DYNAMICAL MODELING OF M13 PROPER MOTIONS

Kyle Cudworth*
Yerkes Observatory
University of Chicago

D.N.C. Lin and Kap-Soo Oh
Lick Observatory
University of California, Santa Cruz

ABSTRACT

Proper motion data of M13 cluster members are very useful for the investigation of the cluster's internal dynamics. We introduce a maximum likelihood method to fit these data in terms of conventional King-Michie models.

Cudworth and Monet (1979) obtained high precision proper motions for some 440 stars in the region of the globular cluster M13 using many plates from the Yerkes 40-inch refractor taken from 1900 to 1977. They derived cluster membership probabilities for individual stars and concluded that about 330 of these stars are highly probable cluster members. Most of these proper motion determinations have errors less than or comparable to the expected cluster internal motion dispersion and are thus useful for an investigation of the cluster's internal dynamics.

In preparation for a dynamical study, these proper motion data have been carefully analysed for any systematic effects. A small net radial motion of the cluster members was found and removed. The most likely cause for this motion is due to the details of the original reduction method. No other significant systematic trends in the proper motion were found. The errors of the motions have also been studied carefully, with only one significant trend noted: there are relatively fewer stars with very accurate motions in the outer parts of the cluster. This effect was expected, since some of the old plates were not full-sized, leading to proper motions based on fewer plates for the outlying stars.

When these proper motion data are fully analysed, they are particularly useful for determining the velocity dispersion ellipsoid and the

*Alfred P. Sloan Foundation Fellow

J. Goodman and P. Hut (eds.), Dynamics of Star Clusters, 65–67.

https://doi.org/10.1017/S0074180900147229 Published online by Cambridge University Press

potential of the cluster as functions of distance from the cluster center. They can also be used to determine the evolutionary history of the cluster. Traditionally, the "velocity dispersion" of the cluster stars is determined by the variance of a Gaussian distribution function which is fitted to the motions of a collection of stars. These stars are binned according to their distance from the center of the cluster. When this method is applied to the M13 data, we find that the "velocity dispersion" of the cluster stars is essentially isotropic near the cluster center and is strongly anisotropic beyond about 7 core radii (300"). These data suggest that the outlying stars are primarily moving in highly eccentric orbits about the cluster core. This behavior is probably associated with two body relaxation process in the cluster.

Although the binning method is very informative, its statistical significance can be questioned. For example, since it is the variance of a Gaussian distribution function, the "velocity dispersions" deduced from the binning analysis may not necessarily have a Gaussian distribution about the values deduced from any theoretical dynamic models. Thus, direct comparison between these "velocity dispersions" and the conventional theoretical models, such as the King-Michie model, may be inappropriate and misleading. Furthermore, the "observed" anisotropy radius could depend upon the choice of bin boundaries. Some of these problems can be overcome if the proper motion data for individual stars are analysed with a maximum likelihood method. The basic principle of this method is to fit the observed proper motion of every member star directly to the stellar distribution function of an _ab initio_ chosen theoretical model such as the King-Michie model. For any given theoretical model, the stellar distribution function at any location in the cluster is essentially determined by a set of structural parameters such as the central potential, anisotropy radius, and stellar mass function. The primary objective for modelling the proper motion data is to determine these structural parameters. When it is properly normalized, the six dimensional phase space distribution function can be used as a probability function for individual stars with particular values of velocity and position. For a given set of structural parameters, the total probability of the model can be evaluated from the product of the individual probability functions. In reality, stellar position in the cluster is measured in terms of projected distance from the cluster center. The spatial projection effect can be accounted for with appropriate integration of the probability function along the line of sight. In addition, if the one or two velocity component data are available, we can still make use of the maximum likelihood method after integration over the missing phase space element. Finally, the measurement error can be accounted for by deconvolution. The major advantage of this method is that the statistical significance of the model can be directly measured from the total probability function. The most probable structural parameters for the theoretical model are selected from the maximum total probability function. At this time we have constructed a numerical code for this purpose. As a test for the method, we generated a set of pseudo data for 1,000 stars and we have successfully fitted relevent theoretical models to these pseudo data. With the testing of the code essentially completed, we expect to run it with the real M13 data in the near future.

This research has been partially supported by the California Space Institute, the National Science Foundation, Korean Government Fellowship, and by an Alfred P. Sloan Foundation Fellowship to KC.

REFERENCE

Cudworth, K.M. and Monet, D.G.: 1979, Astron. J., 84, 774.