EFFECTS OF PRECESSION UNCERTAINTIES ON PLANETARY EPHEMERIDES

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1. Introduction

Several methods of data analysis applied recently to precise astrometric observations give evidence for a correction of the luni-solar precession (ψ) of the order of $\Delta\psi = -3$ milliarcseconds per year (Williams et al., 1994). When studying the motion of bodies of the planetary system from photographic observations, an inertial stellar reference frame is required. Thus, any imperfection of the luni-solar precession would have repercussions on the determination of the orbit dynamics of the celestial body under investigation.

We calculate the change in the determination of the semimajor axis of orbits of planets and asteroids as a function of the luni-solar precession, and compare the estimated shift of the semimajor axis with its precision obtained from radar ranging measurements.

2. The Basic Equations

Relations between orbital elements and angular observations (λ, β) referring to the ecliptic coordinate system can be derived from the equation of elliptical motion. We employ the customary notations of the orbital elements $(\mathrm{a}, \mathrm{e}, \mathrm{i}, \Omega, \omega, \mathrm{M})$. Let $\mathbf{r}$ be a barycentric radius vector to a planet. Expressed by the orbital elements referring to the triad $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ defining the ecliptic coordinate system, three unit vectors are introduced:

$$\begin{aligned}
\mathbf{e}_1 &= \cos\Omega\,\mathbf{i} + \sin\Omega\,\mathbf{j} \\
\mathbf{e}_2 &= -(\sin\Omega\cos i)\,\mathbf{i} + (\cos\Omega\cos i)\,\mathbf{j} + \sin i\,\mathbf{k} \\
\mathbf{e}_3 &= (\sin\Omega\sin i)\,\mathbf{i} - (\cos\Omega\sin i)\,\mathbf{j} + \cos i\,\mathbf{k}\,.
\end{aligned}$$

S. Ferraz-Mello et al. (eds.), Dynamics, Ephemerides and Astrometry of the Solar System, 475–476.

The semimajor axis is expressed by (e.g. McCuskey, 1963)

$$a = \frac{r + B_1(\mathbf{r} \cdot \mathbf{e}_1) + B_2(\mathbf{r} \cdot \mathbf{e}_2)}{1 - e^2}\ , \tag{1}$$

where $B_1 = e\,cos\,\omega$ and $B_2 = e\,sin\,\omega$.

On forming the differential of eq. (1) with respect to the precession one gets for the variation of the semimajor axis

$$\Delta a = B_1/(1 - e^2)(\frac{\partial \mathbf{r}}{\partial \psi} \cdot \mathbf{e}_1)\Delta\psi + B_2/(1 - e^2)(\frac{\partial \mathbf{r}}{\partial \psi} \cdot \mathbf{e}_2)\Delta\psi \tag{2}$$

comprising the terms contributed by the precession.

Note that Δa depends on a through $\mathbf{r}$, and that the total error in the determination of the semimajor axis contains Δa as a systematic part.

3. Numerical Results and Conclusions

On applying eq. (2) to Jupiter ($e \approx 0.05$), Mars ($e \approx 0.10$), and to an asteroid ($e = 0.20$, $a = 4$ AU) we find $\Delta a = 4.0$ km, 2.3 km and 15.4 km, respectively, 10 years after the epoch of reference.

The ratio of the radar measurement to its uncertainty is about 10^8 for asteroids (Ostro et al., 1991) and slightly larger for planets. Accordingly, the precision of radar ranging to a planet at Jupiter distance lies between 1 to 10 km, and amounts to some 10 km for asteroids. Thus, in conclusion, we infer from the above estimates of Δa caused by the imperfections of the luni-solar precession, that estimation of the semimajor axis of a planetary orbit derived from angular measurements contains a systematic error approximately as large as the precision of planetary radar ranging. This fact deserves consideration when semimajor axes are compared after being determined from angular observations and radar ranging.

4. References

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