CORRESPONDENCE

To the Editor, *The Mathematical Gazette*

DEAR SIR,

J. de Meulenaer devotes his note "On certain sums of products of binomial coefficients" (October 1965, p. 284) to the use of Leibniz' Rule as a means of deriving such sums. His three examples, however, can all be obtained just as easily without using this Rule, as I now show:

Equating coefficients of x^n in the two sides of the identity

$$(1 + x)^p(1 + x)^q(1 + x)^r = (1 + x)^{p+q+r}$$

yields de Meulenaer's equation (1) immediately.

Again, replacing r in (1) by $r - u$, multiplying through by $\binom{r}{u}$ and remembering that $\binom{r}{u}\binom{r-u}{k} = \binom{r}{k}\binom{r-k}{u}$, we obtain his equation (2) without further effort. Finally, writing $q - u$ for q and $r - v$ for r in (1), multiplying through by $\binom{q}{u}\binom{r}{v}$ and remembering that $\binom{q}{u}\binom{q-u}{j} = \binom{q}{j}\binom{q-j}{u}$ and $\binom{r}{v}\binom{r-v}{k} = \binom{r}{k}\binom{r-k}{v}$, we have de Meulenaer's (3). The remaining results in the Note are, as the author states, special cases of (1) and (2).

Why use multiple differentiation when a little elementary algebra does the job at least as easily?

Yours faithfully,

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To the Editor, *The Mathematical Gazette*

DEAR SIR,

I was sorry to see only a cursory treatment by your reviewer of Miss E. M. Renwick's *Children Learning Mathematics* in last October's issue of *The Gazette*, (No. 369). I appreciate the fact that reviewers have different tastes and interests, but to devote only some 11 lines to an important empirical study of learning seems to me to be out of keeping with a journal which exists to serve 'an Association of Teachers and Students of Elementary Mathematics'.

The book should prove much more than "of special interest to students in Training Colleges and beginners in the Teaching Profession": it should be read by everyone who thinks they know something about the teaching of mathematics. (The same goes for the author's *The Case Against Arithmetic*, published in 1935.)

Most publications concerned with mathematics in the classroom deal, whether they happen to admit it or not, with mathematics as it is *taught*. We still know very little about mathematics as it is *learned*, and I believe that, in its own small way, *Children Learning Mathematics* will prove to have been a pioneer study in the right direction. On the face of it, the book deals with the unimpressive. Its language is that of the classroom, it does not go in for mental elaborations, neither does it seem to come to any brilliant conclusions. But in this respect it is like any good book on philosophy—it raises more questions than it answers.

Miss Renwick's style shows her to be a person of unusual insight and compassion. Her writing is refreshingly free from jargon and it is probably for this reason that the simplicity of the book is deceptive. "Basic English," wrote Benjamin Whorf, "appeals to people because it seems simple." "But," he went on, "English is anything but simple—it is a bafflingly complex organization, abounding in covert classes, crypto types, taxemes of selection, taxemes of order." From what Miss Renwick reveals it is obvious that similar remarks could be made about Basic Mathematics and the context of our language which surrounds it.

Take for example the first study given in the book. It concerns a girl called Moira.

> 'This incident was described by Moira's nursery governess. The child's age is not given.
>
> The governess hands her a plate on which are three pieces of bread and butter, and says: "Count them."
>
> Moira points to them in turn, saying: "One, two, three."
>
> Governess: "Eat one."
>
> Moira eats one and the governess asks: "How many now?"
>
> Moira: "Three."
>
> Governess: "But you've eaten one!"
>
> Moira, pointing: "Yes, but two and three are left!" '

As is customary, there follows an explanation, and an analysis of the child's difficulties. (Or rather, an analysis which serves to highlight some of the inadequacies of our language-depending upon which way you care to look at it.) The author points out that in any collection of objects "the *name* of the last object counted is transferred to the collection and becomes its *number*." The confusion here, in the child's mind, is of course between ordinal number and cardinal number.

Yet what we constantly have to bear in mind is that there is nothing 'natural' about such a distinction between numbers—indeed, there is nothing 'natural' about numbers at all. The whole of mathematics is a symbolic structure, an intellectual device, which we choose to impose (by common agreement) on physical situations from outside. Knowledge of mathematics *in itself* is not sufficient to explain (a) how children learn mathematics, or (b) why some children find such learning easy and others do not. The answer to these crucial problems must be sought in realms outside the study of mathematics. These realms may be of little interest to the (pure) mathematician, neither, perhaps, much to his taste, but they can hardly be ignored by those who wish to teach the subject. One of these realms is that of psychopathology.

In connection with the above example, I should like to end by drawing attention to a section of an article written by a group of American anthropologists and psychiatrists, Gregory Bateson et al., and published in *Behavioural Science*, Vol. 1, in 1956. Its title, 'Toward a Theory of Schizophrenia,' is self explanatory. The following extract is offered without comment:

'The Base in Communications Theory.

> Our approach is based on that part of communications theory which Russell has called the Theory of Logical Types. The central thesis of this theory is that there is a discontinuity between a class and its members. The class cannot be a member of itself nor can one of the members *be* the class, since the term used for the class is of a *different level of abstraction*—a different Logical Type-from terms used for members. Although in formal logic there is an attempt to maintain this discontinuity between a class and its members, we argue that in the psychology of real communications this discontinuity is continually and inevitably breached, and that a priori we must expect a pathology to occur in the human organism when certain formal patterns of the breaching occur in the communication between mother and child. We shall argue that this pathology at its extreme will have symptoms whose formal characteristics would lead the pathology to be classified as a schizophrenia.'

Yours faithfully,
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To the Editor, *The Mathematical Gazette*

DEAR SIR,

The following statement occurs on page 52 of *Generalized Functions* by Gel'fand and Shilov (p. 81 of the Russian edition):

"The gamma function is defined by the integral

$$\Gamma(\lambda) = \int_0^\infty x^{\lambda-1}e^{-x}\,dx$$

which converges for Re $\lambda > -1$."

Could this be used as an argument for the adoption of the factorial function (as in the British Association Tables) instead of the gamma function? The function defined by

$$\lambda! = \int_0^\infty x^{\lambda}e^{-x}\,dx$$

does converge for Re $\lambda > -1$.

Yours faithfully,
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