

COSMIC MAGNETIC FIELDS AND SUPERCONDUCTING STRINGS

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ABSTRACT. A cosmic magnetic field may play a significant role in the formation of galaxies and large scale structure. In particular, a fossil field of present strength $\sim 10^{-9}$ Gauss is an essential ingredient in the superconducting string model of galaxy formation (Ostriker, Thompson and Witten 1986 (OTW); Thompson 1988a). We discuss the mechanism by which a current is induced on a superconducting string, including recent work on the reconnection of magnetic field lines near the string (Kulsrud and Thompson 1989). A substantial amount of baryonic plasma is trapped on the magnetic field lines which close around the string. The current on a loop almost certainly does not undergo exponential dynamo amplification; an oscillating superconducting loop emits a relativistic MHD wind (Thompson 1988a). Decaying superconducting loops fill most of the intergalactic medium with a relativistic, magnetized fluid. In this model, the gas between galaxies is highly clumped and strongly magnetized, the field strength approaching $1 \mu\text{G}$. The maximum energy of cosmic ray protons accelerated at string-driven shocks is $\sim 10^{20}$ eV (Madau and Thompson 1989).

Kibble (1976) was the first to point out that strings of cosmic dimensions might have formed in the early universe. The objects usually called cosmic strings are vortex lines, formed when a gauge symmetry is broken to a smaller symmetry with a discrete subgroup. This variety of cosmic string *may* be superconducting, capable of carrying a current in excess of 10^{20} amperes if the mass density of the string is $G\mu/c^2 \sim 10^{-6}$ in Planck units (Witten 1985a). Another variety of cosmic string is the superstring itself. A macroscopic superstring is superconducting (Witten 1985b). Although macroscopic superstrings appear to be too heavy to exist in nature, the properties of a gauge string which give rise to superconductivity are very reminiscent of the superstring.

A magnetic field must be present if a superconducting string is to realize its current-carrying potential. An enormous current is induced on a string when it moves a cosmological distance through a tiny magnetic field (Witten 1985a)

$$\frac{dI}{dt} \simeq \frac{2\pi c}{\ln(R/\Delta)} B_{ez} V_{\perp}. \quad (1)$$

We assume the presence of a fossil cosmic magnetic field $B_{ez} \sim 10^{-9} (1+z)^2$ G.

This is within the range required if the Galactic magnetic field is a fossil field which has undergone compression and linear winding, but not exponential dynamo amplification (Kulsrud, these proceedings). In (1), V_{\perp} is the velocity of the string; the logarithm is approximately equal to 100 given that R is a cosmological distance, and that the thickness of the string is only $\Delta \sim 10^{-30}$ cm.

A cosmic string network consists of an infinite random walk (the "long string") and loops which split off this random walk (Vilenkin 1980). A current is induced on the long string as it moves through the magnetic field. The current induced on a loop of radius R_{ℓ} by the flux trapped in that loop is smaller* by the factor $\sim 10^{-1} R_{\ell}/ct$ (Thompson 1988a). The flux through a superconducting loop is conserved, so long as the current on the string is less than the maximum current that the string can carry; otherwise, charges are lost from the string, and the flux decreases.

Let us sketch how a superconducting string acquires its current from a magnetized plasma. Further details can be found in Thompson (1988a) and Kulsrud and Thompson (1989). As the string sweeps up flux, it also sweeps up plasma. When the magnetic field lines reconnect behind the string, plasma is trapped on them. Since the string has no end, the field lines cannot slip around it, and the plasma is locked in the string's magnetocylinder. In this respect, our treatment of the reconnection problem differs significantly from that of Chudnovsky *et al.* (1986), who implicitly treat the case where only a tiny fraction of the current on the string is induced by its motion through the plasma, and very little plasma is trapped.

The outer edge of the magnetocylinder should not be thought of as a rigid surface. The field lines, as they pile up behind the bow shock which separates the string from the ambient medium, move inward toward the string at the rate $dR/dt \simeq -\ln(R/\Delta) R/t$; here t is the age of the universe. Initially, the field lines remain open, oriented like the lines at the end of a running track. As the field lines are compressed, so is the plasma, which, to remain in rough pressure equilibrium with the field, flows out along the open lines. The ratio

$$\frac{\text{plasma rest energy density}}{\text{magnetic energy density}} \simeq \left(\frac{R}{R_{shock}} \right)^{1/2}, \quad (2)$$

decreases as the field lines move closer to the string. Eventually the plasma cools and the ratio (2) stabilizes. Closer still to the string the plasma density is large enough that the field lines can slip through it, and the current is transferred from the plasma to the string.

The plasma trapped near the string efficiently screens electric fields: the number of charges in the plasma greatly exceeds the number of charges on the string, by the ratio $\sim m_{GUT}/m_p$. One may show (Thompson 1988a) that the cutoff frequency of strong electromagnetic waves near a string loop of length L is larger than the oscillation frequency $2c/L$ of the loop by a factor $\sim 10^6$. An oscillating superconducting loop emits a relativistic MHD wind, and *not* ultra-low frequency electromagnetic waves as claimed by OTW. On general grounds, the MHD wind

* Mijic (1989) claims that the current on the long string is larger by many orders of magnitude than the current induced by the flux trapped in the loops. He apparently neglects the fact that loops which chop off the long string carry away charges. When this loss of charges is taken into account, one finds that the current on the long string is dominated not by the first decade of the expansion of the universe, but by the last.

from a rotating[†] magnetized body, and the vacuum electromagnetic waves from the same body in the absence of plasma, carry comparable energy, provided that the rest energy in the plasma is a small fraction of the energy in the wind (Goldreich and Julian 1969). The MHD wind may extract angular momentum from a loop more efficiently than gravitational or vacuum electromagnetic radiation.

The current on a superconducting loop is dominated by the DC current on the string. AC modes on the string are damped by resistive losses in the surrounding plasma (Thompson 1988a). The plasma itself may maintain AC current modes in the form of magnetosonic waves, but the amplitude of these waves is limited to the radius of the loop, and so $I_{\text{plasma}} \lesssim I_{\text{string}}$. It is very doubtful that the current on a superconducting loop undergoes exponential dynamo amplification, as may happen in vacuum (Spergel *et al.* 1989). Clearly, no conventional dynamo can operate, since the rest energy in the plasma is at most comparable to the energy in the magnetic field.

A superconducting loop does, nonetheless, act as a dynamo in the following sense. The MHD wind carries a wound up magnetic field, and in most solutions of the wind equations around a rotating body, the Poynting flux is a significant fraction of the total energy flux. The energy in the magnetic field disgorged by each superconducting loop greatly exceeds the energy of the magnetic field which induced the current on the loop.

It is useful to compare the electromagnetic output $\dot{E}_{\text{wind}} = \Gamma_{\text{wind}} I^2 / 4\pi c$ of a loop with its output of gravity waves $\dot{E}_g = \Gamma_g G \mu^2 c$. Expressed in terms of the present strength of the background magnetic field, the ratio is (Thompson 1988a)

$$f \equiv \frac{\dot{E}_{\text{wind}}}{\dot{E}_g} \simeq 10 \left(\frac{B_{\text{ex}}}{10^{-9} \text{ G}} \right)^2 \left(\frac{G\mu/c^2}{10^{-6}} \right)^{-2} \frac{\Gamma_{\text{wind}}}{\Gamma_g}. \quad (3)$$

Here Γ_{wind} and Γ_g are numerical factors which depend on the shape of the loop. The electromagnetic output $\dot{E}_{\text{wind}} = 2 \cdot 10^{49} f (G\mu c^{-2}/10^{-6})^2 (\Gamma_g/50) \text{ erg s}^{-1}$ is prodigious. If a loop does not form cusps, then in vacuum $\Gamma_{\text{em}} \simeq \Gamma_g$ (R. Y. Cen, private communication).

The wind emitted by a superconducting loop differs qualitatively from the wind emitted by a pulsar. First, the plasma is composed of protons and electrons, not electron-positron pairs. Second, the magnetic field in the vicinity of a loop, where the charges are accelerated, is much weaker than the field at the surface of a neutron star: $\sim 10^{-3}(1+z)^{3/2} \text{ G}$ as opposed to $\sim 10^{12} \text{ G}$. The respective particle Lorentz factors may be similar, however, if the particles carry a significant fraction of the energy in the wind: $\gamma_{\text{wind}} \sim 10^{6-7}$ (Thompson 1988a). See Kulsrud and Thompson (1989) for further details, and Thompson (1989) for estimates of synchrotron emission near a string.

A superconducting loop, if it carries a large current, causes a great disturbance to its surroundings. It drives a blast wave, whose basic structure is similar in some ways the structure of the Crab nebula, although on a much larger scale. An outer shock propagates at a constant velocity $V_S \sim (G\dot{E}_{\text{wind}}/\Omega_b)^{1/5}$ and peculiar velocity $\Delta V_S = \frac{1}{3}V_S$ in an Einstein-de Sitter universe with fractional baryon density Ω_b ; thus, the physical radius of the shock grows linearly with time, $R_S = V_S t$ (Ostriker

† A cosmic string, although not a rigid body, does carry finite angular momentum.

and Thompson 1987). The wind is shocked at a radius $R_S \cdot \sqrt{2V_S/c}$. The region between the inner and outer shocks is filled with a magnetized, relativistic plasma. The equipartition magnetic field strength in this region is quite large,

$$B_{eq} = 0.8 \mu G (1+z)^{3/2} \left(\frac{\Delta V_S}{2,500 \text{ km s}^{-1}} \right) \left(\frac{\Omega_b h_{50}^2}{0.1} \right)^{1/2}. \quad (4)$$

The electron density in this plasma is very low, of order 10^{-10} of the mean cosmic density of electrons. As a result, the relativistic protons suffer only adiabatic losses, and the Faraday rotation through the plasma is negligible. It should be emphasized that the magnetic field lines in this relativistic medium are *disconnected* from the primordial flux which induced the current on the string, and as a result almost no cold plasma is expected to mix with the relativistic plasma.

Superconducting strings heat the universe continually from a very early epoch, and string-driven blasts first fill the universe at $1+z \sim 10^3$ (Ostriker and Thompson 1987). The extraordinary depletion of generally distributed neutral hydrogen in the intergalactic medium (e.g. Steidel and Sargent 1987), as determined from the Gunn-Peterson (1965) effect, has a simple explanation in this model: most of the volume of the IGM is filled with a magnetic field exceeding 10^{-7} G in strength. The gas swept up by the blasts is concentrated in discrete sheets and lumps.

Large scale structure must form at $1+z \gtrsim 10$ in explosion models. The blasts are so large that the gas is shocked to temperatures near 10^8 degrees, and the only effective cooling mechanism is Compton cooling off the background radiation (Ostriker and Thompson 1987). A rule of thumb is that the present Hubble velocity of the last generation of shells which can Compton cool, namely those forming at $1+z \sim 10$, is very nearly the same as ΔV_S , and one requires $\Delta V_S \simeq 2,500 \text{ km s}^{-1}$ (Thompson *et al.* 1989). If we associate the low mass clouds seen in absorption in QSO spectra blueward of Lyman- α with cold pressure-confined fragments of the blast waves*, then we conclude that these Lyman- α clouds form at $1+z \gtrsim 10$ (Thompson 1988b). Some of the gas which cooled at $1+z \gtrsim 10$ is shocked at lower redshifts, forming a hot (non-relativistic) component of the IGM.

One immediate prediction of the superconducting string model, then, is that Lyman- α clouds should contain magnetic fields. As the shock-heated gas cools, the fossil magnetic field which threads it is compressed. The size and shape of the fragments depends on the complicated details of the Rayleigh-Taylor instability suffered by a cooling blast (Vishniac 1983; Bertschinger 1986; Thompson *et al.* 1989). One does know that as a pressure-confined cloud cools, it becomes flatter. In this case, the comoving volume of the clouds decreases by a factor $\sim 10^2$, and a final axis ratio $\sim 10^2$ is plausible. At $1+z \sim 3.5$, the pressure of the compressed fossil field dominates the pressure of the gas by a factor $\sim 30 (T_{gas}/3 \cdot 10^4 \text{ K})^{-1} (B_{ex,0}/10^{-9} \text{ G})$ which depends somewhat on the geometry.

The column density through one Jeans length of cold hydrogen in pressure equilibrium with the blast is

$$N_{Jeans} \simeq \sqrt{\frac{10}{\Omega_b}} \cdot \frac{1}{3} (n_H) R_S = 3 \cdot 10^{20} (1+z)^{3/2} \left(\frac{\Delta V_S}{2,500 \text{ km s}^{-1}} \right) \left(\frac{\Omega_b h_{50}^2}{0.1} \right)^{1/2} \text{ cm}^{-2}. \quad (5)$$

* As originally suggested by Ikeuchi and Ostriker (1986) in the context of explosions on smaller scales and at lower redshifts.

At a redshift $1 + z \sim 3.5$ this critical column density is $N_{Jeans} \simeq 2 \cdot 10^{21} \text{ cm}^{-2}$. The magnetic field inside the cloud is $\sim 3 \mu\text{G}$. These quantities are very similar to the column density and magnetic field strength in our galactic disk, and it is tempting to speculate that a cold fragment with a sheet-like geometry could pose as a galactic disk at a redshift $z \sim 2 - 3$. Recall that QSO absorption line systems of column density $10^{20-21} \text{ cm}^{-2}$ cover an area of the sky ~ 6 times larger than expected for spiral disks of constant size, in an Einstein-de Sitter universe (Wolfe 1988). Moreover, one infers the presence of a magnetic field stronger than $\sim 1 \mu\text{G}$ in these “damped” Lyman- α systems, and absorption line systems of lower column density, from the presence of a large amount of Faraday rotation along the line-of-sight to the quasar (Welter *et al.* 1985; Wolfe 1988). We should emphasize that it is not clear how to hide a large amount of high column density neutral hydrogen nearby. The column density of a flat pressure-confined cloud is at present smaller only by a factor ~ 10 than its column density at $1 + z \sim 3.5$.

We address one other observational consequence of giant string-driven blast waves. Shocks are sites for the acceleration of non-thermal particles, via the first-order Fermi process (Bell 1978; Blandford and Ostriker 1978). Shocks of the size considered here, containing a magnetic field approaching $1 \mu\text{G}$ in strength, may accelerate protons to energies in excess of 10^{20} eV . The maximum energy of the cosmic ray protons is found by equating the diffusion length of the cosmic rays with the radius of the shock (e.g. Blandford 1988),

$$E_{max} \simeq \frac{1}{3} e B_{eq} R_S \left(\frac{\Delta V_S}{c} \right). \quad (6)$$

Here, one assumes that the cosmic rays scatter off waves of strength $(\delta B/B)^2 \sim \frac{1}{10}$. If $\Delta V_S = 2,500 \text{ km s}^{-1}$, then at the present epoch $R_S \simeq 100 h_{50}^{-1} \text{ Mpc}$ and $B_{eq} \simeq 1 \mu\text{G}$; the maximum energy of the protons is $E_{max} = 2 \cdot 10^{20} \text{ eV}$. When one considers the acceleration of cosmic rays at intergalactic shocks (e.g. Hillas 1984), one often assumes that the magnetic field threads a significant amount of plasma, in which case the field must be weaker than $\sim 10^{-9} \text{ G}$, and only shocks as large as the horizon can accelerate protons to energies approaching $\sim 10^{20} \text{ eV}$.

The energy density in a blast is $\simeq 10^{-2} (1 + z)^3 \text{ eV cm}^{-3}$. If the cosmic rays contribute a fraction ϵ_{CR} of the pressure, and if the flux of 10^{20} eV cosmic rays equals the observed flux at the Earth, then the spectral index (defined by $dn_{CR}/dE \propto E^{-\Gamma}$) must be $\Gamma = 2.5 + 0.1 \log_{10}(\epsilon_{CR}/0.1)$ between $\sim 10^9 \text{ eV}$ and $\sim 10^{20} \text{ eV}$ (Thompson and Madau 1989). More detailed calculations of the spectrum of high energy cosmic ray protons accelerated by superconducting strings can be found in Madau and Thompson (1989).

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