trace quantities of H in the DB stars and He in the DA stars provides some pieces of evidence which remain to be successfully interpreted. In particular, Shipman, Liebert, and Green (1987) discussed the DBA stars, a phenomenon which may (or may not) suggest an evolutionary connection between DA and non-DA stars. These stars are predominantly He, with H abundances of order 10^{-4}; about 20 % of the non-DA stars in the temperature range where H and He can be seen simultaneously are DBA stars. Shipman, Liebert, and Green discuss two scenarios for the origin of the H in these stars: convective mixing in the star (in which a DA would mix and turn into a DBA) and accretion. There are problems with both scenarios. A very recent discovery of Ca in two DBA stars (Kenyon, Shipman, Sion, and Aannestad 1987) suggests that, in these cases at least, accretion is the origin of the H seen in their spectra. We still have to explain how objects like GD 40 accrete Ca but no H, while these DBA stars accrete Ca and H in approximately solar proportions.

Thus we have no simple answers which can explain why the lower part of the HR diagram contains such a wide diversity of white dwarf stars. At the present time, white dwarf researchers are endeavoring to sort out what pieces of evidence really are clues to the correct fundamental answer and which are red herrings. If the "primordial" scenario is even partially correct, then the chemical properties of white dwarf stars will provide an important boundary condition for understanding the way that planetary nebulae are formed and evolve. If, on the other hand, "mixing" (and accretion from the interstellar medium) can explain the chemical diversity of white dwarfs, then their chemical composition will be only loosely coupled to prior evolutionary stages.

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