Correspondence

DEAR EDITOR,

I should like to respond to Martin Taylor's thought-provoking article 'Calculators and computer algebra systems' (*Math. Gaz.* **79**, March 1995).

Martin argued that new technology (graphics calculators, computer algebra systems and powerful spreadsheets) is here now and we have to take this into account when designing curricula and assessment arrangements.

At the moment, assessment is decidedly biased in favour of candidates who can afford graphics calculators. Last week I visited an inner-city school and worked with an A level group of six students, none of whom possessed a graphics calculator. The school had just two machines. The first question in a recent exam paper taken by the students was to solve the equation $\sin 2x = -0.58$.

As Martin pointed out, the solutions to questions like this can be checked easily using the hardware possessed by wealthier students.

At the moment, students don't have access to hand-held symbol manipulators, yet the arrival of the Texas TI92 is imminent and the problem about unfairness I describe above will get worse. It will be very difficult to set exam papers which will not advantage candidates with access to the hardware.

The situation is not dissimilar to that prevalent when calculators first appeared back in the seventies. The solution then was easy – ban them. Twenty years on we find almost universal acceptance of scientific calculators as an examination resource. Are we not in a position now where CASs should be banned in examinations? After all, in 20 years *every* student will possess one, and we can then change our assessment regulations and procedures to accommodate the new technology. Maybe we should do as Martin suggests in his conclusion, and have two systems running side by side? This would result in separate examination systems; one for those who have access to CASs and one for those who don't.

All we have to do then is decide which skills we would like our A level students to retain and how we're going to assess them! I'm glad I'm not an A level Chief Examiner!

Yours sincerely,

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DEAR EDITOR,

I am no mathematician, despite Douglas Quadling's best endeavours at Marlborough in the 60s, but I retain enough interest in the subject to read my wife's copy of the *Gazette* from time to time. As a musician I greatly enjoyed John Bowers' piece on the angelic choir and thoroughly agree with his conclusions. I am surprised, however, at his omitting two conclusive, although

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non-mathematical, proofs.

It is well known that angelic choirs are accompanied by trumpets (for a demonstration of this consider Sydney Smith's definition of paradise as 'eating *foie gras* to the sound of trumpets'). Now the natural trumpet – and can we suppose that angels play other than natural instruments? – is tuned in D which in 'white-note' terms is the Dorian mode identified by Bowers as the second of the keys preferred by angelic choirs. Before the development of the modern valve-trumpet most trumpet-accompanied music is therefore in D whatever difficulties this may pose to merely human singers – for example the bass aria 'The trumpet shall sound' in Handel's *Messiah.*

When setting the scene where the angels appear to the shepherds in the fields earlier in the same work, Handel begins with the 'pastoral symphony' in C major, the first of Bowers' angelic keys, creating an atmosphere of almost supernatural calm, but has to perform feats of modulation in the course of the ensuing recitatives in order to bring the angels and trumpets in some twenty bars later with 'Glory to God' in D.

Yours sincerely,

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DEAR EDITOR,

What is the mathematics of bowls, **Note 79.23**

In his note, John Branfield tells of his post-retirement introduction to the game of lawn bowls. In books on bowling he read that for the bowl to finish on the centre line one must project the bowl at an angle α which is independent of the launching speed (but dependent on the state of the green). In practice, however, he found that the angle must be reduced for lower speeds.

My brief exposure to the game of bowls occurred some 45 years ago when, in my early twenties, I might have been the youngest player in the state. In my early games I became aware of the importance of α; and I remember using various off-green features to help me fix α and thus turn it to good account.

A decade later, while browsing journals in the Cambridge Philosophical Library, I came across an article by M. N. Brearley and B. A. Bolt [1]. I did not need to look far to find theoretical support for α: the abstract said 'These give the equation of the path and show that the angle of deflexion from a straight line is independent of the velocity of projection provided the bowling green conditions remain unchanged.'

Within a few years I was to meet Maurice Brearley. I was at RAAF Academy, Point Cook, when he was appointed to the Chair of Mathematics. I learnt then that his work on the dynamics of a bowl had formed part of his Ph.D. thesis, and that there was a second paper which considered the effect of an initial wobble on the final position of the bowl [2].

Maurice now practises his craft as a professor emeritus from his home at Clifton Springs. I sent him a copy of John Branfield's note. He responded with a third, apparently unpublished, paper on 'A mathematician's view of bowling' [3] in which he makes the theory of bowls accessible to a wide range of readers. He also informed me that a few years ago Henselite Bowls Co. introduced a new bowl, the *Classic*, with a new shape which entails a smaller aiming angle for shorter heads.

References

1. M. N. Brearley and B. A. Bolt, The dynamics of a bowl, *Quarterly Journal of Mechanics and Applied Mathematics* **11** (1958) pp. 351-363.
2. M. N. Brearley and B. A. Bolt, The motion of a biased bowl with perturbing projection conditions, *Proc. Camb. Phil. Soc.* **57** (1961) pp. 131-151.
3. M. N. Brearley, A mathematician's view of bowling, unpublished paper.

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Editor's Note: An article on the mathematics of bowls, by Tom Roper, is planned for the July 1996 edition.

DEAR EDITOR,

I very much enjoyed your own and Professor Grassl's articles in the July *Gazette* [1, 2]. The numbers $q(j, k)$ defined in [1] are well-known (but not as well known as they might be!). They are called *Eulerian numbers*, and are discussed in [3], where the notation is

$$q(j, k) = \left\langle {j \atop k-1} \right\rangle.$$

These numbers have a combinatorial interpretation; $\left\langle {n \atop k} \right\rangle$ is the number of permutations of $1, 2, \dots, n$ with k 'ascents'. For instance $\left\langle {5 \atop 1} \right\rangle = 26$ as the following permutations have 1 ascent;

15432	21543	25431	31542	32154	32541	35421	41532	42153
42531	43152	43215	43251	43521	45321	51432	52143	52431
53142	53214	53241	53421	54132	54213	54231	54312	

This interpretation allows an easy proof of the recurrence relation

$$\left\langle {n \atop k} \right\rangle = (n-k)\left\langle {n-1 \atop k-1} \right\rangle + (k+1)\left\langle {n-1 \atop k} \right\rangle.$$

A formula, set as an exercise in [3] is

$$x^n = \sum_{k=0}^{n-1} \left\langle {n \atop k} \right\rangle \binom{x+k}{n}. \tag{1}$$

Substituting j for x and summing from 1 to N gives

$$\sum_{j=1}^{N} j^n = \sum_{k=0}^{n-1} \left\langle {n \atop k} \right\rangle \binom{N+k+1}{n+1} \qquad \text{as in [2].}$$

Substituting m for x in (1), multiplying by p^m, and summing (using $\left\langle {n \atop k} \right\rangle = \left\langle {n \atop n-k-1} \right\rangle$ (e.g. from the interpretation) and $z^n(1-z)^{-n-1} = \sum_{i=n}^{\infty} \binom{i}{n} z^i$ for $|z| < 1$) gives

$$\sum_{m=1}^{\infty} m^n p^m = \sum_{k=0}^{n-1} \left\langle {n \atop k} \right\rangle \frac{p^{k+1}}{(1-p)^{n+1}} \qquad \text{as in [1].}$$

See Graham, Knuth, Patashnik's book for a wealth of information on these, and related, coefficients.

References

1. Steve Abbott, A difference method for $\sum m^k p^m$, *Math. Gaz.* **79** (July 1995) pp. 355-359.
2. Richard Grassl, The squares do fit, *Math. Gaz.* **79** (July 1995) pp. 361-364.
3. Graham, Knuth & Patashnik, *Concrete Mathematics*, Addison-Wesley (1989, 1994).

Yours sincerely,

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At a premium

Sir, In your article, 'Driving down car insurance costs' (Weekend Money, December 10), I note the following discounts applicable to me are available from various companies:

1. Over 50s with five-year no claims bonus – 70 per cent,
2. Use restricted to two named drivers – 20 per cent.
3. Anti-theft devices – 10 per cent.

I am paying £398 for my insurance, why is it not free?

From the *Times*, 18 December 1994, spotted by Frank Tapson.

Safe hiding place?

Proof that mathematics is an unpopular subject came yesterday when an IRA bomb was found hidden behind calculus text books in the Oxford branch of Dillons bookshop.

Discovery of the unexploded device, thought to date back to the Provisionals' mainland bombing campaign two years ago, may also speak volumes about slow stock turnover at Dillons which went into receivership on Wednesday.

Spotted by Nick Lord in the *Guardian*, 3 March 1995.